Simulation of time-dependent positron behavior in neon gas

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The positron-annihilation spectra and decay rate in neon gas have been simulated via the Boltzmann time-dependent equation in the presence of electric and magnetic fields at room temperature using the positron-atom interaction model of McEachran, Ryman, and Stauffer [J. Phys. B 3, 551 (1978)]. The electric and magnetic fields are varied over the ranges 0–10 V cm^{-1} and 0–10 kG, respectively. Equilibrium is reached at about 18000 ns after the initial introduction of a positron swarm when there are no electric and magnetic fields present. The equilibrium value of the annihilation decay rate is 6.97 at zero electric and magnetic fields. The time for attaining equilibrium decreases with the increase in electric field. Its values are 6000, 3600, and 2400 ns for $E=2$, 5, and 10 V cm^{-1}, respectively. The effect of magnetic field is to delay the approach to the equilibrium of the positron distribution.

I. INTRODUCTION

When a positron is emitted from a source, its energy is generally in the (0.1–0.5)-MeV range. These high-energy positrons are moderated in the gas to less than 100 eV in about 1 ns. These positrons lose energy by colliding with the atoms of the gas. At first, positrons lose energy quickly, mainly by inelastic and ionizing collisions. When their energy is less than the positronium formation threshold, they are slowed down by elastic collisions from the energies near the inelastic threshold to thermal energies of 0.025 eV. After a sufficiently long time, greater than the slowing-down time, positrons are believed to approach equilibrium with a constant decay rate, also known as equilibrium annihilation rate. This annihilation decay rate provides a test of models of positron-atom interaction and this is the parameter which can be measured experimentally as well. The annihilation decay rate can be calculated theoretically when the velocity distribution of positrons is known. This velocity distribution can be determined by solving the time-dependent Boltzmann equation[1,2] with appropriate boundary conditions or by using a Monte Carlo[3] approach. Here we have used the Boltzmann time-dependent equation to analyze the transient behavior of the positrons in neon gas. The velocity distribution of positrons is also influenced by the temperature of the gas and applied external electric and magnetic fields. So, an annihilation decay rate calculated under the influence of these fields will provide additional data with which to test the accuracy of the different models of positron-atom interaction.

The equilibrium annihilation decay of positrons in neon gas was investigated earlier, theoretically by Srivastava and Grover[1] and Schrader and Svetic,[4] and experimentally by Coleman *et al.*,[5] Mao and Paul,[6] and Canter and Roellig.[7] The time-dependent annihilation of positrons in neon, argon, krypton, and xenon has been studied by Campeanu,[8] and in helium by Campeanu and Humberston.[8]

In this paper we shall present the detailed study of the time-dependent annihilation decay rate in neon gas and also the effects of electric and magnetic fields on it using the model of McEachran *et al.*[9]

II. METHOD OF STUDY

After entering the gas assembly positrons lose energy very quickly and then enter the energy region below the positronium formation threshold ($E_{\text{th}}=14.7$ eV for Ne). In this region positrons lose energy by elastic collisions with the gas atoms and undergo annihilation. Here the velocity distribution of positrons in a gas at temperature T, and subjected to cross electric field (E) and magnetic field (H), is given by the Boltzmann equation,[1,2]

$$\frac{\partial y(v,t)}{\partial t}=\frac{\partial}{\partial v}\left[\left[\frac{a^2}{3\nu_m(v)[1+\omega^2/\nu_m^2(v)]}+\frac{\nu_m(v)kT}{M}\right]\frac{\partial y(v,t)}{\partial v}+\left[\mu\nu_m(v)v-\frac{2a^2}{3v\nu_m(v)[1+\omega^2/\nu_m^2(v)]}-\frac{2\nu_m(v)kT}{Mv}\right]y(v,t)\right]-\nu_a(v)y(v,t)\ , \quad (1)$$

with the following boundary conditions: $y(0,t)=y(\infty,t)=0$ for all t, where v and $y(v,t)$ are positron velocity and its distribution function, respectively, $a=eE/m$ is the acceleration of the positron, e and m are the charge and mass of the positron, k is the Boltzmann constant, $\omega=eH/mc$ is the cyclotron frequency, c is the velocity of light, $\nu_a(v)$ and $\nu_m(v)$ are the positron-annihilation and scattering rates, respectively, and

$\mu = m/M$, is the ratio of positron mass m and the gas-atom mass M.

