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Finite-Q cavity electrodynamics: Dynamical and statistical aspects

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Solutions of the basic equations of a simplified model of the cavity quantum electrodynamics are presented under the condition that the single-photon Rabi frequency is much larger than the cavity decay rate. Such solutions are used to calculate the quantum-statistical properties of the field and atomic observables under a variety of initial conditions involving the states of the field and atom. Effects of increasing cavity damping and of the addition of thermal photons on collapse and revival phenomena are discussed. Phase-sensitive aspects of the cavity field are also treated. Quantum effects are shown to be most easily isolated by sending an atom initially prepared in a dressed state for such a state does not evolve further under the influence of a classical field. The appearance of squeezing in the cavity field is demonstrated. The squeezing is most prominent for a coherently prepared atom passing through an empty cavity. The quantum features of the complex dipole moment and their detectability are also discussed in detail.

I. INTRODUCTION

The interaction of an atom with the field in an infinitely high-Q cavity leads to several quantum-mechanical effects such as vacuum field Rabi oscillations,[1] collapse and revival[2–5] in the atomic excitation as a function of time. Recent experiments[6–9] have established the existence of such remarkable quantum effects. Other very interesting effects in cavity quantum electrodynamics arise from the competition of coherent and incoherent (which results from the leakage of photons from the cavity) effects. For example, the decay of the atom can be significantly affected by the cavity.[10] The quantum statistics[4] of the photons in the cavity is also crucial for the overall behavior of the atom. It is clear that a complete theory should (a) quantize the cavity field, (b) take into account the quantum statistics of the field, (c) account for the cavity losses. Many aspects of the cavity electrodynamics have been treated in the literature. Exact solutions to the models[11] of cavity-field—atom interaction have been reported. The effect of the cavity damping on the predictions of such models has been disussed in special cases.[12,13] In a recent communication[14] we reported the effect of cavity damping on the collapse and revival phenomena. Analogous results were also reported by Barnett and Knight.[15] In this paper we give detailed results for the various dynamical quantities in cavity electrodynamics. We fully account for quantum statistical and cavity leakage effects.

The organization of this paper is as follows. In Sec. II we present the model equations and their solutions assuming that the cavity damping is much smaller than the single-photon Rabi frequency. In Sec. III we discuss the effect of the cavity damping and field statistics on the collapse and revival phenomena. In Sec. IV we discuss the time-dependent quantum statistics of the cavity field. We also examine the phase-dependent properties of the cavity field. We treat in Sec. V the case when the atom is prepared initially in a coherent state, say, by irradiation by a microwave field. In particular, we discuss the evolution of the classical dressed states. The further evolution of the dressed states is due to the quantized nature of the electromagnetic field in the cavity. In Sec. VI we calculate the complex dipole moment. The real part of the dipole moment shows collapse and revival phenomena on a much larger time scale. The detectability of the collapse and revivals in complex dipole moments is also discussed. Various Appendixes give the supplementary results and some details of the calculations. We also discuss very interesting squeezing properties of the cavity field when an atom prepared coherently passes through an empty cavity.

II. MODEL AND SOLUTION FOR DENSITY MATRIX ELEMENTS IN THE HIGH-Q LIMIT

The master equation for the density matrix ρ of the Jaynes-Cummings model consisting of a two-level atom of transition frequency ω interacting resonantly with a single mode of the radiation field in a cavity is given by

$$\frac{\partial\rho}{\partial t}=-i/\hbar[H,\rho]-\kappa(a^\dagger a\rho-2a\rho a^\dagger+\rho a^\dagger a)\ , \qquad (2.1)$$

where H, the Jaynes-Cummings Hamiltonian in the rotating-wave approximation, is

$$H=\hbar\omega(S^z+a^\dagger a)+\hbar g(S^+a+S^-a^\dagger)\ . \qquad (2.2)$$

Here g denotes coupling between the atom (which is characterized by the spin-$\frac{1}{2}$ angular momentum operators $S^\pm,S^z$) and the field (described by the creation and annihilation operators $a^\dagger$ and a). The last term in Eq. (2.1) arises due to finite Q ($\equiv\omega/2\kappa$) of the cavity. Thus 2κ represents the rate of loss of photons from the cavity. We further assume that the cavity frequency and temperature are such that the mean number of thermal photons is much smaller than unity.

To solve Eq. (2.1) we work in the dressed-states representation, i.e, the representation consisting of the complete set of eigenstates of H which are known to be given by

$$
\begin{aligned}
&H\,|0,-\tfrac{1}{2}\rangle=-\hbar\omega/2\,|0,-\tfrac{1}{2}\rangle\,,\\
&H\,|\Psi_n^{\pm}\rangle=\lambda_n^{\pm}\,|\Psi_n^{\pm}\rangle\,,\\
&|\Psi_n^{\pm}\rangle=(|n,\tfrac{1}{2}\rangle\pm|n+1,-\tfrac{1}{2}\rangle)/\sqrt{2},\quad n=0,1,2,\ldots,\infty\\
&\lambda_n^{\pm}=\hbar[\omega(n+\tfrac{1}{2})\pm g\sqrt{n+1}]\,.
\end{aligned}
\tag{2.3}
$$

Here $|n,\pm\frac{1}{2}\rangle$ refers to a state with n photons in the cavity-field mode and the atom in the excited $|+\frac{1}{2}\rangle$ or in the ground $|-\frac{1}{2}\rangle$ state. Note that H causes transitions only between the eigenstates of $N=a^{\dagger}a+S^{z}$ having the same eigenvalue. Each eigenvalue of N is twofold degenerate since the eigenvalue of S^z can only be $\pm\frac{1}{2}$. Therefore, in the absence of damping, Eq. (2.1) becomes a 2×2 matrix equation which can be easily solved. However, the relaxation of the cavity causes a decrease in the number of photons without a corresponding increase in the atomic population. Consequently, an initial state with an eigenvalue $N-\frac{1}{2}$ of N can be connected with any of the states with eigenvalue $N-\frac{3}{2},N-\frac{5}{2},\ldots,-\frac{1}{2}$. Since an eigenvalue $N-(2m+1)/2$ $(m=1,2,\ldots,N)$ is twofold degenerate it follows that in the case of cavity damping, Eq. (2.1) reduces to a $(2N+1)\times(2N+1)$ matrix equation. For $N=1$ we have reported[13] an exact solution of Eq. (2.1). Evidently, for increasing N, the task of solving Eq. (2.1) becomes increasingly formidable and one needs to take recourse to approximate methods for its solution. The analytic methods for obtaining approximate solutions of Eq. (2.1) distinguish between two limiting cases of a low-Q ($\kappa \gg g$) and a high-Q ($\kappa \ll g$) cavity. In the case of a low-Q cavity, one can obtain an equation for the reduced atomic density matrix by tracing out the fast decaying field modes.[9,16] In the case of a high-Q cavity, Haroche[9] has used the dressed-state representation to derive the equations for the density matrix. We present analytic expressions for the elements of the density matrix in a high-Q cavity which is valid for any initial state of the field. A preliminary account of our work has previously been given.[14]

To derive the equation in the high-Q limit we first write the field operators appearing in Eq. (2.1) in terms of the dressed states. Note that on the basis of the $|n,\pm\frac{1}{2}\rangle$ states we have

$$
a=\sum_{n=0}^{\infty}\sqrt{n}\,(|n-1,\tfrac{1}{2}\rangle\langle n,\tfrac{1}{2}|+|n-1,-\tfrac{1}{2}\rangle\langle n,-\tfrac{1}{2}|)\,,
\tag{2.4}
$$

which upon using Eqs. (2.3) gives

$$
\begin{aligned}
a=&(|0,-\tfrac{1}{2}\rangle\langle\Psi_0^{+}|-|0,-\tfrac{1}{2}\rangle\langle\Psi_0^{-}|)/\sqrt{2}\\
&+\tfrac{1}{2}\sum_{n=1}^{\infty}[(\sqrt{n+1}+\sqrt{n})(|\Psi_{n-1}^{+}\rangle\langle\Psi_n^{+}|+|\Psi_{n-1}^{-}\rangle\langle\Psi_n^{-}|)-(\sqrt{n+1}-\sqrt{n})(|\Psi_{n-1}^{+}\rangle\langle\Psi_n^{-}|+|\Psi_{n-1}^{-}\rangle\langle\Psi_n^{+}|)]\,.
\end{aligned}
\tag{2.5}
$$

The number operator can be similarly expressed as

$$
a^{\dagger}a=\tfrac{1}{2}\sum_{n=0}^{\infty}[(2n+1)(|\Psi_n^{+}\rangle\langle\Psi_n^{+}|+|\Psi_n^{-}\rangle\langle\Psi_n^{-}|)-(|\Psi_n^{+}\rangle\langle\Psi_n^{-}|+|\Psi_n^{-}\rangle\langle\Psi_n^{+}|)]\,.
\tag{2.6}
$$

Next, we go to the interaction picture by defining

$$
W(t)=\exp(iHt)\rho(t)\exp(-iHt)\,,
\tag{2.7}
$$

and obtain an equation for $W(t)$ by substituting Eqs. (2.5) and (2.6) in Eq. (2.1). The exact equation for $W(t)$ is found to be a sum of time-independent and time-dependent terms. The time-dependent terms in this equation oscillate at the frequency proportional to the atom-field coupling g. It can be shown that the contribution of the oscillatory terms is of the order of κ^2/g^2. Hence, if we make the secular approximation, i.e., if we neglect the oscillatory terms by assuming $\kappa \ll g$ then the equation for $W(t)$ is found to be

$$
\begin{aligned}
\dot{W}(t)=&\kappa[\,|0,-\tfrac{1}{2}\rangle\langle\Psi_0^{+}|W(t)|\Psi_0^{+}\rangle\langle 0,-\tfrac{1}{2}|+|0,-\tfrac{1}{2}\rangle\langle\Psi_0^{-}|W(t)|\Psi_0^{-}\rangle\langle 0,-\tfrac{1}{2}|\,]\\
&+\kappa/2\sum_{n=1}^{\infty}\{(\sqrt{n+1}+\sqrt{n})^2[\,|\Psi_{n-1}^{+}\rangle\langle\Psi_n^{+}|W(t)|\Psi_n^{+}\rangle\langle\Psi_{n-1}^{+}|+|\Psi_{n-1}^{-}\rangle\langle\Psi_n^{-}|W(t)|\Psi_n^{-}\rangle\langle\Psi_{n-1}^{-}|\,]\\
&\qquad+(\sqrt{n+1}-\sqrt{n})^2[\,|\Psi_{n-1}^{+}\rangle\langle\Psi_n^{-}|W(t)|\Psi_n^{-}\rangle\langle\Psi_{n-1}^{+}|+|\Psi_{n-1}^{-}\rangle\langle\Psi_n^{+}|W(t)|\Psi_n^{+}\rangle\langle\Psi_{n-1}^{-}|\,]\}\\
&-\kappa/2\sum_{n=0}^{\infty}(2n+1)[(|\Psi_n^{+}\rangle\langle\Psi_n^{+}|+|\Psi_n^{-}\rangle\langle\Psi_n^{-}|)W(t)+\text{H.c.}]\,.
\end{aligned}
\tag{2.8}
$$

We now solve Eq. (2.8) to find various matrix elements of $W(t)$. From Eq. (2.8) it easily follows that

$$\langle \Psi_m^\epsilon | W(t) | \Psi_n^\epsilon \rangle = \exp[-\kappa t(m+n+1)]\langle \Psi_m^\epsilon | W(0) | \Psi_n^\epsilon \rangle \quad (m \neq n)\ (\epsilon = +,-)$$

$$\langle \Psi_m^+ | W(t) | \Psi_n^- \rangle = \exp[-\kappa t(m+n+1)]\langle \Psi_m^+ | W(0) | \Psi_n^- \rangle \quad (\forall m,n) \qquad (2.9)$$

$$\langle 0,-\tfrac{1}{2} | W(t) | \Psi_m^\pm \rangle = \exp[-2\kappa t(m+\tfrac{1}{2})]\langle 0,-\tfrac{1}{2} | W(0) | \Psi_m^\pm \rangle .$$

Equation (2.9) along with its Hermitian conjugates determines all the off-diagonal elements of $W(t)$. These off-diagonal elements describe the evolution of the coherences between different dressed states. It is clear that any initial coherence between the dressed-atom states is eventually destroyed.

