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Generating and detecting short-duration pulses of squeezed light

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Mode-locked lasers provide a train of high-intensity light pulses that can be used to pump nonlinear media in order to produce squeezed light. The squeezed light produced would consist of a train of short-duration pulses. It is shown here that a homodyne detector using a pulsed local oscillator can be used to observe the squeezing even when the response time of the photodetector is much longer than the local oscillator or squeezed light pulse width.

I. INTRODUCTION

Squeezed states have been successfully generated[1-4] using optical materials which exhibit parametric gain (either parametric down conversion or four-wave mixing gain). Since the optical nonlinearities suitable for squeezed-state generation are generally weak, high finesse cavities or long-path-length media (such as optical fibers) were employed. High-intensity lasers to serve as pumps for the optical media could reduce the cavity storage time or interaction length, making the generation of squeezed light a less demanding technical task. In addition, linear losses, which limit the amount of squeezing that can be realized, are reduced by decreasing the effective optical path length.

High-intensity lasers generally emit light pulses whose pulse width is short compared to the characteristic response time of currently available photodetectors. Here a technique is described by which fast time-scale squeezing can be detected using slow photodetectors. A laser source emitting a periodic train of pulses (such as a mode-locked laser) is used to provide both the pump for the parametric gain medium and the local oscillator for the homodyne detector. The pulsed local oscillator in effect stroboscopically samples the pulsed squeezed light generated by the pump. By employing a periodic train of pulses with period T the output current from a homodyne detector will have intense spectral peaks at the frequencies $f_n = n/T$ where n is a positive integer. Between these sharp spectral peaks the power spectrum exhibits a shot noise floor. When squeezed light enters the signal port of the homodyne detector this noise floor will drop below the level due to the vacuum entering the signal port, provided the local oscillator phase is adjusted correctly and the optical path lengths are chosen so that the squeezed-light pulses and local-oscillator light pulses overlap when they arrive at the photodetector surface. An explicit expression for this noise reduction, involving the local oscillator and pump-pulse shape functions, will be given. As a technical point, balanced homodyne detection[5-7] can be used to greatly reduce (and in principle completely eliminate) the intense spectral peaks occurring at f_n.

The pulsed technique which can be used to overcome linear medium losses can be regarded as an AM analog of an FM technique introduced by Shelby *et al.*[2] to generate and detect squeezed light in which they spread the pump energy over many frequency components in order to suppress stimulated Brillouin oscillations. Schumaker[8] has also investigated the use of pump beams comprised of multiple frequency components for the generation of squeezed light and has explored various amplitude correlations that arise among the frequency components of the squeezed light generated in nonlinear media with such pumps.

Operator expressions for the squeezed light generated by a parametric gain medium pumped with a periodic train of light pulses (e.g., obtained by frequency doubling the mode-locked laser output) will now be obtained.

Assume that the signal and pump beams are collinear and phase matched, then (using the undepleted pump approximation) the signal field E_s propagating through a nonlinear medium with a second-order polarizability will be adequately described[9] by the wave equation

$$\frac{\partial^2 E_s}{\partial t^2} - v^2 \frac{\partial^2 E_s}{\partial x^2} = \kappa E_p E_s \ , \tag{1}$$

where E_p is the pump electric field, v is the light velocity in the medium, and κ characterizes the strength of the nonlinearity. The incoming intense pump field can be treated as a classical field of the form

$$E_p(x,t) = A(t - x/v)\sin[2\omega_0(t - x/v) + \phi] \ , \tag{2}$$

where $2\omega_0$ is the angular frequency of the optical carrier and ϕ is the phase of the optical cycle at $(t - x/v) = 0$. The carrier is modulated by the periodic envelope function $A(t - x/v)$ which consists of a train of equally spaced pulses in which each pulse has a temporal width of $\tau (1/\tau \ll \omega_0)$. The temporal periodicity (T) of successive pulses need not be commensurate with a multiple of the optical period. The signal field is split into positive- and negative-frequency components and expressed in the form

