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Structure and properties of the ground state of a two-level system arbitrarily coupled to a boson mode including the counter-rotating terms

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The structure of the ground state of the Hamiltonian of a two-level system linearly coupled to a single mode of a radiation Bose-like field is studied for arbitrary values of the coupling constant including the counter-rotating terms. First of all the Hamiltonian is canonically transformed, introducing a new set of unitary operators in such a way that the eigenstates of the transformed Hamiltonian are exactly factorizable into eigenstates of the new pseudospin $\frac{1}{2}$ and of the new field. The ground state is then found by a variational procedure. The validity of the results obtained by this approach is shown introducing a suitable class of canonical transformations by which it is possible to see that this variational ground state differs from the exact one only for perturbative contributions for which we give explicit expressions. Furthermore, we present investigations on the properties of this system in its ground state based on the calculation of the covariance of suitable pairs of operators. In this way we succeed in obtaining, among other things, a physically transparent meaning for the mathematical variational condition which determines the ground state.

I. INTRODUCTION

It is well known that the simplest model that contains the essential ingredients to understand the physical properties of a system of a few-level object interacting with a radiation Bose-like field is that of a two-level object coupled to a single-mode radiation field by an interaction Hamiltonian which is linear both in the field and in the object coordinates. This model, besides its simplicity, has the value of being quite general; for example, in quantum optics, it describes a two-level atom coupled to a single linearly polarized quantized mode of the electromagnetic field[1] while in solid-state physics it has been used to describe the interaction of a dipolar impurity (paraelectric or paraelastic) with a crystal lattice[2-4] or in connection with the paramagnetic spin-phonon interaction that is a spin $\frac{1}{2}$ in a static external magnetic field, interacting with a single phonon mode.[5] The Hamiltonian assumed to represent this model has the following form:

$$H=\hbar\omega\alpha^{\dagger}\alpha+2\epsilon(\alpha+\alpha^{\dagger})S_x+\hbar\omega_0 S_z \ . \tag{1.1}$$

Here α and $\alpha^{\dagger}$ are the Bose operators for the quantized mode with frequency ω of the radiation field, and S_x and S_z are, respectively, the x and z components of a pseudospin-$\frac{1}{2}$ operator which describes the two levels of the object separated by the energy $\hbar\omega_0$. ϵ is the coupling constant between the two-level system and the field and can be defined as a real non-negative quantity; both its analytical expression in terms of other more direct physical parameters and the range of values which can be assumed depend on the particular system to which (1.1) is applied. In spite of its apparent simplicity, exact closed analytical expressions for the eigenvalues and the eigenvectors of (1.1) for arbitrary values of ω, ω_0, and ϵ are not known. Swain[6] has found a formal exact solution in terms of infinitely continued fractions to the problem of the diagonalization of Hamiltonian (1.1) but its usefulness is greatly limited by the fact that results can be extracted from it only by numerical analysis. Another exact numerical treatment for the low-lying energy levels of (1.1) has been given by Reik *et al.*[7] using an interesting mathematical procedure. Shore and Sanders,[8] using a Hamiltonian as (1.1) to study the problem of an exciton hopping between two sites and coupled to a phonon field, have obtained the ground-state energy of the system but, also in this case, through numerical methods. More recently, Graham and Hohnerbach,[9] investigating the quantum behavior of systems described by (1.1) related to their nonintegrability in the classical limit, have obtained diagrams in which the lowest-lying energy levels are plotted as a function of ω/ω_0 by a numerical diagonalization of the time-independent Schrödinger equation corresponding to (1.1). In order to obtain analytical results numerous approximations have been made on the Hamiltonian model (1.1) according to the specific physical situation to be investigated. In quantum optics the most usual approximation is the rotating-wave approximation which corresponds to the well-known Jaynes-Cummings model;[10] this model can be used under the condition of weak coupling ($\epsilon \ll \hbar\omega_0$) and small detuning ($|\omega-\omega_0| \ll \omega_0$) and shows that the ground state of (1.1) is very similar, in this case, to the empty state. The opposite limit to this approximation, that is, the strong-coupling case ($\epsilon \gg \hbar\omega_0$), has also been investigated in connection with several physical problems using a treatment of (1.1) by which it is possible to see that the ground state in this case is very different from that of the weak-coupling limit.[5] The purpose of this paper is to determine analytical expressions for the eigenvalue and eigenvector of the ground state as well as of some excited states of (1.1) without making any restriction on the value of the coupling constant. Here we shall treat this problem in the case $\omega_0 \leq \omega$; the only reason to do this is the simplification obtained in the presentation of our mathematical procedure which, however, can be generalized to the case $\omega_0 > \omega$. Our method is based on the use of two suitable canonical transformations by which we succeed in rewriting the Hamiltonian (1.1) as a sum of two terms in such a way that one of them can be considered, for any value of ϵ, as a perturbation with respect to the other one while the dominant term can be exactly diagonalized.

II. DECOUPLING OF THE INTERNAL OBJECT MOTION FROM THE FIELD DYNAMICS

We start by making some remarks on the matrix representation of (1.1) on the basis of the simultaneous eigenstates of the operators $\alpha^+\alpha$ and S_z. Let us denote the generic vector of this basis by $|n,\sigma_z\rangle \equiv |n\rangle|\sigma_z\rangle$. It is identified by the conditions $\alpha^\dagger\alpha|n\rangle = n|n\rangle$ and $S_z|\sigma_z\rangle = (\sigma_z/2)|\sigma_z\rangle$ with $\sigma_z = \pm 1$. The excitation number operation $N = \alpha^+\alpha + S_z + \frac{1}{2}$ is diagonal in this basis and its eigenvalues are all the natural numbers. On the contrary, H is not diagonal in the basis $\{|n,\sigma_z\rangle\}$ because of the presence of the interaction term $\epsilon(\alpha+\alpha^+)S_x$; however, $\langle n,\sigma_z|H|n',\sigma_z'\rangle$ vanish if the parity of the number $n+\sigma_z/2+\frac{1}{2}$ is different from the parity of the number $m'+\sigma_z'/2+1/2$. This circumstance implies the possibility of dividing the total Hilbert space into the two subspaces $S_{-1} \equiv \{|n,\sigma_z\rangle$: $n+\sigma_z/2+\frac{1}{2}$ is odd$\}$ and $S_{+1} \equiv \{|n,\sigma_z\rangle$: $n+\sigma_z/2+\frac{1}{2}$ is even$\}$ is such a way that H cannot connect them. It is interesting to observe that within each subspace the value of σ_z is univocally determined by the knowledge of n and this suggests the possibility of finding the eigenstates of H belonging to S_w with $w = \pm 1$ using a purely bosonic w-dependent effective Hamiltonian. To succeed in this objective let's consider the excitation number parity operator $P = e^{i\pi N}$; it is an unitary operator, diagonal in the basis $\{|n,\sigma_z\rangle\}$ and, as its only eigenvalues are ± 1, is also Hermitian. More precisely $P|n,\sigma_z\rangle = w|n,\sigma_z\rangle$ with $w = +1$ for any $|n,\sigma_z\rangle$ belonging to S_{+1} and $w = -1$ for any $|n,\sigma_z\rangle$ belonging to S_{-1}. From these properties it immediately follows that $[P,H]=0$, that is, P is a constant of motion. Then the subdivision of the total Hilbert space into the direct sum of S_{+1} and S_{-1} is equivalent to looking for simultaneous eigenstates of H and P. As $P = -2S_z\cos(\pi\alpha^+\alpha)$ a generic vector belonging to S_w may be written as