For the population of the dressed states, Eq. (2.8) yields

$$\langle \Psi_n^\epsilon | \dot{W}(t) | \Psi_n^\epsilon \rangle = 2\kappa[\Gamma_{n+1}^+ \langle \Psi_{n+1}^\epsilon | W(t) | \Psi_{n+1}^\epsilon \rangle + \Gamma_{n+1}^- \langle \Psi_{n+1}^{-\epsilon} | W(t) | \Psi_{n+1}^{-\epsilon} \rangle - (\Gamma_n^+ + \Gamma_n^-)\langle \Psi_n^\epsilon | W(t) | \Psi_n^\epsilon \rangle] , \quad (\epsilon = +,-) \qquad (2.10)$$

where the Γ's are defined as

$$\Gamma_n^\pm = (\sqrt{n+1} \pm \sqrt{n})^2/4 . \qquad (2.11)$$

It turns out to be convenient to work with the following equation which is derived from Eq. (2.10):

$$\dot{F}_n \equiv \langle \Psi_n^+ | \dot{W}(t) | \Psi_n^+ \rangle + \langle \Psi_n^- | \dot{W}(t) | \Psi_n^- \rangle = 2\kappa[-(n+\tfrac{1}{2})F_n + (n+\tfrac{3}{2})F_{n+1}] . \qquad (2.12)$$

To solve Eq. (2.12) we write it as

$$F_n(t) = \exp[-2\kappa(n+\tfrac{1}{2})t]F_n(0) + 2\kappa(n+\tfrac{3}{2}) \times \int_0^t \exp[-2\kappa(n+\tfrac{1}{2})(t-\tau)]F_{n+1}(\tau)d\tau , \qquad (2.13)$$

and assume that there is an upper limit on the number of photons initially present in the system so that $\langle \Psi_{N+1}^\pm | W(0) | \Psi_{N+1}^\pm \rangle = 0$. This, in turn, implies that $\langle \Psi_{N+1}^\pm | W(t) | \Psi_{N+1}^\pm \rangle = 0$ for all t since the cavity cannot add to the number of photons; it can only absorb them. We then start with $n = N$ and iterate Eq. (2.12) for successively smaller values of n to obtain

$$F_n(t) = \exp[-2\kappa(n+\tfrac{1}{2})t] \times \sum_{j=n}^{N} \frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{(j-n)!(n+\frac{1}{2})!} F_j(0) . \qquad (2.14)$$

If there is no upper limit on the initial number of photons in the system we take the limit $N \to \infty$ in Eq. (2.14).

We can likewise obtain an equation for $G_n(t) = \langle \Psi_n^+ | W(t) | \Psi_n^+ \rangle - \langle \Psi_n^- | W(t) | \Psi_n^- \rangle$. However, in what follows $G_n(t)$ is not needed.

Lastly, for the population of the ground state, the solution of Eq. (2.8) is found to be

$$\langle 0,-\tfrac{1}{2} | W(t) | 0,-\tfrac{1}{2} \rangle = \kappa \int_0^t F_0(\tau)d\tau + \langle 0,-\tfrac{1}{2} | W(0) | 0,-\tfrac{1}{2} \rangle \equiv \langle 0,-\tfrac{1}{2} | \rho(t) | 0,-\tfrac{1}{2} \rangle . \qquad (2.15)$$

The density matrix is thus completely determined for an arbitrary initial state of the system. To specify the initial state, we assume that at $t = 0$ the field and the atom are decoupled so that $\rho(0) = \rho_a(0) \otimes \rho_f(0)$.

For an ideal cavity ($\kappa = 0$) an extensive study of the effects of the field statistics on the dynamics of the system has been made by considering the atom initially in an excited or ground state and the field in a coherent or a chaotic state.[1-5] It is found that a superposition of noncommensurate Rabi frequencies arising due to a distribution in the number of photons, leads to the phenomenon of collapse and revival Rabi oscillations. In the presence of a coherent field the revivals occur at regular intervals but this regularity seems to be missing in the case of a chaotic field.

In this work we investigate the effect of finite but large cavity Q on the collapse and revival phenomena. We assume that the atom is initially in an excited state, i.e.,

$$\rho_a(0) = |\tfrac{1}{2}\rangle\langle\tfrac{1}{2}| . \qquad (2.16)$$

The initial state of the field is taken to be

$$\rho_f(0) = :\exp[-(a^\dagger - z^*)(a-z)/(\bar{n}+1)]:/(\bar{n}+1) , \qquad (2.17)$$

where : : denotes normal ordering of the operators $a^\dagger$ and a. In the number state representation we have

$$\rho_f(0) = \sum_{m,n=0}^{\infty} p_{mn} | m \rangle\langle n | , \qquad (2.18)$$

where[17]

$$p_{mn} = \sqrt{n!/m!}\{\bar{n}^{\,n}\alpha^{m-n}\exp[-|\alpha|^2/(\bar{n}+1)]L_n^{m-n} \times [-|\alpha|^2/\bar{n}(\bar{n}+1)]\}/(\bar{n}+1)^{m+1} , \qquad (2.19)$$

for $m \geq n$ and $p_{mn} = p_{nm}^*$ for $m \leq n$. Here $L_n(x)$ are the associated Laguerre polynomials:

$$L_n^\nu(x) = \sum_{m=0}^{n} \frac{(-1)^m(n+\nu)!}{(n-m)!m!(m+\nu)!} x^m . \qquad (2.20)$$

The coherent state $|\alpha\rangle$ of the field corresponds to the case $\bar{n} = 0$ when Eq. (2.19) reduces to

$$p_{mn} = \alpha^m \alpha^{*n} \exp(-|\alpha|^2)/\sqrt{m!n!} . \qquad (2.21)$$

For a chaotic field ($\alpha = 0$) Eq. (2.19) results in

$$p_{mn} = [\bar{n}^{\,m}/(\bar{n}+1)^{m+1}]\delta_{mn} . \qquad (2.22)$$

Thus the state (2.17) is general enough. Note that experimentally it is feasible[9] to put in the cavity photons in a variety of states such as coherent, thermal, as well as in a state obtained by superposing coherent and thermal photons. It should be noted that we are considering the interaction of the field, in a variety of initial states, with atoms in a cavity at zero temperature. If the cavity is at finite temperature, then the basic Eq. (2.1) gets modified which in turn modifies Eq. (2.12). However, so far we have not succeeded in obtaining solutions of such modified equations. In what follows we assume that the cavity temperatures and frequencies are such that $(e^{\hbar\omega/kT}-1)^{-1}\approx 0$. This would, for example, be the case with experiments at subkelvin temperatures.

For $\rho_a(0)$ and $\rho_f(0)$ as given above, the expression for $W(0)$ in the dressed-state representation becomes

$$W(0)=\frac{1}{2}\sum_{m,n=0}^{\infty} p_{mn}(\,|\Psi_m^+\rangle\langle\Psi_n^+| + |\Psi_m^-\rangle\langle\Psi_n^-| + |\Psi_m^+\rangle\langle\Psi_n^-| + |\Psi_m^-\rangle\langle\Psi_n^+|\,)\,. \tag{2.23}$$

In the following sections we use the density matrix elements derived here to study the dynamical and statistical properties of the system.

III. COLLAPSE AND REVIVAL OF RABI OSCILLATIONS IN THE POPULATIONS OF THE ATOMIC STATES

In the experiments on cavity electrodynamics, one monitors the excited state as a function of time t. In terms of the density matrix ρ, the excited-state population is

$$P(t)=\sum_{n=0}^{\infty}\langle n,\tfrac{1}{2}\,|\rho(t)|\,n,\tfrac{1}{2}\rangle\,, \tag{3.1}$$

which when using the dressed atom representation [Eq. (2.3)] can be written as

$$\langle n,\tfrac{1}{2}\,|\rho(t)|\,n,\tfrac{1}{2}\rangle = \tfrac{1}{2}\{F_n(t)+[\exp(-2igt\sqrt{n+1}) \times\langle\Psi_n^+|W(t)|\Psi_n^-\rangle+\text{c.c.}]\}\,, \tag{3.2}$$

with $\langle\Psi_n^+|W(t)|\Psi_n^-\rangle$ and $F_n(t)$ given by Eqs. (2.9) and (2.14), respectively. For $W(0)$ given by Eq. (2.23) we have

$$\langle n,\tfrac{1}{2}\,|\rho(t)|\,n,\tfrac{1}{2}\rangle = \frac{\exp[-2\kappa t(n+\frac{1}{2})]}{2} \times\left[p_n\cos(2gt\sqrt{n+1}) + \sum_{j=n}^{\infty}\frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{(j-n)!(n+\frac{1}{2})!}p_j\right]\,, \tag{3.3}$$

where $p_j\equiv p_{jj}$ is the photon number distribution function. Barnett and Knight[15] have also studied the effects of cavity damping on the atomic excitation. However, the form of their expression appears different because of the difference in the method of derivation. In Appendix A we show that the expression for $\langle n,\frac{1}{2}\,|\rho(t)|\,n,\frac{1}{2}\rangle$ derived by following the approach of Ref. 15 is equivalent to Eq. (3.3). We now use Eqs. (3.1) to (3.3) to study the effects of field statistics on the atomic excitation.