$$E_s(x,t) = E_s^{(+)}(x,t) e^{-i\omega_0(t - x/v)} + E_s^{(-)}(x,t) e^{i\omega_0(t - x/v)} \ , \tag{3}$$

where $E_s^{(-)}$ is the Hermitian conjugate of $E_s^{(+)}$. The goal of the exercise here is to determine the response of a homodyne detector to the squeezed light. The temporal range over which the variations in $E_s^{(-)}$ and $E_s^{(+)}$ can be detected is determined by the shortest of the three characteristic response times given by τ, $\tau_{\rm LO}$ (local oscillator pulse width), and $1/B$ (inverse bandwidth of the homodyne detector electronics). All characteristic times are assumed to be long compared to an optical period, therefore only frequency components encompassing the greatest value of $1/\tau$, $1/\tau_{LO}$, or B are kept in the Fourier expansion of the electromagnetic field $E_s^{(+)}$. Hence, neglecting weak frequency dependences,[10-13] the positive frequency components $E_s^{(+)}$, of the electromagnetic field incident on the nonlinear medium ($x=0$) can be expressed by

$$E_s^{(+)}(0,t)=\varepsilon_0\int_\Omega d\omega\, a(\omega)e^{-i\omega t} ,$$

where ε_0 converts the right-hand side to the electric field units and $a(\omega)$ denotes the annihilation operator for a photon of frequency $\omega_0+\omega$ and satisfies the usual boson commutation relations:

$$[a(\omega),a^\dagger(\omega')]=\delta(\omega-\omega') , \quad [a(\omega),a(\omega')]=0 . \tag{5}$$

The integration is carried out over the frequency interval $-\Omega<\omega<\Omega$, where $2\omega_0\gg\Omega$ but $\Omega/2\pi$ is somewhat greater than the greatest of $1/\tau$, $1/\tau_{LO}$, or B.

Substituting Eqs. (2) and (3) into Eq. (1) and making the slowly varying amplitude approximation which neglects second-order derivatives in $E^{(+)}(x,t)$ and $E^{(-)}(x,t)$, one obtains the expression

$$\left[\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right]E_s^{(+)}(x,t) = -\frac{\kappa A(t-x/v)e^{-i\phi}}{4\omega_0}E_s^{(-)}(x,t) , \tag{6}$$

where perfect phase matching has been assumed.

Transforming to the coordinates

$$\xi=t+x/v , \quad \eta=t-x/v . \tag{7}$$

Equation (6) can be further simplified to

$$\frac{\partial}{\partial\xi}E_s^{(+)}(\xi,\eta)=-K(\eta)e^{-i\phi}E_s^{(-)}(\xi,\eta) , \tag{8}$$

where $K(\eta)=\kappa A(\eta)/8\omega_0$. Between this equation and its Hermitian conjugate, $E_s^{(-)}$ can be eliminated to yield the equation

$$\frac{\partial^2}{\partial\xi^2}E_s^{(+)}(\xi,\eta)=K^2(\eta)E_s^{(+)}(\xi,\eta) , \tag{9}$$

which has the general solution

$$E_s^{(+)}(\xi,\eta)=C_1(\eta)e^{K(\eta)\xi}+C_2(\eta)e^{-K(\eta)\xi} . \tag{10}$$

The C_1 and C_2, in terms of the initial field operators $E_s^{(+)}(0,t)$ and $E_s^{(-)}(0,t)$, are

$$C_1(t)=\frac{e^{-K(t)t}}{2}[E_s^{(+)}(0,t)-e^{-i\phi}E_s^{(-)}(0,t)] ,$$
$$C_2(t)=\frac{e^{K(t)t}}{2}[E_s^{(+)}(0,t)+e^{-i\phi}E_s^{(-)}(0,t)] . \tag{11}$$