$$|\psi_w\rangle = \sum_{n=0}^{\infty} a_n |n, -w\cos(\pi n)\rangle = \sum_{n=0}^{\infty} a_n |n\rangle |\sigma_z = -w\cos(\pi n)\rangle \tag{2.1}$$

and is univocally determined by w and by the bosonic field state $\sum_{n=0}^{\infty} a_n|n\rangle$. We are interested in finding an operator which transforms $|\psi_w\rangle$ into the product of a bosonic field state and a spin state. It is easy to convince oneself that a coordinate rotation about an arbitrary axis perpendicular to the z axis through an angle π or 2π according to the parity of n is the transformation which leads to the factorization of $|\psi_w\rangle$. The operator which accomplishes this coordinate transformation may be chosen in the following form:

$$T = e^{i(\pi/2)\alpha^\dagger\alpha} e^{-i\pi u\cdot s\alpha^\dagger\alpha} , \tag{2.2}$$

where $u = (\cos\varphi, \sin\varphi, 0)$ is the rotation axis versor. Applying T^{-1} to $|\psi_w\rangle$ we obtain

$$T^{-1}|\psi_w\rangle = \sum_{n=0}^{\infty} e^{-i(\pi/2)\alpha^\dagger\alpha} e^{i\pi u\cdot s\alpha^\dagger\alpha} a_n |n\rangle \times |\sigma_z = -w\cos(\pi n)\rangle = \left[\sum_{n=0}^{\infty} e^{-iw\varphi\sin^2(n\pi/2)} a_n |n\rangle\right] |\sigma_z = -w\rangle . \tag{2.3}$$

From the properties of T it follows that by submitting H to the canonical transformation $T^{-1}HT$ we shall obtain a transformed Hamiltonian $\tilde{H}$ whose eigenvectors can be expressed in a factorized form. In other words, by this canonical transformation we may realize exactly the passage from the system of a two-level atom coupled to a radiation field, described by (1.1), to a system of a new pseudospin $\frac{1}{2}$ interacting with a new bosonic field described by the transformed Hamiltonian where the internal motion of the atom can be easily and exactly separated from the field dynamics. In fact, using (2.2) we immediately obtain

$$T^{-1}\alpha^\dagger\alpha T = \alpha^\dagger\alpha , \tag{2.4}$$

$$T^{-1}(\alpha+\alpha^\dagger)S_x T = (\alpha^\dagger+\alpha)[\tfrac{1}{2}\cos\varphi - iS_z\sin\varphi\cos(\pi\alpha^\dagger\alpha)] , \tag{2.5}$$

$$T^{-1}S_z T = S_z\cos(\pi\alpha^\dagger\alpha) . \tag{2.6}$$

From (2.4), (2.5), and (2.6) immediately follows the explicit expression of $\tilde{H}(\varphi) = T^{-1}HT$,

$$\tilde{H}(\varphi) = \hbar\omega\alpha^\dagger\alpha + 2\epsilon(\alpha+\alpha^\dagger)[\tfrac{1}{2}\cos\varphi - iS_z\sin\varphi\cos(\pi\alpha^\dagger\alpha)] + \hbar\omega_0 S_z\cos(\pi\alpha^\dagger\alpha) . \tag{2.7}$$

(2.7) shows that $[\tilde{H}(\varphi), S_z] = 0$ for any φ so that the eigenvectors of $\tilde{H}(\varphi)$ can be written as the product of an eigenstate of S_z and an eigenstate of the following bosonic Hamiltonian:

$$\tilde{H}(\varphi) = \hbar\omega\alpha^\dagger\alpha + \epsilon(\alpha+\alpha^\dagger)[\cos\varphi - i\sigma_z\sin\varphi\cos(\pi\alpha^\dagger\alpha)] + \frac{\hbar\omega_0}{2}\sigma_z\cos(\pi\alpha^\dagger\alpha) , \tag{2.8}$$

which depends parametrically on σ_z. As $T^{-1}PT = -2S_z$ the possibility of classifing the eigenstates of (2.7) by σ_z is equivalent to the possibility of classifing the eigenstates of H by w. Up to now no restriction has been imposed on φ. If, for some particular value of φ, $\tilde{H}(\varphi)$ assumes a more tractable form, we may give this value to φ in order to work with a simpler Hamiltonian. It is easy to convince oneself that the most convenient value of φ is $\varphi=0$ which corresponds to rotating the coordinate system about the x axis. Setting $\varphi=0$ we finally obtain

$$\widetilde{H}=\widetilde{H}(\varphi)=\hbar\omega\alpha^\dagger\alpha+\epsilon(\alpha+\alpha^\dagger)+\frac{\hbar\omega_0}{2}\sigma_z\cos(\pi\alpha^\dagger\alpha) . \tag{2.9}$$

Hamiltonian (2.9) has been obtained in a very similar form by Shore and Sanders[8] and successively proposed again by other authors[9,11] without any modification. The way through which we succeed in getting (2.9) clearly shows that it is a member of the more general family (2.8).