A. Coherent state

In Fig. 1 we have plotted $P(t)$ for $|\alpha|^2=5$, $\bar{n}=0$ and for different values of the cavity damping. For $\kappa=0$ the plot shows the familiar phenomenon of collapse and revival of the Rabi oscillations around $P(t)=\frac{1}{2}$. This phenomenon can be studied analytically by going over to an integral representation for the sum in Eq. (3.3) and evaluating the integral by the method of steepest descent by assuming that $|\alpha|\gg 1$. Following the methods of Ref. 2 (see also Ref. 18) we present the details of the derivation of the asymptotic expression for $P(t)$ in Appendix B. It is found that the revivals are regularly placed. The period of the revivals is given by $T_R=2\pi|\alpha|/g$. The envelope of the revivals is almost a Gaussian. The width of the envelope of the kth Gaussian is given by

$$\Delta s_k=\sqrt{2}(1+\pi^2k^2)^{1/2}/\pi|\alpha| \tag{3.4}$$

which increases with k. Thus, as the time increases, the neighboring revivals overlap increasingly. In the overlap region, the Rabi oscillations are a result of the superposition of the oscillations from different overlapping revivals. For long times, therefore, when the overlap occurs between increasingly more revivals, the oscillations then are due to the superposition of many frequencies, and $P(t)$ exhibits an apparently chaotic behavior. As dis-

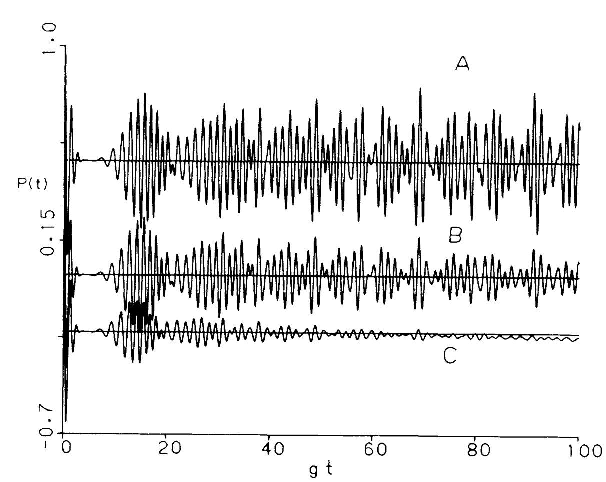

FIG. 1. The probability of finding the atom in the excited state as a function of time for $|\alpha|^2=5$ and $\bar{n}=0$. The curve A is for the cavity relaxation parameter $\kappa=0$; the curves B and C represent the excitation probability for $\kappa/g=0.001[P(t)-\frac{1}{2}]$ and $\kappa/g=0.005[P(t)-\frac{3}{4}]$. Note that gt is dimensionless.

cussed by Yoo and Eberly,[2] the asymptotic expression is found to be good even for the values of $|\alpha|^2$ as low as 3.

For $\kappa \neq 0$ it is seen from Fig. 1 that the amplitude of the oscillations in each revival decreases with virtually no oscillations for large values of gt. Also, the mean value of the oscillations in $P(t)$ keeps dropping below $\frac{1}{2}$ to approach the steady-state value zero.

B. The effects of the chaotic field

Let us now consider the initial state to be a superposition of the coherent and the chaotic fields. In this case the p_{mn}'s are given by Eq. (2.19). In Figs. 2 and 3 we have plotted $P(t)$ for $|\alpha|^2=5$ and increasingly large values of $\bar{n}$, whereas Fig. 4 is for the case of a purely chaotic field. A comparison of Figs. 2–4 with Fig. 1 shows that as the proportion of the chaotic photons is increased, the envelope of the revivals deviates increasingly from its Gaussian shape. The addition of the chaotic field seems to cause an interference between different revivals even for smaller times much like what happens in the case of a purely coherent field for longer times. For large $\bar{n}$, therefore, the oscillations appear to be more and more irregular because of an increase in the overlap between different revivals. The increased interference between the revivals in a chaotic field may perhaps be attributed to the fact that its photon number distribution function is much more broad as compared with that in the case of a coherent field.

For a purely chaotic field ($|\alpha|=0$) in an ideal cavity ($\kappa=0$) the atomic excitation probability $P(t)$ can be represented in terms of an integral. In Appendix C we present an analytic expression for $P(t)$ valid for large $\bar{n}$ and for the time range within the first collapse. This is also the time range in which the classical description of the chaotic field gives results in agreement with the exact quantum analysis.[4] The classical description of the field fails to account for the revivals of the oscillations.

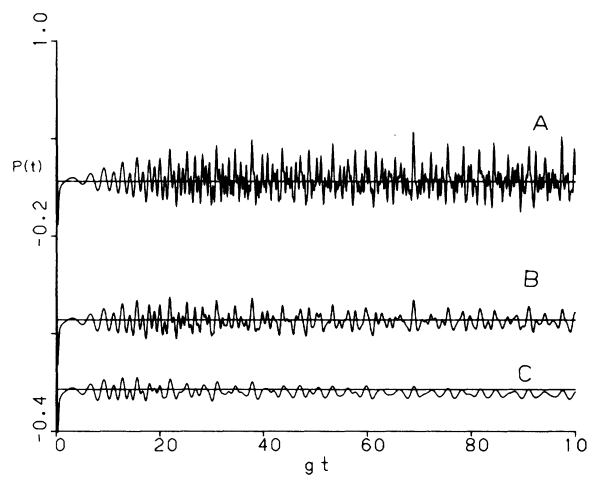

FIG. 3. The same as Fig. 1 but with $|\alpha|^2=5$ and $\bar{n}=20$.

IV. DYNAMICAL PROPERTIES OF THE CAVITY FIELD

The properties of the cavity field can be determined by evaluating the average of a normally ordered product of the field operators:

$$A_{mn}(t) \equiv \langle a^{\dagger m} a^{n} \rangle = \sum_{p=0}^{\infty} \frac{\sqrt{p}!\sqrt{(p-n+m)!}}{(p-n)!} \times [\langle p, \tfrac{1}{2} | \rho(t) | p-n+m, \tfrac{1}{2} \rangle + \langle p, -\tfrac{1}{2} | \rho(t) | p-n+m, -\tfrac{1}{2} \rangle] \quad (m \geq n) . \quad (4.1)$$

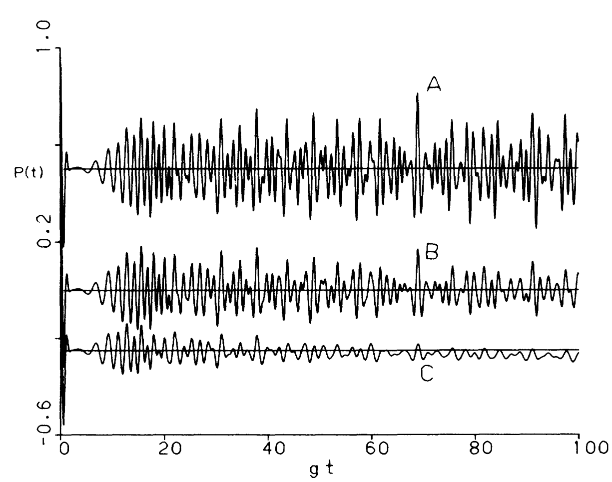

FIG. 2. The same as Fig. 1 but with $|\alpha|^2=5$ and $\bar{n}=1$.

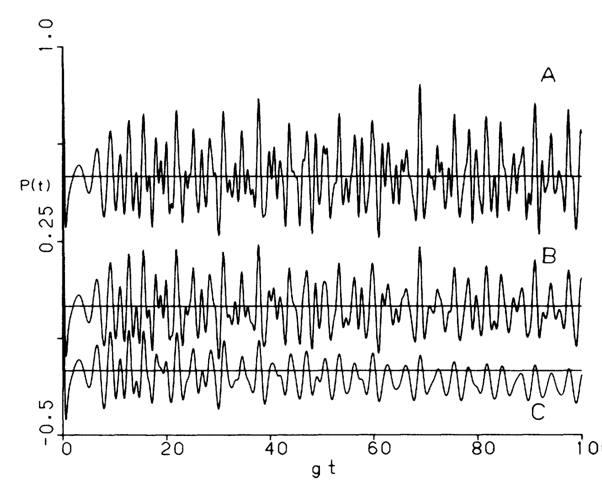

FIG. 4. The same as Fig. 1 but with $|\alpha|^2=0$ and $\bar{n}=5$.

The matrix elements relevant for the evaluation of Eq. (4.1) are as follows:

$$\langle p,\epsilon/2\,|\,\rho(t)\,|\,q,\epsilon/2\rangle=\tfrac{1}{2}(\exp[igt(\sqrt{q-\epsilon'+1}-\sqrt{p-\epsilon'+1})]\langle\Psi^{+}_{p-\epsilon'}\,|\,W(t)\,|\,\Psi^{+}_{q-\epsilon'}\rangle$$
$$+\exp[-igt(\sqrt{q-\epsilon'+1}-\sqrt{p-\epsilon'+1})]\langle\Psi^{-}_{p-\epsilon'}\,|\,W(t)\,|\,\Psi^{-}_{q-\epsilon'}\rangle$$
$$+\epsilon\{\exp[igt(\sqrt{q-\epsilon'+1}+\sqrt{p-\epsilon'+1})]\langle\Psi^{-}_{p-\epsilon'}\,|\,W(t)\,|\,\Psi^{+}_{q-\epsilon'}\rangle$$
$$+\exp[-igt(\sqrt{q-\epsilon'+1}+\sqrt{p-\epsilon'+1})]\langle\Psi^{+}_{p-\epsilon'}\,|\,W(t)\,|\,\Psi^{-}_{q-\epsilon'}\rangle\})$$
$$(\epsilon=+,-)\quad[\epsilon'=(1-\epsilon)/2]\,, \tag{4.2}$$

$$\langle 0,-\tfrac{1}{2}\,|\,\rho(t)\,|\,p,-\tfrac{1}{2}\rangle=1/\sqrt{2}[\exp(igt\sqrt{p})\langle 0,-\tfrac{1}{2}\,|\,W(t)\,|\,\Psi^{+}_{p-1}\rangle+\exp(-igt\sqrt{p})\langle 0,-\tfrac{1}{2}\,|\,W(t)\,|\,\Psi^{-}_{p-1}\rangle]\quad(p\neq 0)\,. \tag{4.3}$$

We will use the convention $\pm\epsilon=\pm$ or ± 1 if $\epsilon=+$ and $\epsilon=\mp$ or ∓ 1 if $\epsilon=-$. Again, using the results of Sec. II, it follows that

$$A_{nn}(t)=\frac{\exp(-\kappa t)}{2}\sum_{p=0}^{\infty}p!\exp(-2p\kappa t)\left[\left[\frac{1}{(p-n)!}+\frac{p+1}{(p-n+1)!}\right]\sum_{j=p}^{\infty}\frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-p}}{(j-p)!(p+\frac{1}{2})!}F_j(0)\right.$$
$$\left.+\left[\frac{1}{(p-n)!}-\frac{p+1}{(p-n+1)!}\right][\exp(-2igt\sqrt{p+1})\langle\Psi^{+}_{p}\,|\,W(0)\,|\,\Psi^{-}_{p}\rangle+\text{c.c.}]\right]\,, \tag{4.4}$$

and for $m>n\geq 0$ we have

$$A_{mn}(t)=\frac{\exp(-\kappa t)}{2}\sum_{p=0}^{\infty}\sqrt{p!}\sqrt{(p-n+m)!}\exp[-\kappa t(2p+m-n)]$$
$$\times\left[\left[\frac{1}{(p-n)!}+\frac{\sqrt{p+1}\sqrt{p-n+m+1}}{(p-n+1)!}\right]\right.$$
$$\times(\exp\{igt[\sqrt{p-n+m+1}-\sqrt{p+1}]\}\langle\Psi^{+}_{p}\,|\,W(0)\,|\,\Psi^{+}_{p-n+m}\rangle$$
$$+\exp\{-igt[\sqrt{p-n+m+1}-\sqrt{p+1}]\}\langle\Psi^{-}_{p}\,|\,W(0)\,|\,\Psi^{-}_{p-n+m}\rangle)$$
$$+\left[\frac{1}{(p-n)!}-\frac{\sqrt{p+1}\sqrt{p-n+m+1}}{(p-n+1)!}\right]\exp[-igt(\sqrt{p-n+m+1}+\sqrt{p+1})]$$
$$\left.\times\langle\Psi^{+}_{p}\,|\,W(0)\,|\,\Psi^{-}_{p-n+m}\rangle+\exp[igt(\sqrt{p-n+m+1}+\sqrt{p+1})]\langle\Psi^{-}_{p}\,|\,W(0)\,|\,\Psi^{+}_{p-n+m}\rangle\right]$$
$$+\sqrt{m!/2}\delta_{n0}\exp[-2\kappa t(m-\tfrac{1}{2})][\langle 0,-\tfrac{1}{2}\,|\,W(0)\,|\,\Psi^{+}_{m-1}\rangle\exp(igt\sqrt{m})$$
$$-\langle 0,-\tfrac{1}{2}\,|\,W(0)\,|\,\Psi^{-}_{m-1}\rangle\exp(-igt\sqrt{m})]\,, \tag{4.5}$$

with $A_{mn}=A^{*}_{nm}$ for $n>m$. We use these expressions to show the time evolution of the mean photon number $n(t)=\langle a^{\dagger}(t)a(t)\rangle=A_{11}(t)$ in Figs. 5 and 6. Note that the photons decay to the steady state much faster than the atomic population inversion. This is understandable because even one photon is sufficient to maintain the Rabi oscillations between the atomic levels and, besides, the cavity permits the leakage of photons.