Having obtained the functional form of $C_1(t)$ and $C_2(t)$, Eqs. (10) and (11) give $E^{(+)}(x,t)$ and $E^{(-)}(x,t)$ inside the nonlinear medium $0\le x\le L$,

$$E_s^{(+)}(x,t)=\cosh\left[\frac{2K(t-x/v)x}{v}\right]E_s^{(+)}(0,t-x/v)-e^{-i\phi}\sinh\left[\frac{2K(t-x/v)x}{v}\right]E_s^{(-)}(0,t-x/v) . \tag{12}$$

Equation (12) relates the electromagnetic field of the light emerging from the nonlinear medium at $x=L$ and time t to the field entering the medium at $x=0$ at the earlier time $t-L/v$. Equation (12) is a generalization of the canonical transformation[14]

$$b=\mu a+\nu a^\dagger , \tag{13}$$

which converts coherent light into squeezed light. After emerging from the nonlinear medium the light beam will be governed by the wave equation

$$\frac{\partial^2 E}{\partial t^2}-c^2\frac{\partial^2 E}{\partial x^2}=0 \tag{14}$$

and propagates to the homodyne detector ($x=l$). The time t_d it takes for light to propagate from $x=0$ to $x=l$ is given by $t_d=(l-L)/c+L/v$ and the phase ψ accumulated by the optical carrier is given by $\psi=\omega_0 t_d$. The electric field of the light arriving at the homodyne detector is then given by

$$E_s(l,t)=\{\cosh[2K(t-t_d)L/v]E_s^{(+)}(0,t-t_d)-e^{-i\phi}\sinh[2K(t-t_d)L/v]E_s^{(-)}(0,t-t_d)e^{i\psi}e^{-\omega_0 t}+\text{H.c.}\} . \tag{15}$$

The response of a homodyne detector to light emitted by the parametric gain medium will now be evaluated. The analysis will be carried out for a balanced homodyne detector since this detector[5-7] has the advantage of being blind to intensity fluctuations of the pump or signal. It responds only to the interference between the signal and the local oscillator. To a good approximation,[13] the output current delivered by a balanced homodyne detector is given by

$$I(t)=\int_{-\infty}^{t}d\tau H(t-\tau)I_p(\tau) . \tag{16}$$

$I_p(t)$ is proportional to the instantaneous current and is given by

$$I_p(t)=E_{LO}^{(-)}(t)E_s^{(+)}(t)+E_s^{(-)}(t)E_{LO}^{(+)}(t) , \tag{17}$$

and $H(t)$ is the response function of the detector electronics.

The intense coherent local oscillator light can be treated classically and has the form

$$E_{LO}(t)=F(t)\cos(\omega_0 t+\theta) , \tag{18}$$

where θ is the local oscillator phase and $F(t)$ is the periodic (T) amplitude function. Assuming $\tau_{LO} \gg 1/\omega_0$, one has, to a good approximation,

$$E_{LO}^{(+)}(t)=\frac{e^{-i\theta}}{2}F(t) \ . \tag{19}$$

From Eq. (15) one has

$$E_s^{(+)}(t)=\{\cosh[2K(t-t_d)L/v]E_s^{(+)}(0,t-t_d)-e^{-i\phi}\sinh[2K(t-t_d)L/v]E_s^{(-)}(0,t-t_d)\}e^{i\psi} \ . \tag{20}$$

Substituting Eqs. (19) and (20) into Eq. (17), $I_p(t)$ becomes

$$I_p(t)=\frac{e^{i(\theta+\psi)}}{2}[g(t)E_s^{(+)}(0,t-t_d)-e^{-i\phi}h(t)E_s^{(-)}(0,t-t_d)]+\text{H.c.} \ , \tag{21}$$

where $g(t)$ and $h(t)$ are given by

$$g(t)=F(t)\cosh[2K(t-t_d)L/v] \ ,$$
$$h(t)=F(t)\sinh[2K(t-t_d)L/v] \ . \tag{22}$$