After Eq. (1) is solved to yield the positron distribution function at any time t, the average annihilation rate at time t can be obtained from

$$\bar{\lambda}(t) = \frac{\int_0^\infty y(v,t)\nu_a(v)dy}{\int_0^\infty y(v,t)dv}, \quad (2)$$

and it is this quantity which is compared with the experimentally observed annihilation rate.

We define a dimensionless annihilation decay rate $\bar{Z}_{\text{eff}}(t) = \bar{\lambda}(t)/\pi r_0^2 cn$, where $r_0 = e^2/mc^2$ is the classical electron radius. We shall take the gas density n to be one amagat (one amagat $= 2.687 \times 10^{19}$ atoms/cm^3).

The time-dependent average energy (in units of kT_0; T_0 is room temperature 300 K) is obtained from

$$\bar{\varepsilon}(t) = \frac{1}{2} \frac{\int_0^\infty v^2 y(v,t)dv}{\int_0^\infty y(v,t)dv}. \quad (3)$$

Thus, we see that the annihilation decay rate ($\bar{\lambda}$) and the average energy ($\bar{\varepsilon}$) can be obtained, provided the positron velocity distribution function is known. This function can be obtained by solving the Boltzmann equation (1). To do this, it is essential that $\nu_a(v)$ and $\nu_m(v)$ be known. We have taken $\nu_a(v)$ and $\nu_m(v)$ from the recent calculations of McEachran *et al.*[9] These quantities have a complicated dependence on velocity, so the Boltzmann equation cannot be solved analytically. We have solved it numerically by use of the Crank-Nicolson technique[10,11] and have obtained the distribution function by performing extensive computer calculations.

In order to start the solution of Eq. (1), we need to assume a form for the initial positron distribution function. Campeanu and Hamperston[8] assumed the following forms for initial velocity distributions:

(1) $y(v,t=0) = v^2$, uniform distribution in momentum space.

(2) $y(v,t=0) = v$, uniform distribution in energy.

They concluded that both forms lead to the same equilibrium distribution, so we have taken $y(v,0) = v^2$.

Equation (1) has been solved for time $(0\text{—}24) \times 10^{-6}$ s. Different time intervals—$\Delta t = 100$, 200, and 400 ns—have been tried, but all lead to a similar result. The time interval $\Delta t = 200$ ns provides the best convergence to and stability for the distribution function,[2] so in our calculations we have used this time interval. The electric and magnetic fields are varied over the ranges 0—10 V cm^{-1} and 0—10 kG, respectively. We have confined ourselves to such low electric and magnetic fields as the positron behavior in neon gas is quite sensitive in this region.[12]

III. RESULTS AND DISCUSSION

Figure 1 shows the variation of the distribution function $y(v,t)$ with velocity at different times ($t = 13\,800$, 16 200, and 18 000 ns). The electric and magnetic fields are kept at zero. The dotted curve represents the Maxwel-