The statistical properties of the field can be studied by examining the second-order correlation function

$$g^{(2)}(t)=\frac{\langle a^{\dagger 2}(t)a^{2}(t)\rangle-\langle a^{\dagger}(t)a(t)\rangle^{2}}{\langle a^{\dagger}(t)a(t)\rangle^{2}}\,. \tag{4.6}$$

Figures 7 and 8 exhibit the behavior of $g^{(2)}(t)$ for $\kappa=0.005$ and various values of $\bar{n}$ and $|\alpha|^2$. It is clear that the antibunching present for the case of a pure coherent field is destroyed by the presence of on the average of even one chaotic photon.

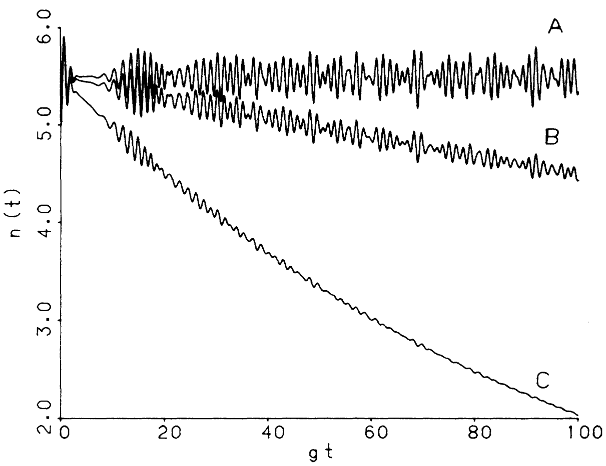

FIG. 5. The mean photon number $n(t)=\langle a^{\dagger}(t)a(t)\rangle$ as a function of time for $|\alpha|^2=5$ and $\bar{n}=0$. The curves A, B, and C are for $\kappa/g=0,0.001$, and 0.005, respectively.

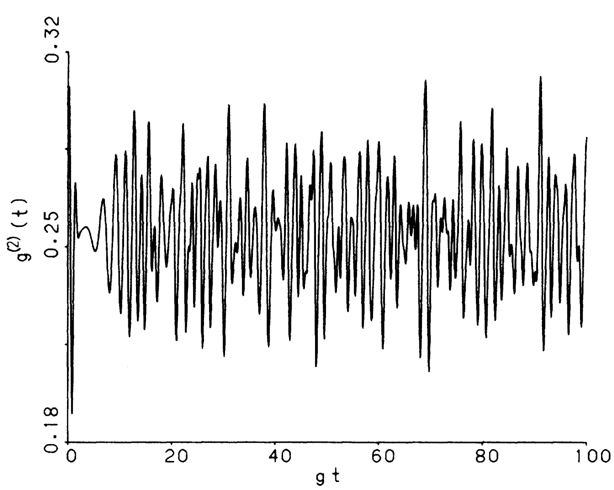

FIG. 7. The normalized intensity correlation function $g^{(2)}(t)$ as a function of time for $|\alpha|^2=5$, $\bar{n}=1$, and $\kappa=0$.

Note that Eq. (4.6) gives the noise in the field intensity (or the dispersion in the photon number distribution) in the process of direct detection of radiation. This noise is clearly insensitive to the phase of the field. Alternatively, we may use the method of homodyne detection[19] in which the signal field is mixed with a strong local oscillator field before its photodetection. This process, for a detector of unity efficiency, measures the field quadrature $a\exp(i\theta)+a^{\dagger}\exp(-i\theta)$ where θ is the phase difference between the signal and the local oscillator field. Moreover, it can be shown that if the local oscillator field is very strong compared with the signal field then the second-order correlation function $g^{(2)}(\theta,t)$ for the superposed field is given by[20]

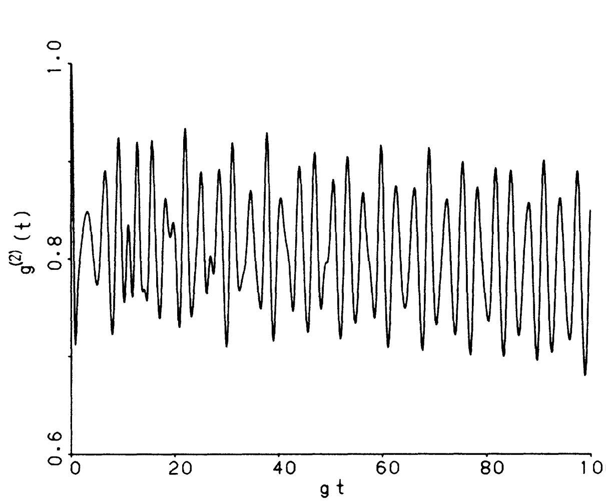

FIG. 8. The same as Fig. 7 but with $|\alpha|^2=0$, $\bar{n}=5$.

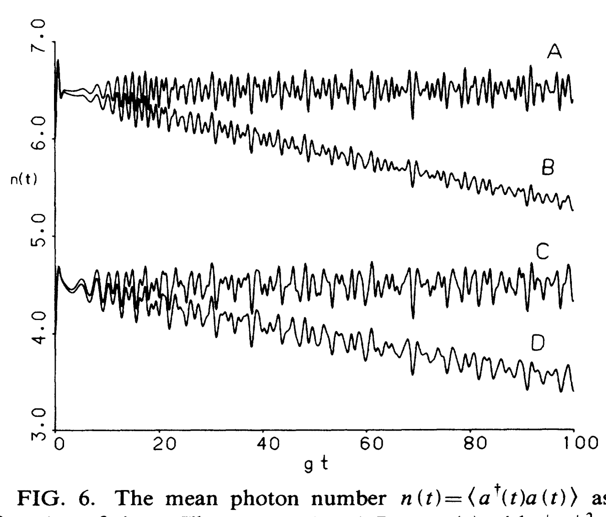

FIG. 6. The mean photon number $n(t)=\langle a^{\dagger}(t)a(t)\rangle$ as a function of time. The curves A and B are $n(t)$ with $|\alpha|^2=5$, $\bar{n}=1$, and for $\kappa/g=0$ and 0.001, respectively. The curves C and D are $n(t)-1$ with $|\alpha|^2=0$, $\bar{n}=5$, and for $\kappa/g=0$ and 0.001, respectively.

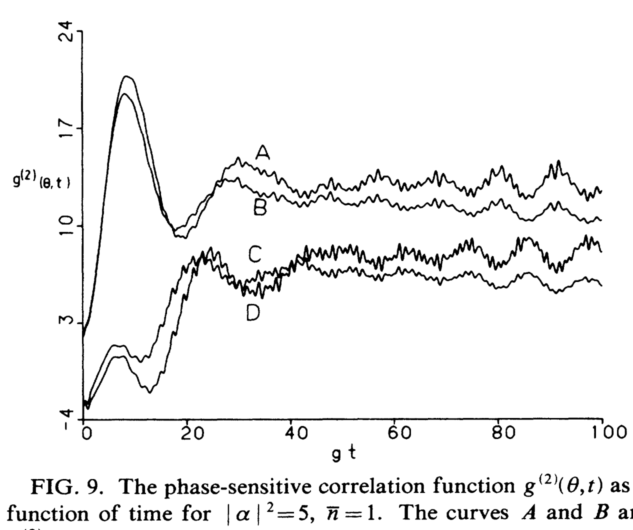

FIG. 9. The phase-sensitive correlation function $g^{(2)}(\theta,t)$ as a function of time for $|\alpha|^2=5$, $\bar{n}=1$. The curves A and B are $g^{(2)}(\pi/2,t)$ for $\kappa/g=0$ and 0.001, respectively. The curves C and D are $g^{(2)}(0,t)-4$ for $\kappa/g=0$ and 0.001, respectively.

$$g^{(2)}(\theta,t)=(\langle :E_1^2:\rangle-\langle E_1\rangle^2)\cos^2(\theta)+(\langle :E_2^2:\rangle-\langle E_2\rangle^2)\sin^2(\theta)+\tfrac{1}{2}(\langle E_1E_2\rangle+\langle E_2E_1\rangle-2\langle E_1\rangle\langle E_2\rangle)\sin(2\theta)\ , \qquad (4.7)$$

where $E_1=a+a^\dagger$ and $E_2=i(a-a^\dagger)$. In Fig. 9 we have plotted $g^{(2)}(\theta,t)$ for $\theta=0$ and $\pi/2$ to show the effect of thermal field on the phase sensitive noise. Note that the thermal field destroys antibunching in the homodyned radiation or equivalently the squeezing in the signal field. Note also the sensitive dependence on the relative phase θ.

We can also use our results for studying the properties of the photon number distribution function $p_n(t)$:

$$p_n(t)=\langle n,+\tfrac{1}{2}\,|\rho(t)|\,n,+\tfrac{1}{2}\rangle+\langle n,-\tfrac{1}{2}\,|\rho(t)|\,n,-\tfrac{1}{2}\rangle\ . \qquad (4.8)$$

Using Eqs. (4.2), (4.3), and the results of Sec. II we get

$$p_n(t)=\frac{\exp[-2\kappa t(n+\frac{1}{2})]}{2}\Bigg[[\exp(-2igt\sqrt{n+1})\langle\Psi_n^+|W(0)|\Psi_n^-\rangle+\exp(2igt\sqrt{n+1})\langle\Psi_n^-|W(0)|\Psi_n^+\rangle-(1-\delta_{n0})(\text{terms with } n\to n-1)]+\Bigg[\sum_{j=n}^{\infty}\frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{(j-n)!(n+\frac{1}{2})!}F_j(0)+(1-\delta_{n0})\ (\text{terms with } n\to n-1)\Bigg]\Bigg]+\delta_{n0}\kappa\int_0^t F_0(\tau)d\tau\ . \qquad (4.9)$$

We have already given[14] a discussion of $p_n(t)$ for the case of an input field in coherent state. We have found that $p_n(t)$ was very close to a Poisson distribution centered around $n(t)\equiv\langle a^\dagger(t)a(t)\rangle$.