III. VARIATIONAL GROUND STATE OF $\widetilde{H}$

In this section we look for the ground state of $\widetilde{H}$ by using a variational approach. The unitary operator

$$V(\eta)=e^{\eta(\alpha^\dagger-\alpha)} , \tag{3.1}$$

where η is a real number, is the creation operator for the coherent radiation state $|\eta\rangle$, that is,

$$|\eta\rangle=V(\eta)|0\rangle . \tag{3.2}$$

If $\epsilon=0$ the ground state of (2.9) is $|0\rangle$ and then it has the form (3.2) with $\eta=0$; if, on the other hand, we consider $\epsilon\gg\hbar\omega_0$ and $\hbar\omega\gg\hbar\omega_0$ the ground state can be approximately written in the form (3.2) with $\eta=-\epsilon/\hbar\omega$. It is then natural to use as a trial state for the ground state of $\widetilde{H}$ the coherent state $|\eta\rangle$ (3.2) taking the parameter η as a variational parameter. The expectation value of $\widetilde{H}$ on this state has the following form:

$$\begin{aligned} E(\eta)&=\langle 0|V^\dagger(\eta)[\hbar\omega\alpha^\dagger\alpha+\epsilon(\alpha+\alpha^\dagger)\\ &\qquad+\frac{\hbar\omega_0}{2}\sigma_z\cos(\pi\alpha^\dagger\alpha)]V(\eta)|0\rangle\\ &=\langle\eta|\hbar\omega\alpha^\dagger\alpha+\epsilon(\alpha+\alpha^\dagger)|\eta\rangle\\ &\qquad+\left\langle\eta\left|\frac{\hbar\omega_0}{2}\sigma_z\cos(\pi\alpha^\dagger\alpha)\right|\eta\right\rangle . \end{aligned} \tag{3.3}$$

We immediately have

$$\langle\eta|\hbar\omega\alpha^\dagger\alpha+\epsilon(\alpha+\alpha^\dagger)|\eta\rangle=\hbar\omega\eta^2+2\epsilon\eta . \tag{3.4}$$

To evaluate the mean value of $\cos(\pi\alpha^\dagger\alpha)$ we first transform this operator as follows:

$$\cos(\pi\alpha^\dagger\alpha)=\sum_{l=0}^{\infty}\frac{2^l(-1)^l}{l!}(\alpha^\dagger)^l(\alpha)^l . \tag{3.5}$$

We then have

$$\begin{aligned}\langle\eta|\cos(\pi\alpha^\dagger\alpha)|\eta\rangle&=\sum_{l=0}^{\infty}(-1)^l\frac{2^l}{l!}\langle\eta|(\alpha^\dagger)^l(\alpha)^l|\eta\rangle\\ &=\sum_{l=0}^{\infty}(-1)^l\frac{2^l}{l!}\eta^{2l}=e^{-2\eta^2} . \end{aligned} \tag{3.6}$$

Using (3.4) and (3.6) in (3.3) we obtain

$$\overline{E}(\eta)=\eta^2-2\gamma\eta+\tfrac{1}{2}\beta e^{-2\eta^2} , \tag{3.7}$$

where

$$\overline{E}(\eta)=\frac{E(\eta)}{\hbar\omega},\quad \gamma=-\frac{\epsilon}{\hbar\omega},\quad \beta=\frac{\omega_0}{\omega}\sigma_z . \tag{3.8}$$

There certainly exist a value of η for which $\overline{E}(\eta)$ reaches its absolute minimum. This value of η is among the roots of the following equation:

$$\frac{d\overline{E}(\eta)}{d\eta}=0 \quad\text{that is}\quad \gamma=\eta(1-\beta e^{-2\eta^2}) . \tag{3.9}$$

The number and the sign of the solutions of Eq. (3.9) depends on the value of γ and β. It is easy to convince oneself that, for $|\beta|\le 1$, (3.9) has one and only one root for any value of γ and that this root is always a nonpositive real number. If we denote this root by $\overline{\eta}=\overline{\eta}(\beta,\gamma)$ it may be shown directly from (3.9) that when $|\gamma|$ increases from 0 to $+\infty$, $\overline{\eta}(\beta,\gamma)$ monotonically decreases from 0 to $\gamma<0$. From (3.9) we immediately deduce

$$\begin{aligned}&\overline{\eta}(|\beta|,\gamma)-\overline{\eta}(-|\beta|,\gamma)\\ &\quad=|\beta|[\overline{\eta}(|\beta|,\gamma)e^{-2\overline{\eta}^2(|\beta|,\gamma)}\\ &\qquad\qquad+\overline{\eta}(-|\beta|,\gamma)e^{-2\overline{\eta}^2(-|\beta|,\gamma)}] ,\end{aligned} \tag{3.10}$$

which clearly shows that

$$\overline{\eta}(|\beta|,\gamma)\le\overline{\eta}(-|\beta|,\gamma) , \tag{3.11}$$

both being nonpositive numbers. Using (3.7) we may easily write

$$\begin{aligned}\overline{E}_+-\overline{E}_-&=\eta_+^2-\eta_-^2-2\gamma(\eta_+-\eta_-)\\ &\quad+\tfrac{1}{2}|\beta|(e^{-2\eta_+^2}+e^{-2\eta_-^2}) ,\end{aligned} \tag{3.12}$$

where, to save some writing, we set

$$\begin{aligned}&\eta_\pm=\overline{\eta}(\pm|\beta|,\gamma) ,\\ &\overline{E}_\pm=\overline{E}(\overline{\eta}_\pm) .\end{aligned} \tag{3.13}$$

From (3.9) it is not difficult to obtain

$$\eta_+^2-\eta_-^2=\beta^2(\eta_+^2e^{-4\eta_+^2}-\eta_-^2e^{-4\eta_-^2})+2\gamma(\eta_+-\eta_-) . \tag{3.14}$$

Inserting (3.14) into (3.12) yields

$$\begin{aligned}\overline{E}_+-\overline{E}_-&=|\beta|(|\beta|\eta_+^2e^{-4\eta_+^2}+\tfrac{1}{2}e^{-2\eta_+^2})\\ &\quad+\tfrac{1}{2}|\beta|e^{-2\eta_-^2}(1-2|\beta|\eta_-^2e^{-2\eta_-^2}) .\end{aligned} \tag{3.15}$$

As $1-2|\beta|\eta_-^2e^{-2\eta_-^2}>0$ we obtain

$$\overline{E}_+>\overline{E}_- .$$

Thus, in the limit of this variational approach, we deduce that the ground state of (2.9) belongs, for $\omega_0\le\omega$, to the subspace with $\sigma_z=-1$, whatever the strength of the coupling. When $|\beta|>1$ the situation is more complicated because now there exist values of η for which Eq. (3.9) admits three solutions. Generally speaking, a very serious difficulty connected with any variational calculation is the fact that, in the obvious absence of the exact solution of the problem, it is almost impossible to judge the validity of the results obtained by this approach. In particular, even if it gives good values for the energy, the approximate state vector may preset certain completely unpredictable erroneous features which may be very difficult to check. An important consequence of this fact is that when we calculate the expectation value of an operator other than the Hamiltonian using the trial ket, we cannot in general be sure of the physical validity of the result obtained. In Sec. IV by the introduction of a suitable class of unitary transformations, we shall show that the trial ket (3.2), with η given by (3.9), gives a good approximation of the ground-state vector of (2.9) for $|\beta| \leq 1$.