The functions $g(t)$ and $h(t)$ have periodicity T and will be expanded in a Fourier series to determine the noise power spectrum of $I_p(t)$. Therefore,

$$g(t)=\sum_{n=-\infty}^{\infty} g_n e^{-in\omega_s t} \ ,$$
$$h(t)=\sum_{n=-\infty}^{\infty} h_n e^{-in\omega_s t} \ , \tag{23}$$

where $\omega_s=2\pi/T$. Furthermore, $g(t)$ and $h(t)$ are real, yielding the relations

$$g_{-n}=g_n^*, \ h_{-n}=h_n^* \ . \tag{24}$$

From Eq. (4), $E_s^{(+)}(0,t-t_d)$ can be written in the form

$$E_s^{(+)}(0,t-t_d)=\varepsilon_0\int_\Omega d\omega\, b(\omega)e^{-i\omega t} \ , \tag{25}$$

where $b(\omega)=a(\omega)e^{i\omega t_d}$. The Fourier transform of $I_p(t)$,

$$\hat{I}_p(\omega)=1/\sqrt{2\pi}\int_{-\infty}^{\infty}dt\, e^{+i\omega t}I_p(t) \ , \tag{26}$$

is then given by

$$\hat{I}_p(\omega)=\left(\frac{\pi}{2}\right)^{1/2}\varepsilon_0\sum_{n=-\infty}^{\infty}[(e^{i(\theta+\psi)}g_n-e^{-i(\theta+\psi-\phi)}h_n)b(\omega-n\omega_s)+(e^{-i(\theta+\psi)}g_n-e^{i(\theta+\psi-\phi)}h_n)b^+(n\omega_s-\omega)] \ . \tag{27}$$

For simplicity one assumes that $H(\omega)$ [the Fourier transform of $H(\tau)$] has the value h_0 out to frequency $\Delta\omega$ and then rapidly drops to zero so that $I(t)$ can be approximated by

$$I(t)=\frac{h_0}{\sqrt{2\pi}}\int_{-\Delta\omega}^{\Delta\omega}d\omega\,\hat{I}_p(\omega)e^{-i\omega t} \ . \tag{28}$$

The moments of $I(t)$ can now be evaluated for a given state of incoming light. Consider the case in which the incident light is in the vacuum state $|0\rangle$. From Eqs. (28) and (27) it is evident that the expectation value of $I(t)$ is zero $\langle 0|I(t)|0\rangle=0$. The first moment of $I(t)$ is, keeping Eq. (24) in mind,

$$\langle 0|I^2(t)|0\rangle=\left(\frac{\varepsilon_0 h_0}{2}\right)^2\int_{-\Delta\omega}^{\Delta\omega}d\omega\int_{-\Delta\omega}^{\Delta\omega}d\omega'\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}(e^{i(\theta+\psi)}g_n-e^{-i(\theta+\psi-\phi)}h_n)(e^{-i(\theta+\psi)}g_m^*-e^{i(\theta+\psi-\phi)}h_m^*)$$
$$\times e^{-i(n-m)\omega_s t}\delta(\omega-\omega'-(n-m)\omega_s) \ . \tag{29}$$

The time average of this quantity can be written more compactly as

$$\langle\langle I^2(t)\rangle\rangle=2\left(\frac{\varepsilon_0 h_0}{2}\right)^2\sum_{n=-\infty}^{\infty}|e^{i(\theta+\psi)}g_n-e^{-i(\theta+\psi-\phi)}h_n|^2\Delta\omega \ . \tag{30}$$

Using the experimental variability of the electronic bandwidth $(\Delta\omega)$ of the homodyne detector, and realizing that the average power is equal to the integral of the power spectrum, one concludes that the power spectrum $S(\omega)$ is given by

$$S(\omega)=2\left(\frac{\varepsilon_0 h_0}{2}\right)^2\sum_{n=-\infty}^{\infty}|e^{i(\theta+\psi)}g_n-e^{-i(\theta+\psi-\phi)}h_n|^2 \ . \tag{31}$$