V. EFFECT OF ATOMIC COHERENCE ON COLLAPSE AND REVIVAL PHENOMENA

So far we have assumed the atom to be initially in an excited state. However, when an atom interacts with an external coherent field, it is left, in general, in a coherent superposition of the two states. The extent of the superposition depends on the duration of the interaction (see Sec. VI for the details). Hence, it is interesting to investigate the effects of the coherence between the atomic levels on the dynamical properties of the system. Let us then consider the atom in an atomic coherent state $|\mu\rangle$ where

$$|\mu\rangle=(1+|\mu|^2)^{-1/2}(|\tfrac{1}{2}\rangle+\mu|-\tfrac{1}{2}\rangle)\ , \qquad (5.1)$$

and let the initial state of the field be given by Eq. (2.18). The expression for $W(0)$ in the dressed-state representation is then found to be

$$\begin{aligned}W(0)=\tfrac{1}{2}\sum_{m,n=0}^{\infty}[&(p_{m\,n}+|\mu|^2p_{m+1\,n+1}+\mu p_{m+1\,n}+\mu^*p_{m\,n+1})|\Psi_m^+\rangle\langle\Psi_n^+|\\&+(p_{m\,n}+|\mu|^2p_{m+1\,n+1}-\mu p_{m+1\,n}-\mu^*p_{m\,n+1})|\Psi_m^-\rangle\langle\Psi_n^-|\\&+(p_{m\,n}-|\mu|^2p_{m+1\,n+1}+\mu p_{m+1\,n}-\mu^*p_{m\,n+1})|\Psi_m^+\rangle\langle\Psi_n^-|\\&+(p_{m\,n}-|\mu|^2p_{m+1\,n+1}-\mu p_{m+1\,n}+\mu^*p_{m\,n+1})|\Psi_m^-\rangle\langle\Psi_n^+|\,]\\&+1/\sqrt{2}\sum_{n=0}^{\infty}\{[p_{0n}\mu(|0,-\tfrac{1}{2}\rangle\langle\Psi_n^+|+|0,-\tfrac{1}{2}\rangle\langle\Psi_n^-|)+\text{H.c}]\\&+|\mu|^2[p_{0\,n+1}(|0,-\tfrac{1}{2}\rangle\langle\Psi_n^+|-|0,-\tfrac{1}{2}\rangle\langle\Psi_n^-|)+\text{H.c}]\}+p_{00}|\mu|^2|0,-\tfrac{1}{2}\rangle\langle0,-\tfrac{1}{2}|\ .\end{aligned} \qquad (5.2)$$

For the sake of simplicity we consider the initial state of the field to be a coherent state so that the $p_{m\,n}$ are given by Eq. (2.21). Using this expression for $W(0)$ in Eqs. (3.1) and (3.2) the excited-state population is found to be given by

$$P(t)=\frac{\exp(-\kappa t)\exp(-|\alpha|^2)}{2(1+|\mu|^2)}\sum_{n=0}^{\infty}\exp(-2n\kappa t)\left\{\frac{|\alpha|^{2n}}{n!}\left[\left[1-\frac{|\alpha|^2|\mu|^2}{n+1}\right]\cos(2gt\sqrt{n+1})+2\frac{|\mu||\alpha|}{\sqrt{n+1}}\sin(\theta+\varphi)\sin(2gt\sqrt{n+1})\right]+\sum_{j=n}^{\infty}\frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{(j-n)!(n+\frac{1}{2})!}(|\alpha|^{2j}/j!)\times\left[1+\frac{|\mu|^2|\alpha|^2}{j+1}\right]\right\}, \tag{5.3}$$

where $\mu=|\mu|\exp(i\varphi)$ and $z=|z|\exp(i\theta)$. For $\mu=0$, $P(t)$ gives the excitation probability for an initially excited atom whereas for $\mu\to\infty$ it gives that for an initially unexcited atom. Thus, for $\mu\to\infty$, we do not expect any dependence on θ or φ of $P(t)$. We have found that even for $|\mu|\sim 5$, $P(t)$ is phase independent and its behavior is very close to that for an initially unexcited atom. In Appendix B we have derived an asymptotic expression for $P(t)$ in the case of an ideal cavity ($\kappa=0$). An interesting special case of the asymptotic expression Eq. (B29) arises for $\theta+\varphi=0$. In this case the population inversion at the kth revival ($s_k=0$) is

$$P(k)=\tfrac{1}{2}+\{(1-|\mu|^2)\cos[2\pi|\alpha|^2k+\tfrac{1}{2}\tan^{-1}(\pi k)]\}/[2(1+\pi^2k^2)^{1/4}(1+|\mu|^2)]\ . \tag{5.4}$$

For $|\mu|=1$ we find that $P(k)=\frac{1}{2}$. Thus, in this case, the population is equally distributed between the two levels at each revival time. In Fig. 10 we show the effect of cavity damping on $P(t)$ when the population is initially equally distributed between the two levels, i.e., $|\mu|=1$. We show the effect of level coherence by plotting $P(t)$ for $\varphi=0$ and $\varphi=\pi/2$. Note that $\varphi=0$ is the classical dressed state $|\frac{1}{2}\rangle+|-\frac{1}{2}\rangle$ of the atom which is a stationary state of the semiclassical Hamiltonian in the interaction picture. Hence, the oscillations exhibited in the figure for $\varphi=0$ are purely due to the quantum effects. The other semiclassical dressed state $|\frac{1}{2}\rangle-|-\frac{1}{2}\rangle$ corresponding to $|\mu|=1,\varphi=\pi$ gives $P(t)$ identical to that for $\varphi=0$ as is evident from the expression (5.3). Thus a clean way of studying the quantum effects is to send the atom initially prepared in either of the dressed states $|\frac{1}{2}\rangle\pm|-\frac{1}{2}\rangle$.

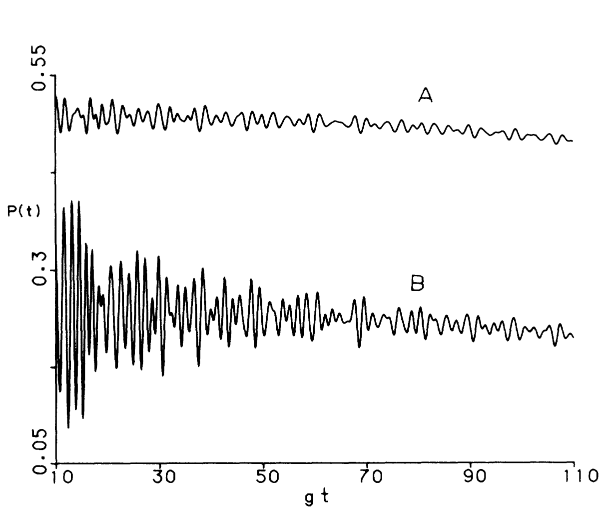

FIG. 10. The probability of finding the atom in the excited state as a function of time for $|\alpha|^2=5$, $|\mu|=1$, $\theta=0$, and $\kappa/g=0.005$. The curve A is $P(t)$ for $\varphi=0$ and the curve B is $P(t)-\frac{1}{4}$ for $\varphi=\pi/2$.

If the phase of the atomic coherent state is a random variable then the atomic excitation is obtained by averaging Eq. (5.3) over φ. The value of $P(t)$ in this case is the same as that in the case of $\theta+\varphi=0$.

In Ref. 14 we have discussed the effects of field coherence on the squeezing characteristics of the output field. Here we study the effects of atomic level coherence on squeezing. To isolate the effects of level coherence from those of the field coherence, we consider the field to be initially in the vacuum state. In Appendix D we present the details of the derivation of the squeezing in the in-phase component of the field. We find that if the atom is prepared in a suitable superposition of the two states, the field is indeed squeezed. For example, for the resonant interaction between the field in an ideal cavity ($\kappa=0$) and the atom, the field is found to be squeezed if $\mu_i>1+\mu_r$ where μ_r and μ_i are, respectively, the real and imaginary parts of μ. In fact, the maximum squeezing in this case is obtained when the probability of finding the atom in the ground state is three times than that in the excited state.

VI. THE DIPOLE MOMENT

A complete description of the atomic dynamical properties involves, besides the population, a knowledge of the dynamics of the coherence between the two atomic levels which can be studied by evaluating the dipole matrix element $D(t)$:

$$D(t)=\sum_{n=0}^{\infty}\langle n,-\tfrac{1}{2}|\rho(t)|n,\tfrac{1}{2}\rangle\exp(-i\omega t)\ . \tag{6.1}$$

In terms of the dressed states we get

$$D(t)=\frac{1}{2}\sum_{n=0}^{\infty}\{\langle\Psi_{n-1}^{+}|W(t)|\Psi_n^{+}\rangle\exp[i(\sqrt{n+1}-\sqrt{n})gt]-\langle\Psi_{n-1}^{-}|W(t)|\Psi_n^{-}\rangle\exp[-i(\sqrt{n+1}-\sqrt{n})gt]$$
$$+\langle\Psi_{n-1}^{-}|W(t)|\Psi_n^{+}\rangle\exp[i(\sqrt{n+1}+\sqrt{n})gt]-\langle\Psi_{n-1}^{+}|W(t)|\Psi_n^{-}\rangle\exp[-i(\sqrt{n+1}+\sqrt{n})gt]\}$$
$$+1/\sqrt{2}[\exp(igt)\langle 0,-\tfrac{1}{2}|W(t)|\Psi_0^{+}\rangle+\exp(-igt)\langle 0,-\tfrac{1}{2}|W(t)|\Psi_0^{-}\rangle]\ . \tag{6.2}$$

If the atom is initially in the excited state then it follows from Eqs. (2.9), (2.23), and (6.2) that

$$D(t)=-i\sum_{n=1}^{\infty}p_{n-1n}\{\sin[gt(\sqrt{n+1}+\sqrt{n})]-\sin[gt(\sqrt{n+1}-\sqrt{n})]\}\exp(-2n\kappa t)2\ . \tag{6.3}$$

For $\kappa=0$, Eq. (6.3) reproduces the results of Narozhny *et al.*[2] If the field is initially in a chaotic state, then $p_{mn}=0$ for $m\neq n$ which implies that $D(t)=0$.

If, on the other hand, the atom is in the coherent state $|\mu\rangle$ and the field is in the coherent state $|\alpha\rangle$ then the real and imaginary parts of $D(t)$ are given, respectively, by

$$D_R(t)=\frac{\exp(-|\alpha|^2)}{2(1+|\mu|^2)}\sum_{n=1}^{\infty}\frac{|\alpha|^{2n-1}}{n!}\exp(-2n\kappa t)$$
$$\times[\,|\mu||\alpha|(\cos(\varphi)\{\cos[gt(\sqrt{n+1}-\sqrt{n})]+\cos[gt(\sqrt{n+1}+\sqrt{n})]\}$$
$$+\sqrt{n/(n+1)}\cos(2\theta+\varphi)\{\cos[gt(\sqrt{n+1}-\sqrt{n})]-\cos[gt(\sqrt{n+1}+\sqrt{n})]\})$$
$$+\sin(\theta)(\sqrt{n}\{\sin[gt(\sqrt{n+1}-\sqrt{n})]-\sin[gt(\sqrt{n+1}+\sqrt{n})]\}$$
$$+|\mu|^2|\alpha|^2\{\sin[gt(\sqrt{n+1}-\sqrt{n})]+\sin[gt(\sqrt{n+1}+\sqrt{n})]\}/\sqrt{(n+1)})]$$
$$+\frac{\exp(-|\alpha|^2)}{(1+|\mu|^2)}\exp(-\kappa t)|\mu|[\cos(gt)\cos(\varphi)+|\mu|\,|\alpha|\sin(gt)\sin(\theta)]\ , \tag{6.4}$$

and

$$D_I(t)=-\frac{\exp(-|\alpha|^2)}{2(1+|\mu|^2)}\sum_{n=1}^{\infty}\frac{|\alpha|^{2n-1}}{n!}\exp(-2n\kappa t)$$
$$\times[\,|\mu||\alpha|(-\sin(\varphi)\{\cos[gt(\sqrt{n+1}-\sqrt{n})]+\cos[gt(\sqrt{n+1}+\sqrt{n})]\}$$
$$+\sqrt{n/(n+1)}\sin(2\theta+\varphi)$$
$$\times\{\cos[gt(\sqrt{n+1}-\sqrt{n})]-\cos[gt(\sqrt{n+1}+\sqrt{n})]\})$$
$$-\cos(\theta)(\sqrt{n}\{\sin[gt(\sqrt{n+1}-\sqrt{n})]-\sin[gt(\sqrt{n+1}+\sqrt{n})]\}$$
$$+|\mu|^2|\alpha|^2\{\sin[gt(\sqrt{n+1}-\sqrt{n})]+\sin[gt(\sqrt{n+1}+\sqrt{n})]\}/\sqrt{n+1})]$$
$$+\frac{\exp(-|\alpha|^2)}{(1+|\mu|^2)}\exp(-\kappa t)|\mu|[\cos(gt)\sin(\varphi)+|\mu|\,|\alpha|\sin(gt)\cos(\theta)]\ , \tag{6.5}$$