IV. CONNECTION BETWEEN THE VARIATIONAL GROUND STATE AND THE EXACT ONE

To investigate the validity of our variational procedure let us consider the following unitary transformation:

$$V(\overline{\eta}(\beta,\gamma)) \equiv V(\overline{\eta}) = e^{\overline{\eta}(\alpha^\dagger - \alpha)} \tag{4.1}$$

obtained from (3.1) substituting for η with the root of (3.9). If we transform $\widetilde{H}$ by $V(\overline{\eta})$ we easily have

$$V^{-1}(\overline{\eta})\widetilde{H}V(\overline{\eta}) \equiv \mathscr{H} = \mathscr{H}_0 + \mathscr{H}_1 \,, \tag{4.2}$$

where

$$\mathscr{H}_0 = \hbar\omega\alpha^\dagger\alpha + \hbar\omega\overline{\eta}^2 + 2\epsilon\overline{\eta} + \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2} \,, \tag{4.3}$$

$$\mathscr{H}_1 = (\hbar\omega\overline{\eta} + \epsilon)(\alpha^\dagger + \alpha) + \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}(\alpha^\dagger - \alpha)}\cos(\pi\alpha^\dagger\alpha) - \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2} \,. \tag{4.4}$$

From (4.3) we see that $\mathscr{H}_0$ is in diagonal form and that its eigenstates are the eigenstates $|n\rangle$ of $\alpha^\dagger\alpha$ while its eigenvalues E_n have the following expression:

$$E_n = n\hbar\omega + \hbar\omega\overline{\eta}^2 + 2\epsilon\overline{\eta} + \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2} \,. \tag{4.5}$$

In particular the ground state of $\mathscr{H}_0$ has energy E_0 given by

$$E_0 = \hbar\omega\overline{\eta}^2 + 2\epsilon\overline{\eta} + \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2} = \hbar\omega\overline{E}(\overline{\eta}) \tag{4.6}$$

and then coincides with the variationally calculated energy $\hbar\omega\overline{E}(\overline{\eta})$ obtained from (3.7) setting $\eta = \overline{\eta}$ according to (3.9). This coincidence is not limited to the energy eigenvalue; in fact, the ground state of $\mathscr{H}_0$ is the radiation vacuum state which, transformed by $V(\overline{\eta})$, gives just $V(\overline{\eta})|0\rangle$, that is, the state variationally obtained from (3.2) setting $\eta = \overline{\eta}$. Obviously the presence of $\mathscr{H}_1$ in (4.2) does not allow us to say that $V(\overline{\eta})|0\rangle$ is exactly the ground state of $\widetilde{H}$. However, it is possible to see that there are very strong suggestions to believe that $V(\overline{\eta})|0\rangle$ differs very little from the exact ground state of $\widetilde{H}$; in fact, we shall show that, whatever the intensity of $|\gamma|$, $\mathscr{H}_1$ satisfies a general necessary condition in order that conventional perturbation theory is applicable to it with respect to $\mathscr{H}_0$. To prove this let us begin by observing that, being that $E_{n+1} - E_n = \hbar\omega$ from (4.5), the energy spectrum of $\mathscr{H}_1$ does not present any degeneration, in particular none in its ground-state eigenvalue. Thus, if perturbation theory is applicable to treat the effect of $\mathscr{H}_1$ on the ground state of $\mathscr{H}_0$, we can refer to the theory for the nondegenerate case and this for any value of $|\gamma|$.

The first-order correction $E_0^{(1)}$ to the energy E_0 is simply equal to the mean value of $\mathscr{H}_1$ in the ground state of $\mathscr{H}_0$:

$$E_0^{(1)} = \langle 0|\mathscr{H}_1|0\rangle = \frac{\hbar\omega_0}{2}\sigma_z(\langle 0|e^{-2\overline{\eta}(\alpha^\dagger - \alpha)}|0\rangle - e^{-2\overline{\eta}^2}) \,. \tag{4.7}$$

As

$$e^{-2\overline{\eta}(\alpha^\dagger - \alpha)} = e^{-2\overline{\eta}^2} e^{-2\overline{\eta}\alpha^\dagger} e^{2\overline{\eta}\alpha} \,, \tag{4.8}$$

we easily have

$$\langle 0|e^{-2\overline{\eta}(\alpha^\dagger - \alpha)}|0\rangle = e^{-2\overline{\eta}^2}\langle 0|e^{-2\overline{\eta}\alpha^\dagger}e^{2\overline{\eta}\alpha}|0\rangle = e^{-2\overline{\eta}^2} \tag{4.9}$$

so that

$$E_0^{(1)} = 0 \,. \tag{4.10}$$

The first-order correction $|\psi_0^{(1)}\rangle$ to the ground state $|0\rangle$ of $\mathscr{H}_0$ has the form

$$|\psi_0^{(1)}\rangle = \sum_{m=1}^{\infty} \frac{\langle m|\mathscr{H}_1|0\rangle}{E_0 - E_m}|m\rangle \,. \tag{4.11}$$