It is useful to express this in the form

$$S(\omega)=2\left(\frac{\varepsilon_0 h_0}{2}\right)^2\sum_{n=-\infty}^{\infty}[|g_n|^2+|h_n|^2-2\cos(2\theta+2\psi-\phi)g_n h_n^*] \ . \tag{32}$$

From Eq. (23) one can readily show that

$$\langle g(t)h(t)\rangle=\sum_{n=-\infty}^{\infty}g_n h_n^* \ , \tag{33}$$

where the $\langle\rangle$ denote time average. Hence the power spectrum can be written in the form

$$S(\omega)=2\left[\frac{\varepsilon_0 h_0}{2}\right]^2[\langle g^2(t)\rangle+\langle h^2(t)\rangle-2\cos(2\theta+2\psi-\phi)\langle g(t)h(t)\rangle] \ . \tag{34}$$

Since $g(t)$ and $h(t)$ are real functions of t, the power spectrum is maximized or minimized when $\cos(2\theta+2\psi-\phi)=\pm 1$. This condition can be met by adjusting any one of the three phases θ,ψ,ϕ which could be accomplished by using a phase shifter in the local oscillator, signal, or pump beam, respectively. Without loss of generality $\langle g(t)h(t)\rangle$ can be chosen to be positive, then the maximum and minimum values of $S(\omega)$ are, using Eq. (22),

$$S(\omega)_{\substack{\max\\ \min}}=2\left[\frac{\varepsilon_0 h}{2}\right]^2\langle F(t)^2\exp[\pm 4K(t-t_d)L/v]\rangle \ . \tag{35}$$

From Eq. (37) the noise power spectrum normalized to the vacuum noise power spectrum is

$$S_n(\omega)_{\substack{\max\\ \min}}=\frac{\int_{-T/2}^{T/2}F^2(t)\exp[\pm 4K(t-t_d)L/v]dt}{\int_{-T/2}^{T/2}F^2(t)dt} \ . \tag{36}$$

Equation (36) constitutes the major result of this paper. The maximum increase or decrease in the noise power spectrum is directly related to the homodyne detector pulse shape $F(t)$ and pump pulse shape $K(t)$. To maximize the squeezing observed, it is evident that the pulse shape $F(t)$ of the local oscillator light should be chosen so that the function $\exp[4K(t-t_d)L/v]$ is sampled only near its maximum, that is, the local oscillator pulse width τ_{LO} should be of the same order as the pump pulse width τ, or shorter. The analysis assumed an ideal balanced homodyne detector. For a real detector, imperfect balancing will give rise to spectral peaks at frequency $f_n=n/T$. Between these peaks the power spectrum is still given by Eq. (36). That $S_n(\omega)$ of Eq. (36) is independent of frequency is a consequence of the assumption of ideal phase matching over the frequency interval $|\omega|<\Omega$ of Eq. (4). The effect of phase mismatches on $S_n(\omega)$ will be described elsewhere.[15]

An explicit expression, Eq. (36), has been obtained for the degree of noise reduction observed in the power spectrum of a homodyne detector's output when a periodically pulsed local oscillator is used to observe the squeezing in periodically pulsed squeezed light. To see a large effect the local oscillator and squeezed light pulses should have comparable pulse widths. It may thus be necessary to employ pulse compression techniques[16] to the local oscillator light before it reaches the homodyne detector. Alternatively, a cavity could be used to lengthen the pump pulse.

More generally, the technique described here could be used to stroboscopically sample portions of the squeezed light where the degree of squeezing and the amplitude component which is squeezed varies along the pulse envelope. For example, the degree of squeezing at various positions along a soliton pulse propagating through an optical fiber[17-19] could be explored in this manner.

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