Note that the dipole moment is a sum of the fast and the slow oscillating terms which have a sum or difference of frequencies in the argument of the trigonometric functions. This is in contrast with the population inversion which contains only the fast oscillating terms. In Appendix B we have outlined a method for the asymptotic evaluation of these terms. It is interesting to see from Eqs. (6.4) and (6.5) that the dominant contribution to $D_R(t)$ comes from the slow terms whereas $D_I(t)$ is dominated by the fast term. This is also borne out by Figs. 11

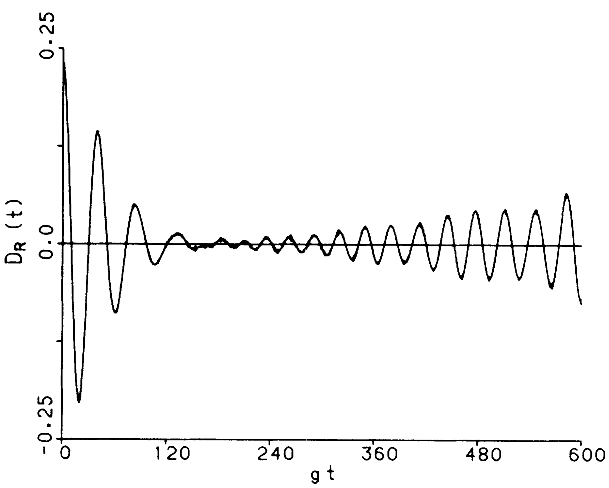

FIG. 11. The real part of the atomic dipole moment as a function of time for $|\alpha|^2=10$, $|\mu|=0.25$, $\theta=\varphi=0$, and $\kappa=0$.

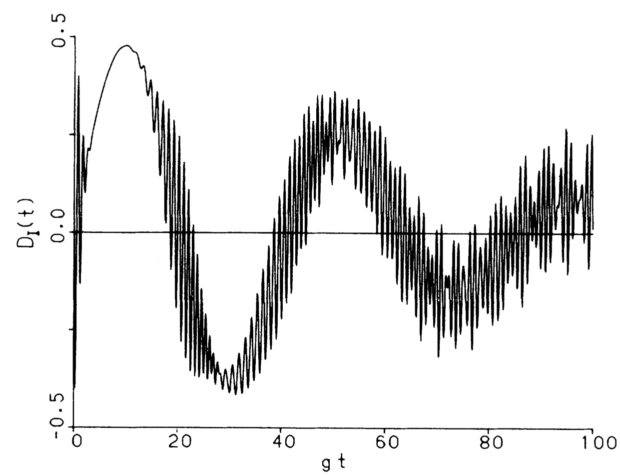

FIG. 12. The imaginary part of the atomic dipole moment as a function of time for $|\alpha|^2=10$, $|\mu|=0.25$, $\theta=\varphi=0$, and $\kappa=0$.

and 12 for $D_R(t)$ and $D_I(t)$, respectively.

The dipole matrix elements are useful in studying the properties of the fluorescent radiation emitted in a direction perpendicular to the cavity field. The positive frequency part of the fluorescent radiation is given by $\beta\langle S^-\rangle$ where β is a constant. This weak field can be detected by making it interfere with a part of the incident field used for preparing the atoms in the superposed state. Thus, during the initial preparation stage of the atom, if E is the part of the incident field interacting with the atoms for time τ then using the Heisenberg equations it can be shown that

$$\langle S^+(\tau)\rangle=\tfrac{1}{2}\sin(2g\tau)\exp[-i(\nu+\pi/2)]\ , \quad (6.6)$$

$$\langle S^z(\tau)\rangle=-\tfrac{1}{2}\cos(2g\tau)\ , \quad (6.7)$$

where

$$g\exp(i\nu)=(dE^*)/\hbar\ .$$

The initial state of the atoms is, therefore, the atomic coherent state $|\mu\rangle$ [Eq. (6.1)] with μ given by

$$\mu=\cot(\theta/2)\exp(i\phi)\ , \quad (6.8)$$

where

$$\theta=2gt,\ \ \phi=-(\nu+\pi/2)\ . \quad (6.9)$$

Note that the phase factor ϕ can have an additional contribution of the form ωT where T is the time during which the atom evolves freely before entering the cavity. The fluorescent radiation from the atoms, after they have interacted with the cavity field, is then made to interfere with the other part $\mathscr{E}$ of the incident field after shifting

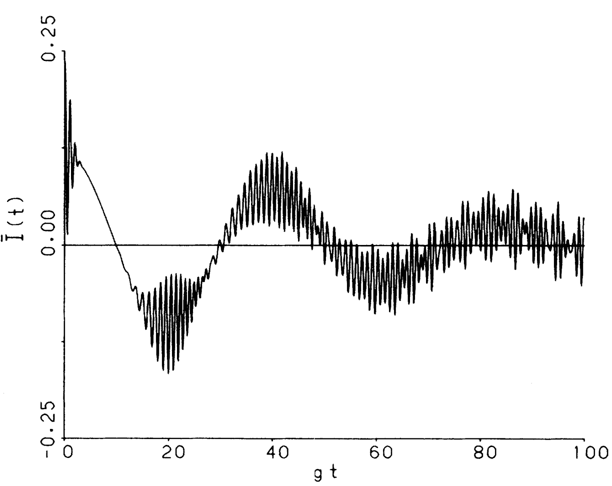

FIG. 13. The interference term $\bar{I}(t)$ as a function of time for $|\alpha|^2=10$, $|\mu|=0.25$, $\Omega=\pi/2$, and $\kappa=0$.

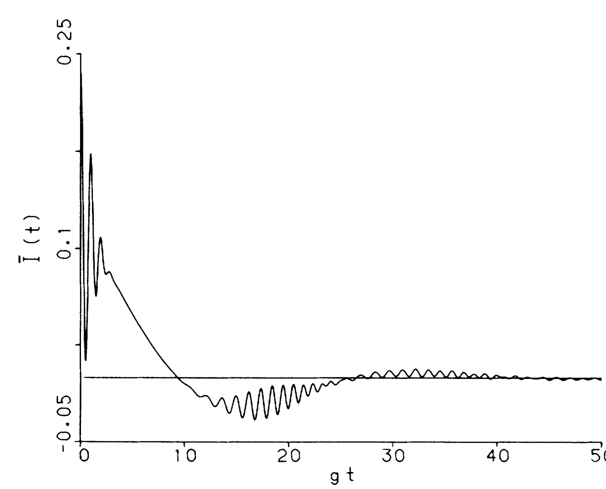

FIG. 14. The same as Fig. 13 but with $\kappa/g=0.005$.

its phase by Ω. The intensity of the superposed field is given by

$$\langle(\mathscr{E}^*-\beta^*S^+)(\mathscr{E}-\beta S^-)\rangle$$
$$=\langle||\mathscr{E}|\exp[i(\nu+\Omega)]-\beta S^-|^2\rangle$$
$$\cong|\mathscr{E}|^2+i\beta|\mathscr{E}|\{\langle S^+\rangle\exp[-i(\phi-\Omega)]-\langle S^-\rangle\exp[i(\phi-\Omega)]\}\ . \qquad (6.10)$$

Here we have neglected the terms of the order β^2 as the fluorescent field is assumed to be weak. In terms of the dipole moment matrix elements D_R and D_I the interference term in Eq. (6.10) may be written as

$$I=[D_I(t)\cos(\phi-\Omega)-D_R(t)\sin(\phi-\Omega)]\ , \qquad (6.11)$$

where we have used the relation $\langle S_\pm\rangle=D_R\pm iD_I$. Now, if the initial field has a random phase we must average over ϕ in which case for an initially coherent state $|\alpha\rangle$ of the cavity field it follows that

$$\bar{I}=\sin(\Omega)|\mu|\exp(-|\alpha|^2)\left[\exp(-\kappa t)\cos(2gt)+\sum_{n=1}^{\infty}\frac{|\alpha|^{2n}\exp(-2n\kappa t)}{n!}\cos(gt\sqrt{n+1})\cos(gt\sqrt{n})\right]\Big/(1+|\mu|^2)\ , \qquad (6.12)$$

where overbar denotes the average over the random distribution of ϕ. It is seen that the state of the atom enters the expression for $\bar{I}$ just as a multiplying factor. In Fig. 13 we have plotted $\bar{I}$ for $|\alpha|^2=10$, $|\mu|=0.25$, and $\kappa=0$. The effect of cavity damping is shown in Fig. 14. In this case, damping is found to suppress the collapse and revival phenomena drastically.

ACKNOWLEDGMENT

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APPENDIX A: RELATION WITH THE WORK OF BARNETT AND KNIGHT (REF. 15)

In this appendix we show that the expression

$$\langle n,\tfrac{1}{2}|\rho(t)|n,\tfrac{1}{2}\rangle=\sum_{m=n}^{\infty}\left[\frac{m!}{n!(m-n)!}[\exp(2\kappa t)-1]^{m-n}\exp(-2m\kappa t)\right]\chi_m\ , \qquad (A1)$$

where

$$\chi_m=\frac{\exp(-2\kappa t)}{2}\left[\sum_{j=m+1}^{\infty}\frac{(j-m-\frac{1}{2})![1-\exp(-2\kappa t)]^{j-m}}{(j-m)!}p_j+p_m+\sum_{j=m}^{\infty}\frac{(-1)^{j-m}j![1-\exp(-2\kappa t)]^{j-m}}{m!(j-m)!}p_j\times\cos(2gt\sqrt{j+1})\right]\ , \qquad (A2)$$

for the probability of finding the atom in the excited state with field in an n-photon state derived in Ref. 15 is equivalent to the one obtained in Eq. (3.3) of this paper.