From (4.4) and (4.8) we have for $m \neq 0$,

$$\langle m|\mathscr{H}_1|0\rangle = \left\langle m \left| (\hbar\omega\overline{\eta} + \epsilon)\alpha^\dagger + \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2} e^{-2\overline{\eta}\alpha^\dagger} \right| 0 \right\rangle \,, \tag{4.12}$$

when $m = 1$ (4.12) gives

$$\begin{aligned}\langle 1|\mathscr{H}_1|0\rangle &= \left\langle 1 \left| (\hbar\omega\overline{\eta} + \varepsilon)\alpha^\dagger + \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2}(-2\overline{\eta}\alpha^\dagger) \right| 0 \right\rangle \\ &= \langle 1|(\hbar\omega\overline{\eta} + \epsilon - \hbar\omega_0\sigma_z\overline{\eta}e^{-2\overline{\eta}^2})\alpha^\dagger|0\rangle = 0\end{aligned} \tag{4.13}$$

using (3.9). Thus we see that the variational calculation gives just that value of $\overline{\eta}$ which eliminates the mixing of $|0\rangle$ and $|1\rangle$ from $\mathscr{H}_1$. When $m > 1$ we have from (4.12)

$$\begin{aligned}\langle m|\mathscr{H}_1|0\rangle &= \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2}\langle m|e^{-2\overline{\eta}\alpha^\dagger}|0\rangle \\ &= \frac{\hbar\omega_0}{2}\sigma_z e^{-2\overline{\eta}^2}\left\langle m \left| \frac{(-2\overline{\eta})^m(\alpha^\dagger)^m}{m!} \right| 0 \right\rangle \\ &= \frac{\hbar\omega_0}{2}\frac{\sigma_z}{\sqrt{m!}}e^{-2\overline{\eta}^2}(-2\overline{\eta})^m \\ &= \frac{\hbar\omega}{2}\beta\frac{(-2\overline{\eta})^m}{\sqrt{m!}}e^{-2\overline{\eta}^2} \,.\end{aligned} \tag{4.14}$$

Substituting (4.13) and (4.14) in (4.11) we obtain

$$|\psi_0^{(1)}\rangle=\sum_{m=2}^{\infty}\beta\frac{e^{-2\bar\eta^2}(-2\bar\eta)^m}{2m\sqrt{m!}}|m\rangle\ . \tag{4.15}$$

At this point we have to investigate the applicability of the perturbation theory to our problem. To this end it is usual to limit oneself to verify that the nondiagonal matrix element of $\mathscr{H}_1$ are much smaller then the corresponding unperturbed energy differences. It is important to observe, however, that this condition is very weak because not only it is obviously not sufficient for the convergence of the perturbative method, but it is not even sufficient to assure that the vector $|0\rangle+|\psi_0^{(1)}\rangle$ is normalized to first order in $\mathscr{H}_1$. To satisfy this last condition we must in fact prove that

$$\sum_{m=2}^{\infty}\left|\frac{\beta e^{-2\bar\eta^2}(-2\bar\eta)^m}{2m\sqrt{m!}}\right|^2\ll 1\ , \tag{4.16}$$

which imply the most usual condition $|\langle m|\mathscr{H}_1|0\rangle|\ll|E_n-E_0|$ for any $m>0$. We now show that condition (4.16) is satisfied whatever the value of $\bar\eta$. Setting $4\bar\eta^2=|x|$, condition (4.16) assumes the following form:

$$f(|x|)\equiv\sum_{m=2}^{\infty}\frac{e^{-|x|}}{4}\frac{|x|}{m^2m!}\beta^2\ll 1\ . \tag{4.17}$$

The series in (4.17) is convergent for any x. Moreover, since $m^2>m+1$ when $m>1$, we have

$$\sum_{m=2}^{\infty}\frac{e^{-|x|}}{4}\frac{|x|^m}{m^2m!}\le\sum_{m=2}^{\infty}\frac{e^{-|x|}}{4}\frac{|x|^m}{(m+1)m!}$$
$$=\frac{e^{-|x|}}{4}\frac{e^{-x}-1-|x|-\frac{x^2}{2}}{|x|}\ . \tag{4.18}$$

Then we have

$$0\le f(x)\le|\beta|\frac{1-e^{-|x|}\left[1+|x|+\frac{x^2}{2}\right]}{4|x|}\equiv|\beta|g(x)\ . \tag{4.19}$$

The function $g(x)$ is convergent to zero when $x\to 0$ and when $x\to\infty$ and consequently, being non-negative, has an absolute maximum which represent a superior limit for $f(|x|)$. By using standard methods it is not difficult to see that $g(x)$ reaches its absolute maximum for $x\simeq 5.63$ and that $g(5.63)\simeq 4\times 10^{-2}$. Then we have $f(|x|)<4\times 10^{-2}|\beta|$, that is,

$$\sum_{m=2}^{\infty}\left|\frac{\beta e^{-2\bar\eta^2}(-2\bar\eta)^m}{2m\sqrt{m!}}\right|^2<4|\beta|\times 10^{-2}\ll 1\ , \tag{4.20}$$

since $|\beta|\le 1$ whatever the value of $\bar\eta$ and then whatever the intensity of the coupling. Condition (4.20), although not sufficient to assure the convergence of the perturbative series, is at least sufficient to say that $|\psi_0^{(1)}\rangle$ is much smaller than $|0\rangle$ and this fact is usually accepted as a strong reason to believe reasonable the application of perturbation methods in situations analogous to our case. It is possible to obtain a closed exact expression for the second-order correction $E_0^{(2)}$ to the energy. We have

$$E_0^{(2)}=\sum_{m=2}^{\infty}\frac{|\langle m|\mathscr{H}_1|0\rangle|^2}{E_0^{(0)}-E_m^{(0)}}$$
$$=-\sum_{m=2}^{\infty}\beta^2\hbar\omega\frac{e^{-|x|}}{4}\frac{|x|^m}{mm!}\ . \tag{4.21}$$

It is easy to show that

$$E_0^{(2)}=-\beta^2\frac{\hbar\omega}{4}e^{-4\bar\eta^2}\int_0^{4\bar\eta^2}\frac{e^t-1-t}{t}dt\ . \tag{4.22}$$

The meaning of the results obtained in this section is that the variationally ground state of $\tilde H$, that is, $V(\bar\eta)|0\rangle$, although approximate, differs from the exact one for contributions which can be considered as perturbative for any value of the coupling constant and for $|\beta|\le 1$. Thus if we denote by $|\psi_w^g\rangle$ the lowest energy eigenstate of H given by (1.1) belonging to S_w, we may write

$$|\psi_w^g\rangle=TV(\eta_{-w})|0\rangle|\sigma_z=-w\rangle+\cdots\ , \tag{4.23}$$

where the ellipse represents the perturbative correction, $\eta_{-w}=\eta_{-1}$ if $w=+1$ [(4.23) in this case gives the ground state of H], and $\eta_{-w}=\eta_{+1}$ if $w=-1$. This circumstance makes legitimate an investigation on the physical properties of the system in its ground state based on the calculation of the mean value of operators other then the Hamiltonian. We shall consider this point in detail in Sec. V.