The contribution from the first term in Eq. (A2) to Eq. (A1) is

$$I_1=\frac{\exp[-2\kappa t(n+\frac{1}{2})]}{2}\sum_{j=n+1}^{\infty}\sum_{m=n}^{j-1}\left[\frac{m!(j-m-\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{n!(m-n)!(j-m)!}\right]p_j\ , \qquad (A3)$$

where in writing Eq. (A3) we have interchanged the order of j and m summations. Using the beta function representation for the factorials in Eq. (A3) it follows that

$$I_1=\frac{\exp[-2\kappa t(n+\frac{1}{2})]}{2}\left[\sum_{j=n+1}^{\infty}\frac{[1-\exp(-2\kappa t)]^{j-n}(j+\frac{1}{2})!}{n!}p_j\sum_{m=n}^{j-1}\left[\frac{1}{(j-m)!(m-n)!}\right]\int_0^1 dx[x^m(1-x)^{j-m-1/2}]\right]$$

$$=\frac{\exp[-2\kappa t(n+\frac{1}{2})]}{2}\left\{\sum_{j=n+1}^{\infty}\frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{n!}p_j\right.$$

$$\left.\times\left[\sum_{m=0}^{j-n}\int_0^1\left[\frac{x^{n+m}(1-x)^{j-m-n-1/2}}{m!(j-m-n)!}\right]-\int_0^1 dx\, x^j(1-x)^{-1/2}/(j-n)!\right]\right\}. \tag{A4}$$

Now, by summing the binomial series in Eq. (A4) and using the definition of beta function it can be shown that

$$I_1=\frac{\exp[-2\kappa t(n+\frac{1}{2})]}{2}\left[\sum_{j=n}^{\infty}\frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{(n+\frac{1}{2})!(j-n)!}p_j-\sum_{j=n}^{\infty}\frac{[1-\exp(-2\kappa t)]^{j-n}j!}{n!(j-n)!}p_j\right]. \tag{A5}$$

The last term in Eq. (A5) cancels with the contribution to Eq. (A1) from the second term of Eq. (A2). Next, the contribution to Eq. (A1) from the last term in Eq. (A2) is

$$I_2=\frac{\exp[-2\kappa t(n+\frac{1}{2})]}{2}\left[\sum_{j=n}^{\infty}\frac{j![1-\exp(-2\kappa t)]^{j-n}}{n!}p_j\cos(2gt\sqrt{j+1})\sum_{m=n}^{j}\frac{(-1)^{j-m}}{(m-n)!(j-m)!}\right], \tag{A6}$$

where, as before, the j and m summations have been interchanged. Clearly, the nonzero contribution to the summation over m in Eq. (A6) is obtained only when $j=n$ so that

$$I_2=\exp[-2\kappa t(n+\tfrac{1}{2})]p_n\cos[2gt\sqrt{(n+1)}]. \tag{A7}$$

Thus, by combining all the contributions to Eq. (A1) it follows that

$$\langle n,\tfrac{1}{2}|\rho(t)|n,\tfrac{1}{2}\rangle=\exp[-2\kappa t(n+\tfrac{1}{2})]\left[p_n\cos(2gt\sqrt{n+1})+\sum_{j=n}^{\infty}\frac{(j+\frac{1}{2})![1-\exp(-2\kappa t)]^{j-n}}{(j-n)!(n+\frac{1}{2})!}p_j\right], \tag{A8}$$

which is the same as the expression in Eq. (3.3).

APPENDIX B: ASYMPTOTIC FORM OF OBSERVABLES FOR AN ATOM INITIALLY PREPARED IN A SUPERPOSITION STATE

In this appendix we present an asymptotic evaluation of the excitation probability $P(t)$ and the dipole moment for an atom in a superposed state in an initially coherent field in an ideal cavity. Here we follow Ref. 2 where $P(t)$ has been evaluated for an atom initially in ground state.

From Eqs. (3.1) and (3.3) it follows that for $\kappa=0$

$$P(t)=\tfrac{1}{2}-[\,\mathrm{Re}(|\mu|^2I_1-I_2)-2\sin(\theta+\phi)|\mu|\,\mathrm{Im}(I_3)]/[2(1+|\mu|^2)], \tag{B1}$$

where

$$I_1=\exp(-|\alpha|^2)\sum_{n=0}^{\infty}\left[\frac{|\alpha|^{2n}}{n!}\exp(2igt\sqrt{n})\right],$$

$$I_2=\exp(-|\alpha|^2)\sum_{n=0}^{\infty}\left[\frac{n|\alpha|^{2n-2}}{n!}\exp(2igt\sqrt{n})\right],$$

$$I_3=\exp(-|\alpha|^2)\sum_{n=0}^{\infty}\left[\frac{\sqrt{n}|\alpha|^{2n-1}}{n!}\exp(2igt\sqrt{n})\right]. \tag{B2}$$

Let us first evaluate I_1. As discussed in Ref. 2, the dominant contribution to I_1 comes from the values of n near $|\alpha|^2$. Therefore, for large $|\alpha|^2$, we may use Stirling's approximation for $n!$ and also replace the summation over n by an integral to obtain

$$I_1=|\alpha|\sqrt{(2/\pi)}\int_0^\infty\exp[-|\alpha|^2Q(z,t)]dz, \tag{B3}$$

where we have substituted $z=\sqrt{n}/|\alpha|$ and $Q(z,t)$ is defined as

$$Q(z,t)=1-z^2+2z^2\ln(z)-2igtz/|\alpha|. \tag{B4}$$

We now evaluate I_1 by the method of steepest descent. Note that the saddle points z_s which are the roots of $Q'(z_s,t)=0$ $[Q'(z,t)\equiv\partial Q(z,t)/\partial z]$ satisfy the equation

$$z_s\ln(z_s)=i\pi T, \tag{B5}$$

with

$$T\equiv gt/2\pi|\alpha|. \tag{B6}$$

Let

$$z_s=\rho_s\exp(i\psi_s), \tag{B7}$$

so that

$$Q''(z_s,t)=|Q_s''(z,t)|\exp(i\phi_s)\ , \tag{B8}$$

where

$$|Q_s''(z,t)|=4\{\psi_s^2+[1+\ln(\rho_s)]^2\}^{1/2}\ , \tag{B9}$$

$$\phi_s=\tan^{-1}\{\psi_s/[1+\ln(\rho_s)]\}\ . \tag{B10}$$

Hence it follows that the angle of steepest descent at z_s in the complex plane is given by

$$\alpha_s=-\phi_s/2=-\tfrac{1}{2}\tan^{-1}\{\psi_s/[1+\ln(\rho_s)]\}\ . \tag{B11}$$

It can now be shown that the asymptotic expression for I_1 is

$$I_1=\sum_s\{\psi_s^2+[1+\ln(\rho_s)]^2\}^{-1/4}\times\exp[-|\alpha|^2Q(z_s,t)+i\alpha_s]\ , \tag{B12}$$

where the summation is over all the solutions of Eq. (B5). We now determine these solutions. Note that from Eqs. (B5) and (B7) we have

$$\begin{aligned}&\rho_s[\ln(\rho_s)\cos(\psi_s)-\psi_s\sin(\psi_s)]=0\ ,\\&\rho_s[\ln(\rho_s)\sin(\psi_s)+\psi_s\cos(\psi_s)]=\pi T\ ,\end{aligned} \tag{B13}$$

from which we get

$$\rho_s\psi_s=\pi T\cos(\psi_s)\ . \tag{B14}$$

Clearly, for $T=k$ $(k=0,1,2,\ldots)$, $\rho_s=1$, and $\psi_s=(-1)^k\pi k$ are the exact roots of these equations. At each such scaled time, k, the inversion has the same phase. Hence $T=k$ is the revival time. We now determine the approximate solution of Eq. (B13) near these revival times by letting

$$T=k+s_k,\quad \rho_s=1+\rho_k,\quad \psi_s=(-1)^k\pi k+\psi_k\ , \tag{B15}$$

where s_k, ρ_k, and ψ_k are the small increments. It then follows from Eq. (B13) that

$$\psi_k=\pi(-1)^ks_k/(1+\pi^2k^2),\quad \rho_k=(-1)^k\pi k\psi_k\ , \tag{B16}$$

so that Eq. (B12) yields

$$I_1=\sum_k((1+\pi^2T^2)^{-1/4}\exp\{-|\alpha|^2[\Psi_k+i(-1)^k\Phi_k]-i(-1)^k\phi_k\})\ , \tag{B17}$$

where

$$\begin{aligned}&\Psi_k=2\pi^2s_k^2/(1+\pi^2k^2)\ ,\\&\Phi_k=2\pi\{k+2s_k+[k\pi^2s_k^2/(1+\pi^2k^2)]\}\ ,\\&\phi_k=\tfrac{1}{2}\tan^{-1}(\pi\{T-[2\pi^2k^2s_k/(1+\pi^2k^2)]\})\ .\end{aligned} \tag{B18}$$

An alternate method of obtaining the integral representation for $I_1(t)$ is given in Ref. (18).

In a similar way it is can be shown that

$$I_2=\sum_k[(1+2\rho_k)+i\psi_k](1+\pi^2T^2)^{-1/4}\times\exp\{-|\alpha|^2[\Psi_k+i(-1)^k\Phi_k]-i(-1)^k\phi_k\}\ , \tag{B19}$$

$$I_3=\sum_k(-1)^k(1+\rho_k+i\psi_k)(1+\pi^2T^2)^{-1/4}\times\exp\{-|\alpha|^2[\Psi_k+i(-1)^k\Phi_k]-i(-1)^k\phi_k\}\ . \tag{B20}$$

Finally, substituting the expressions for I_1, I_2, and I_3 in Eq. (B1), the asymptotic expression for $P(t)$ is obtained as

$$\begin{aligned}P(t)=\tfrac{1}{2}+\sum_k\exp(-|\alpha|^2\Psi_k)(\{&1+(2\pi^2ks_k)/(1+\pi^2k^2)-|\mu|^2\\&+[2\sin(\theta+\phi)|\mu|\pi s_k]/(1+\pi^2k^2)\}\cos(|\alpha|^2\Phi_k+\phi_k)\\&+\{\pi s_k[1-2\pi k|\mu|\sin(\theta+\phi)]/(1+\pi^2k^2)-2|\mu|\sin(\theta+\phi)\}\\&\times\sin(|\alpha|^2\Phi_k+\phi_k))/[2(1+\pi^2T^2)^{1/4}(1+|\mu|^2)]\ .\end{aligned} \tag{B21}$$

Note that $\mu=0$ corresponds to the case of the atom initially in an excited state whereas $\mu\to\infty$ represents an unexcited atom.

Next, we derive the asymptotic expression for the dipole moment. For the sake of simplicity we restrict our attention to the case of an initially excited atom. The other terms in the expression for the dipole moment for the atom in the superposed state can be similarly evaluated. From Eqs. (6.4) and (6.5) we find that the dipole moment in the presence of coherent field in an ideal cavity is given by

$$D(t)=i\alpha^*\exp(-|\alpha|^2)/|\alpha|\sum_{n=1}^{\infty}\frac{\sqrt{n}\,|\alpha|^{2n-1}}{n!}\{\sin[gt(\sqrt{n+1}+\sqrt{n})t]-\sin[gt(\sqrt{n+1}-\sqrt{n})t]\}\ . \tag{B22}$$

Following the same procedure which leads us from Eq. (B2) to Eq. (B3) we find that the fast oscillating term in $D(t)$ has the integral representation given by

$$D_f(t)=i\alpha^*\sqrt{2/\pi}\,\mathrm{Im}\int_0^\infty dz\,z\exp[-|\alpha|^2Q(z,t)]\ , \tag{B23}$$

where $Q(z,t)$ is defined in Eq. (B4). Again, by following the procedure which leads to Eq. (B17) it can be shown that

$$D_f(t)=(i\alpha/2|\alpha|)\sum_k((1+\pi^2T^2)^{-1/4}\exp(-|\alpha|^2\Psi_k)\times\{\pi s_k(1+\pi^2k^2)^{-1}\cos(|\alpha|^2\Phi_k+\phi_k)-[1+(\pi^2ks_k)/(1+\pi^2k^2)]\sin(|\alpha|^2\Phi_k+\phi_k)\}) \quad \text{(B24)}$$

Now we evaluate the slow term of $D(t)$ which, for large $|\alpha|^2$, is given by

$$D_s(t)\simeq -i\alpha^*/|\alpha|\sum_{n=1}^{\infty}\frac{\sqrt{n}\,|\alpha|^{2n-1}}{n!}\sin(gt/2n) . \quad \text{(B25)}$$

Its integral representation is easily found to be of the form

$$D_s(t)=-i(\alpha^*/|\alpha|)\text{Im}(I_s) , \quad \text{(B26)}$$

where

$$I_s=|\alpha|\sqrt{2/\pi}\int_0^\infty z\exp[-|\alpha|^2Q_s(z,t)]dz , \quad \text{(B27)}$$

$$Q_s(z,t)=1-z^2+2z^2\ln(z)-igt/2|\alpha|^3z . \quad \text{(B28)}$$

The roots z_s of $Q'(z,t)=0$ satisfy the equation

$$z_s^3\ln(z_s)=-igt/(8|\alpha|^3)\equiv -i\pi T_s . \quad \text{(B29)}$$

If we let

$$z_s=\rho_s\exp(i\psi_s) , \quad \text{(B30)}$$

then we find that Eq. (B29) reduces to

$$\rho_s^3[\cos(3\psi_s)\ln(\rho_s)-\psi_s\sin(3\psi_s)]=0 ,$$
$$\rho_s^3[\sin(3\psi_s)\ln(\rho_s)+\psi_s\cos(3\psi_s)]=-\pi T_s , \quad \text{(B31)}$$

which also implies that

$$\rho_s^3\psi_s=-\pi T_s\cos(3\psi_s) . \quad \text{(B32)}$$

If $T_s=k$ then it is seen that $\psi_s=(-1)^{k+1}\pi k, \rho_s=1$ are the solutions of the Eq. (B31). At these time points, the phase of I_s remains the same. Hence, $T_s=k$ is the revival time for the slow oscillations. Note that $T_s=k/3$, $\rho_s=1$, and $\psi_s=(-1)^k\pi k/3$ (k not necessarily a multiple of 3) also satisfy Eqs. (B31). However, unless k is a multiple of 3 the phase of I_s at the points $T_s=k/3$ is different. Hence, $T_s=k/3$ is not a revival time if k is not a multiple of 3. As before, we determine the roots of Eqs. (B31) around the revival time $T_s=k$ by letting

$$T_s=k+s_k, \quad \psi_s=(-1)^{k+1}\pi k+\psi_s^k, \quad \rho_s=1+\rho_s^k . \quad \text{(B33)}$$

It is now straightforward to show that

$$\psi_s^k=(-1)^{k+1}\pi s_k/(1+9\pi^2k^2) ,$$
$$\rho_s^k=3(-1)^{k+1}\pi k\psi_s^k . \quad \text{(B34)}$$

Note that

$$|Q''(z_s,t)|^{1/2}\simeq 2(1+9\pi^2T^2)^{1/4} , \quad \text{(B35)}$$

and the angle of the path of steepest descent at z_s is given by

$$\alpha_s=-\tfrac{1}{2}\tan^{-1}\{3\psi_s/[1+3\ln(\rho_s)]\}$$
$$\approx\tfrac{1}{2}(-1)^k\tan^{-1}\{3\pi[T_s-18\pi^2k^2s_k/(1+9\pi^2k^2)]\}$$
$$\equiv(-1)^{k+1}\phi_s . \quad \text{(B36)}$$

The asymptotic expression for $D_s(t)$ is now found to be given by

$$D_s(t)=-i\alpha^*/2|\alpha|\sum_k\exp(-|\alpha|^2\Psi_{ks})(1+9\pi^2T^2)^{-1/4}\times(\{1+[3\pi^2ks_k/(1+9\pi^2k^2)]\}\sin(|\alpha|^2\Phi_{ks}+\phi_{ks})-[\pi s_k/(1+9\pi^2k^2)]\cos(|\alpha|^2\Phi_{ks}+\phi_{ks})) , \quad \text{(B37)}$$

where

$$\Psi_{ks}=6\pi^2s_k^2/(1+9\pi^2k^2) ,$$
$$\Phi_{ks}=2\pi[3k+2s_k-(9\pi^2ks_k^2)/(1+9\pi^2k^2)] . \quad \text{(B38)}$$

APPENDIX C: ASYMPTOTIC PROPERTIES OF $P(t)$ FOR AN ATOM DRIVEN BY A THERMAL FIELD

In this appendix we evaluate the asymptotic expression for $P(t)$ in the case of the atom in a strong chaotic field in an ideal cavity. In this case we have

$$P(t)=(\tfrac{1}{2})[(\bar{n}-1)/\bar{n}+I] , \quad \text{(C1)}$$

where

$$I=1/\bar{n}\sum_{n=0}^{\infty}[\bar{n}/(1+\bar{n})]^n\cos(2gt\sqrt{n}) . \quad \text{(C2)}$$

By replacing the summation with an integral we find that for $\bar{n}\gg 1$

$$I\approx 1/\bar{n}\,\text{Re}\int_0^\infty dn\exp[-n\ln(1+1/\bar{n})+2igt\sqrt{n}] ,$$

$$\approx 1/\bar{n}\,\text{Re}\int_0^\infty dn\exp[-(n/\bar{n}-2igt\sqrt{n})] . \quad \text{(C3)}$$

Now let $z=n/\bar{n}$ so that

$$I=\text{Re}\int_0^\infty dz\exp(-z)\exp(-2igt\sqrt{\bar{n}z}) ,$$
$$=\sum_{n=0}^{\infty}(-1)^n(2gt\sqrt{\bar{n}})^{2n}\frac{n!}{(2n)!} . \quad \text{(C4)}$$

This result is the same as that of Riti and Vetri[5] who have derived it by a direct summation of the series. Note that in Eq. (C4) the $\bar{n}$ dependence comes only as a scaling factor for time. Also, the replacement of the sum by an integral results in the loss of phase information[2] and hence the integral representation is valid only within the time range when the first collapse occurs. This is in contrast with the case of the coherent field where the loss of the phase information in going over to the integral representation somehow seems to be regained in the subsequent saddle point analysis.[2]

APPENDIX D: SQUEEZING PROPERTIES OF THE CAVITY FIELD

In this appendix we present the calculations for squeezing in the case when the atom is in a coherent state and is interacting with the vacuum of the cavity field.

In the context of the squeezed states of the field, we need to evaluate the function defined by

$$S(t)=\langle :(a^\dagger+a)^2: \rangle - \langle a^\dagger + a\rangle^2 \tag{D1}$$

so that $S(t)<0$ signifies squeezing. Since the field is initially empty and since there is at the most one atomic excitation we get $\langle a^2\rangle=\langle (a^\dagger)^2\rangle=0$. Hence, in terms of the elements of the density matrix $\rho(t)$, we may write

$$S(t)=\{\,2\langle 1,-\tfrac{1}{2}\,|\rho(t)|\,1,-\tfrac{1}{2}\rangle - [\langle 1,-\tfrac{1}{2}\,|\rho(t)|\,0,-\tfrac{1}{2}\rangle+\text{c.c.}]^2\}\ . \tag{D2}$$

All the calculations are being done in a frame rotating with the frequency of the cavity field. Now, using the results of Ref. 13 with the initial atomic state given by Eq. (5.1), we find that

$$\langle 1,-\tfrac{1}{2}\,|\rho(t)|\,1,-\tfrac{1}{2}\rangle = g^2\exp(-\kappa t)\sinh(\Gamma t)\sinh(\Gamma^* t)/[(1+|\mu|^2)|\Gamma|^2]\ , \tag{D3}$$

$$\langle 1,-\tfrac{1}{2}\,|\rho(t)|\,0,-\tfrac{1}{2}\rangle = -i\mu^* g\exp(-\kappa t/2)\sinh(\Gamma^* t)\exp(-i\Delta t/2)/[(1+|\mu|^2)\Gamma^*]\ , \tag{D4}$$

where

$$\Gamma=\{[F+(F^2+4\Delta^2\kappa^2)^{1/2}]^{1/2}+i[-F+(F^2+4\Delta^2\kappa^2)^{1/2}]^{1/2}\}/(2\sqrt{2})\ ,$$
$$\equiv x+iy\ , \tag{D5}$$

$$F=\kappa^2-\Delta^2-4g^2,\quad \Delta=\omega_0-\omega\ . \tag{D6}$$

It can be now shown that

$$\langle 1,-\tfrac{1}{2}\,|\rho(t)|\,0,-\tfrac{1}{2}\rangle+\text{c.c.}=2g\exp(-\kappa t/2)(AC+BD)/[(1+|\mu|^2)(x^2+y^2)]\ , \tag{D7}$$

with

$$\begin{aligned}A&=|\mu|[x\cos(\Delta t/2+\varphi)+y\sin(\Delta t/2+\varphi)]\ ,\\ B&=|\mu|[y\cos(\Delta t/2+\varphi)-x\sin(\Delta t/2+\varphi)]\ ,\\ C&=-\cosh(xt)\sin(yt)\ ,\\ D&=\sinh(xt)\cos(yt)\ ,\end{aligned} \tag{D8}$$

and $\mu=|\mu|\exp(i\varphi)$. $S(t)$ can now be calculated by substituting Eq. (D3) and (D7) in Eq. (D2). In particular, if $\Delta=0$, we find[21] that for $\kappa<2g$

$$S(t)=2g^2\exp(-\kappa t)\sin^2(yt)\times[1+|\mu|^2\cos(2\varphi)]/[y^2(1+|\mu|^2)^2]\ , \tag{D9}$$

where $y=(4g^2-\kappa^2)^{1/2}/2$ and for $\kappa>2g$

$$S(t)=2g^2\exp(-\kappa t)\sinh^2(xt)\times[1+|\mu|^2\cos(2\varphi)]/[x^2(1+|\mu|^2)^2]\ , \tag{D10}$$

where $x=(\kappa^2-4g^2)^{1/2}/2$. In either case there is squeezing for all times if $\cos(2\varphi)<0$ and $|\mu|>1$. It is readily found that at any time t, the maximum squeezing is obtained if $\varphi=\pi/2$ and $|\mu|=\sqrt{3}$, i.e., if the probability of finding the atom in the ground state is three times than that in the excited state. Also, for $\kappa=0$ it follows from Eq. (D9) that the minimum attainable value of $S(t)$ is -0.25.

We can also investigate the effects of detuning on squeezing in the case of an ideal cavity ($\kappa=0$). In this case it follows that

$$S(t)=g^2\sin^2(yt)\times[1+|\mu|^2\cos(\Delta t+2\varphi)]/[y^2(1+|\mu|^2)^2]\ , \tag{D11}$$

with $y=(\Delta^2+4g^2)^{1/2}/2$. In particular, for $\Delta\gg 2g$, we have

$$S(t)=4g^2\sin^2(\Delta t/2)\times[1+|\mu|^2\cos(\Delta t+2\varphi)]/(1+|\mu|^2)^2\ . \tag{D12}$$

For $\varphi=0$ and π, it is readily seen that the maximum squeezing occurs when $\Delta t=(2n+1)\pi$ with $|\mu|>1$. The squeezing at each such times is maximum if $|\mu|=\sqrt{3}$ with $S(t)=-g^2/(2\Delta^2)$. However, since Δ is assumed to be much larger than g, the amount of squeezing obtainable is not large in this case.

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