V. PHYSICAL PROPERTIES OF THE STATES $|\psi_w^g\rangle$

In this section we want to take advantage of the knowledge of the ground state of (1.1) in accordance with (4.23) to study the effects of the interaction between the two-level system and the field mode on the dynamics of these two subsystems as a function of ϵ. To reach this objective we shall introduce suitable operators whose expectation values on $|\psi_w^g\rangle$ prove themselves to be very useful to put into evidence the mutual influence of the two interacting subsystems. In order to make simpler the physical meaning of the analytical results which we shall obtain later, it is convenient to describe the field coordinates in (1.1) referring to a mechanical harmonic oscillator. This procedure is justified by the fact that, passing in the Hamiltonian

$$H=\frac{p^2}{2m}+\frac{K}{2}q^2+2F_0qS_x+\hbar\omega_0S_z \tag{5.1}$$

from the variables q and p to the new variables α and $\alpha^\dagger$ using

$$q=\left[\frac{\hbar}{2m\omega}\right]^{1/2}(\alpha+\alpha^\dagger)\ ,\quad p=i\left[\frac{m\omega\hbar}{2}\right]^{1/2}(\alpha^\dagger-\alpha)\ , \tag{5.2}$$

we obtain the Hamiltonian model (1.1) apart from a dynamically inessential constant term provided that

$$F_0=\varepsilon\left[\frac{2m\omega}{\hbar}\right]^{1/2}, \quad \omega=\left[\frac{k}{m}\right]^{1/2} . \tag{5.3}$$

Using (5.1) in the Heisenberg equation of motion for the coordinate operator q of the harmonic oscillator, we immediately get the following expression for the velocity and acceleration operators:

$$\frac{dq}{dt}=\frac{i}{\hbar}[H,q]=\frac{p}{m} , \tag{5.4}$$

$$\frac{d^2q}{dt^2}=\frac{1}{m}\frac{dp}{dt}=-\omega^2 q-\frac{2F_0S_x}{m} . \tag{5.5}$$

Looking at (5.5) we may see that the first term in its right-hand side is proportional to the operator associated with the elastic force of the spring,

$$F_1=-kq , \tag{5.6}$$

while the second term must represent the operator associated with the force that the two-level system exerts on the oscillator through their interaction. We shall denote this force by F_2, setting

$$F_2=-2F_0S_x \tag{5.7}$$

and sometimes we shall refer to it as to the external force on the harmonic oscillator. It is also useful to consider the operator

$$\frac{dF_2}{dt}=2F_0\omega_0 S_y , \tag{5.8}$$

which describes the variation for unitary time of the external force acting on the oscillator. In (5.8) S_y is the y component of the pseudospin operator **S**. All the operators introduced above change their sign when transformed by P; this fact, together with the commutation between H and P, implies that the expectation values of these operators, calculated on arbitrary eigenstates of P, are zero at any time. To get informations on the properties of the system in these states we then calculate the variances of these operators obtaining, in general, results which are not null. However, while the variances of q and p are easily interpretable from a physical point of view, the first being related to the mean modulus of the oscillator's elongation and the second to its mean kinetic energy, the variance of the operators $-2F_0S_x$ and $-d(2F_0S_x)/dt$, although not null, do not give any characteristic indication, being independent of the eigenstate of P used for the calculation. These difficulties, related to the specific symmetry of our Hamiltonian model, can be circumvented by calculating the quantum covariance of suitable pairs of operators, say A and B, with A relative to the two-level subsystem and B relative to the harmonic oscillator subsystem. It is useful to remember[12] that the quantum covariance C_{AB} of two operators A and B on an arbitrary state, on which their mean values are $\langle A\rangle$ and $\langle B\rangle$, is defined in the following way:

$$C_{AB}=\langle \tfrac{1}{2}(AB+BA)\rangle-\langle A\rangle\langle B\rangle . \tag{5.9}$$

(5.9) shows that if the physical quantities associated with A and B are not correlated on a given state then $C_{AB}=0$. This implies that to know the quantum covariance of A and B may give useful indications of how A and B are correlated. It is convenient to introduce the three following pairs of operators:

$$A_1=-2F_0S_x, \quad B_1=-Kq , \tag{5.10}$$

$$A_2=\frac{d(-2F_0S_x)}{dt}, \quad B_2=p , \tag{5.11}$$

$$A_3=\hbar\omega_0 S_z, \quad B_3=\hbar\omega a^\dagger a . \tag{5.12}$$

The calculation of the covariance of the three pairs of operators (5.10), (5.11), and (5.12) on the state $|\psi_w^g\rangle$ yields

$$C_{A_1B_1}=\langle\psi_w^g|2F_0KqS_X|\psi_w^g\rangle=2m\epsilon\omega^2\eta_{-w} , \tag{5.13}$$

$$C_{A_2B_2}=\langle\psi_w^g|2pF_0\omega_0S_y|\psi_w^g\rangle=2wm\omega\omega_0\eta_{-w}e^{-2\eta_w^2} , \tag{5.14}$$

$$C_{A_3B_3}=\hbar^2\omega\omega_0\eta_{-w}^2 e^{-2\eta_w^2} . \tag{5.15}$$

To obtain (5.13) and (5.14) we have used the fact that in every eigenstate of P (as $|\psi_w^g\rangle$) the mean values of the operators (5.10) and (5.11) are null. From (5.13) and (5.14) we easily deduce that, while the physical quantities represented by F_1 and F_2 are correlated whatever coupling regime we consider, those represented by the operators (5.11) reach a maximum of correlation for $|\eta_w|<0.5$ becoming less and less correlated when $|\eta_w|\gg 0.5$. Deductions very similar to the last one can be inferred from (5.15) concerning the correlation between the two subsystem's energy fluctuations. Moreover, (5.13) clearly shows that in the state $|\psi_w^g\rangle$, with $w=\pm 1$, the external force on the oscillator and the elastic force have opposite directions; (5.14), on the other hand, makes it plain that the signs of the time derivative of F_2 and of the oscillator's velocity are opposite in the state $|\psi_{+1}^g\rangle$ and equal in the state $|\psi_{-1}^g\rangle$. The interest toward the calculation of the expectation values of the operators $(-Kq)(-2F_0S_x)$ and $p[d(-2F_0S_x)/dt]$, given, respectively, by (5.13) and (5.14) is twofold: on the one hand, as we have shown, it gives useful indications on the behavior of the correlation existing between the field dynamics and the two-level dynamics in the ground state in the transition from the weak to the strong coupling; on the other hand, as we shall see, it allows us to give a physical interpretation of the mathematical variational condition on η expressed by (3.9). To obtain this result let us start by showing that the expectation values of $(-kq)(-2F_0S_x)$ and $p[d(-2F_0S_x)/dt]$ on exact eigenstates of (5.1) are not independent but are connected by an interesting physical condition. Let us consider, in fact, the following operators:

$$(1) \quad \frac{d\hbar\omega a^\dagger a}{dt}=\frac{p}{m}F_2 , \tag{5.16}$$

which represent the power exchanged by the harmonic oscillator subsystem;

$$(2) \quad \frac{d(\hbar\omega_0 S_x)}{dt}=q\frac{dF_2}{dt} , \tag{5.17}$$

which gives the power exchanged by the two-level subsystem;

$$(3)\quad \frac{d^2(\hbar\omega\alpha^\dagger\alpha)}{dt^2}=-2F_0S_x\frac{d\left[\frac{p}{m}\right]}{dt}+\frac{p}{m}\frac{d(-2F_0S_x)}{dt}\ , \tag{5.18}$$

which describes the time derivative of the power exchanged by the harmonic oscillator subsystem. From (5.18) we immediately deduce that this power may change with time either in connection with changes in the harmonic oscillator's velocity or in connection with changes in the external force. Substituting (5.5) into (5.18) we have

$$\frac{d^2\hbar\omega\alpha^\dagger\alpha}{dt^2}=\frac{d\left[-2F_0S_x\frac{p}{m}\right]}{dt}=\frac{2KF_0S_x}{m}q+\frac{F_0^2}{m}+\frac{p}{m}\frac{d(-2F_0S_x)}{dt}\ . \tag{5.19}$$

Let us point out the physical meaning of the three operators which appear in the right-hand side of (5.19). The operators

$$\frac{2KF_0S_x}{m}q \tag{5.20}$$

and

$$\frac{F_0^2(2S_x)^2}{m}=\frac{F_0^2}{m} \tag{5.21}$$

represent the contributions to the operator $d[-p(F_0 2S_x/m)]/dt$ which stem from that part of the acceleration operator due, respectively, to the harmonic oscillator's restoring force and to the external force F_2 acting on the oscillator subsystem. The operator

$$\frac{p}{m}\frac{d(-2F_0S_x)}{dt}=\frac{2\omega_0F_0}{m}S_yp \tag{5.22}$$

is due to the presence of the term $\hbar\omega_0S_z$ in the Hamiltonian (5.1); this operator, in fact, does not commute with $F_2=-2F_0S_x$ and therefore is responsible for the appearance in (5.16) of the time derivative of the external force on the oscillator. Looking at the operators (5.20) and (5.22) we immediately see that they are proportional to the operators $(-kq)(-2F_0S_x)$ and $p[d(-2F_0S_x)/dt]$ so that we may see that Eq. (5.19) introduces a physical connection between them. In particular, if we evaluate the mean value of both sides of (5.19) on an exact simultaneous eigenstate of (5.1) and P, here denoted by $|\psi_w\rangle_{\text{ex}}$, we immediately deduce the existence of the following simple and physically transparent relation between the expectation values of these two operators:

$$\left[\left\langle\psi_w\left|\frac{d^2\hbar\omega\alpha^\dagger\alpha}{dt^2}\right|\psi_w\right\rangle\right]_{\text{ex}}=\left[\left\langle\psi_w\left|\frac{2KS_x}{m}q\right|\psi_w\right\rangle\right]_{\text{ex}}+\frac{F_0^2}{m}-\left[\left\langle\psi_w\left|\frac{p}{m}\frac{d(2F_0S_x)}{dt}\right|\psi_w\right\rangle\right]_{\text{ex}}=0\ . \tag{5.23}$$

(5.23) clearly shows that if the oscillator velocity and the time derivative of F_2 are not correlated then

$$(\langle\psi_w\,|\,2F_0S_xkq\,|\,\psi_w\rangle)_{\text{ex}}=-F_0^2\ , \tag{5.24}$$

which should be compared with (5.13). Moreover, if there exist suitable values of the parameters of the system such that F_1 and F_2 are not correlated, then necessarily we obtain

$$\left\langle\psi_w\left|\frac{p}{m}\frac{d(-2F_0S_x)}{dt}\right|\psi_w\right\rangle_{\text{ex}}=-\frac{F_0^2}{m}\ , \tag{5.25}$$

that is, the oscillator's velocity and the time derivative of F_2 are remarkably correlated and always have opposite signs. Comparing (5.25) with (5.14) and simultaneously looking at (5.13) we see that for $\omega_0\le\omega$ and in the ground state of (5.1) the system cannot reach conditions similar to those expressed by (5.25) for any coupling regime. It is, however, legitimate to ask whether an opposite behavior should be expected when $\omega_0>\omega$ and for a suitable value of the coupling strength, being impossible to exclude that for such values of the parameters the contribution due to the time derivative of F_2 may exercise a remarkable influence on the physical properties of the system. After this digression let us return to our original problem which consists of expressing the variational condition for the ground state of (5.1) in terms of a physically clear relation among expectation values of operators relative to the two interacting subsystems. Let us note that the states $|\psi_w^g\rangle$ have at $t=0$ the following properties:

$$\left\langle\psi_w^g\left|\frac{d\hbar\omega\alpha^\dagger\alpha}{dt}\right|\psi_w^g\right\rangle=0\ , \tag{5.26}$$

$$\left\langle\psi_w^g\left|\frac{d\hbar\omega_0S_z}{dt}\right|\psi_w^g\right\rangle=0\ , \tag{5.27}$$

$$\left\langle\psi_w^g\left|\frac{d^2\hbar\omega\alpha^\dagger\alpha}{dt^2}\right|\psi_w^g\right\rangle=0\ , \tag{5.28}$$

which can be considered as necessary conditions for the stationariety of the state $|\psi_w^g\rangle$. It is possible to show that the conditions (5.26) and (5.27) can be satisfied from a larger class of variational states than that examined in Sec. III. The condition (5.28), on the contrary, is much more restrictive; calculating, in fact, the mean value of both members of (5.19) on the states $|\psi_w^g\rangle$ at $t=0$ we have

$$\left\langle\psi_w^g\left|\frac{d^2\hbar\omega\alpha^\dagger\alpha}{dt^2}\right|\psi_w^g\right\rangle=2\epsilon\omega^2\eta_{-w}+2\epsilon^2\frac{\omega}{\hbar}+2w\omega_0\omega\epsilon\eta_{-w}e^{-2\eta_{-w}^2}=0\ , \tag{5.29}$$

where we have

$$\left\langle \psi_w^g \left| \frac{2KF_0S_x}{m} q \right| \psi_w^g \right\rangle = 2\epsilon\omega^2\eta_{-w} , \tag{5.30}$$

$$\left\langle \psi_w^2 \left| \frac{F_{0_2}}{m} \right| \psi_w^g \right\rangle = \frac{2\epsilon^2\omega}{\hbar} , \tag{5.31}$$

$$\left\langle \psi_w^g \left| \frac{p}{m} \frac{d(-2F_0S_x)}{dt} \right| \psi_w^g \right\rangle = 2w\omega_0\omega\epsilon\eta_{-w} e^{-2\eta_{-w}^2} . \tag{5.32}$$

As Eq. (5.29) coincides with the variational condition (3.9) for the unknown η, we immediately deduce that the necessary condition (5.28) should not be satisfied by the states (4.23) at the zeroth order if the parameter η were not just η_{-w}. We may therefore say, having in mind (5.29), that the variational condition (3.9) is interpretable from a physical point of view in terms of a condition on the expectation value of the time derivative of the power exchanged by the harmonic oscillator subsystem. Moreover, using (5.30), (5.31), and (5.32), we may associate a well-defined physical meaning with each one of the three contributions appearing in (5.29) or (3.9) and this circumstance, in conclusion, implies that the states $|\psi_w^g\rangle$ can satisfy (5.28) at the zeroth order only because there is a balance among the covariances of pairs of suitable operators relative to the two interacting subsystems.

VI. CONCLUSION

In this concluding section we wish to sum up the results obtained in this paper and indicate some possible developments. We have taken up the problem of finding the ground state of a two-level object interacting with a single resonant or nonresonant mode of a Bose-like field without making any assumption on the coupling regime and taking into account the counter-rotating terms. The reasons which led us to reject the rotating-wave approximation in this paper are two: the first one is that our principal objective consists in obtaining analytical results useful to describe physical properties of all those systems which have the common feature to be well represented by Hamiltonian (1.1) while it is well known that not all such systems could be accurately described by the Hamiltonian in which only the rotating terms are taken into account; the second one is that, also for those systems for which neglecting the nonconserving energy terms introduces quantitatively very small differences, only the complete model (1.1) enables us to explain important physical effects such as the Bloch-Siegert shift[13,14] or the modification of the Landé g factor,[14] for which the counter-rotating terms play an essential role. In Sec. II we have presented a new canonical transformation by which we succeed in transforming exactly H into $\tilde{H}$ where the pseudospin operators are immediately separable from the field operators. The advantages of working on $\tilde{H}$ rather than on H are two: the first one is that we have at our disposal an exact purely bosonic effective Hamiltonian, and the other one is the certainty that, whatever the approximation made on treating $\tilde{H}$, we may take exactly into account the correlations existing in each stationary state of H between the atom and the field due to the commutation of H with P. In Sec. III we have shown variationally that the lowest energy eigenstate of H and P for $\omega_0 \le \omega$ can be written as

$$|\psi_w^g\rangle = e^{-(1/2)\eta_{-w}^2} \sum_{n=0}^{\infty} \frac{(2\eta_{-w})^n}{\sqrt{n!}} (S_x)^n |n,\sigma_z\rangle , \tag{6.1}$$

while the eigenvalue of H on (6.1) has the form

$$E_w^g = \hbar\omega\eta_{-w}^2 + 2\epsilon\eta_{-w} - \frac{\hbar\omega_0}{2} w e^{-2\eta_{-w}^2} , \tag{6.2}$$

where η_{-w} is the root of Eq. (3.9). (6.1) and (6.2) give right results both in the limit of weak coupling and in the limit of strong coupling while for the intermediate case no analytical comparison is possible. In Sec. IV we succeed in establishing the most relevant result of this paper that is a direct relation between the exact ground state of H and our variational solution; precisely, we show that, whatever the coupling regime, (6.1) can be considered as the zeroth-order term of a perturbative series of which the first-order term is explicitly calculated in (4.15); (6.2), analogously, can be considered as the zeroth-order term of a perturbative expansion for the exact ground-state energy and, also in this case, we calculate and give in (4.22) the first non-null term that is the second-order correction. The importance of having established this connection consists in the fact that we may believe convincingly those physical predictions on the behavior of the system inferred from the calculation of the mean values of the operators different from the Hamiltonian. Following this line of reasoning Sec. V is just devoted to an investigation of how some physical properties of the system in the state $|\psi_w^g\rangle$, with $w=\pm 1$, vary in the transition from the weak- to the strong-coupling regime. To this end we have calculated and discussed from a physical point of view the covariance of the operators which represent the restoring force of the harmonic oscillator and the external force on the same oscillator and also the covariance of the operators which are associated with the momentum of the oscillator and the time derivative of the force exerted from the two-level system on the oscillator. We have, moreover, brought to light that the mean values of the operators $(-Kq)(-2F_0S_x)$ and $p[d(-2F_0S_x)/dt]$, evaluated on exact eigenstates (5.1), are not independent but that, on the contrary, there exist a physically transparent relation between them. It is precisely this circumstance that permitted us to show that the variational condition (3.9) which determines the form of the variational ground state $|\psi_w^g\rangle$ can be interpreted in terms of a condition on the mean value of the time derivative of the power associated with the harmonic oscillator or, equivalently, in terms of a balance among the expectation values of suitable operators relative to both the subsystems. The assumption $|\beta| \le 1$ under which we have obtained the results in this paper must be considered, above all, as a mathematical condition imposed to simplify the analytical deductions. It is, in fact, possible to develop suitably the technique used here in such a way as to treat not only the case $|\beta| > 1$ but also to extend it to the multimode model which recently has been considered with much interest as a model useful for describing phenomena of macroscopic quantum coherence with dissipation.[15–17]

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