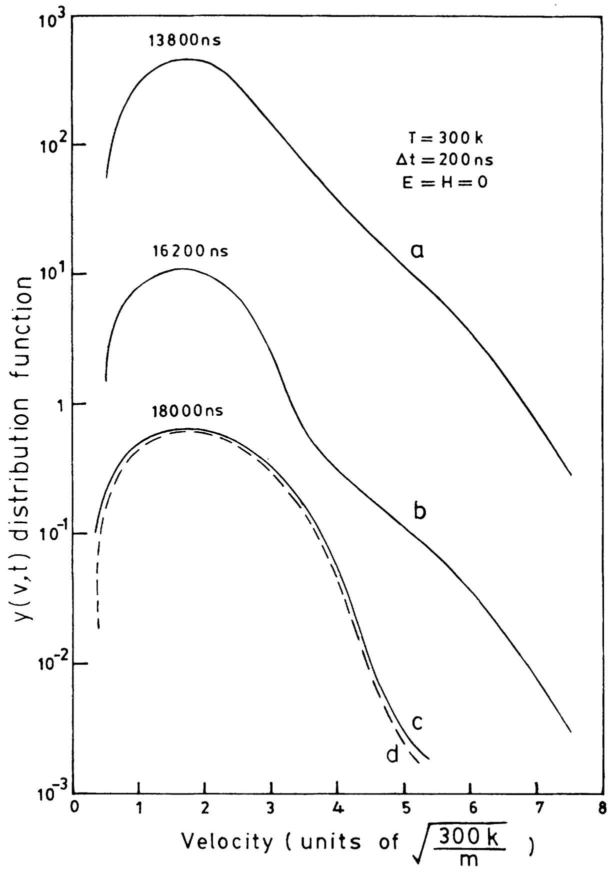

FIG. 1. Variation of the distribution function with velocity (in units of $\sqrt{300k/m}$, where k is the Boltzmann constant and m is the mass of the positron) at different times marked on the curves (13 800, 16 200, and 18 000 ns). The electric and magnetic fields are zero. The dotted curve d is the Maxwellian velocity distribution curve.

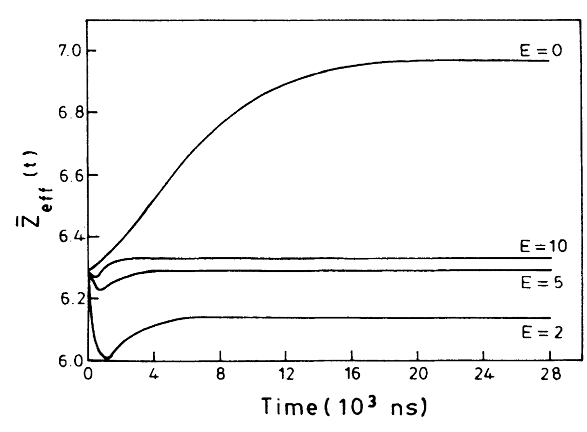

FIG. 2. Variation of annihilation decay rate [$\bar{Z}_{\text{eff}}(t)$] with times at various electric fields $E = 0$, 2, 5, and 10 V cm^{-1}, respectively.

lian velocity distribution function. From curve a we observe that the velocity distribution function increases with velocity, acquires a maximum, and then decreases. The other curves are also shaped like this. But as the time is increased, the shape of the curve tends to the Maxwellian shape (curve d). At $t \approx 18\,000$ ns, the shape of the distribution function approaches Maxwellian, and, hence, equilibrium.

Figure 2 shows the dependence of the annihilation decay rate $\overline{Z}_{\rm eff}(t)$ on time at various electric fields. The electric field values considered are 0, 2, 5, and 10 $\rm V\,cm^{-1}$, respectively. At zero electric field, $\overline{Z}_{\rm eff}(t)$ first increases and then acquires thermal equilibrium. When higher electric fields are applied, $\overline{Z}_{\rm eff}(t)$ goes through a minimum, then increases and acquires equilibrium. The minima occur at $t \approx 1200$, 600, and 400 ns for $E=2$, 5, and 10 $\rm V\,cm^{-1}$, respectively. The minimum in the decay rate (or maximum in lifetime) occurs at lower times as we increase the electric fields. The times for attaining thermal equilibrium also decrease with the increase in electric fields. These times are $\approx 18\,000$, 6000, 3600, and 2400 ns for $E=0$, 2, 5, and 10 $\rm V\,cm^{-1}$, respectively. The equilibrium value of $\overline{Z}_{\rm eff}(t)$ at zero electric and magnetic fields comes out to be 6.97. We have also computed the annihilation decay rate of positrons by the perturbation-iteration technique[12] for zero electric and magnetic fields at $T=300$ K. The value of $\overline{Z}_{\rm eff}$, by this technique, comes out to be 6.99. The agreement between the values found by the two different methods proves the accuracy of the results and computer codes. The experimental and theoretical equilibrium values of $\overline{Z}_{\rm eff}$ from different workers have been shown in Table I. Our result is also in good agreement with theoretical calculations of other workers. The theoretical values are rather higher than the experimental ones. The explanation of the cause of this has been elaborated on by McEachran *et al.*[9]

Variation of the annihilation decay rate $\overline{Z}_{\rm eff}(t)$ with times at various combined electric and magnetic fields is presented in Fig. 3. The values of the electric and magnetic fields are marked on the curves. We observe (curves b and d) that the magnetic field delays the approach to thermal equilibrium. In curve b, when there is no magnetic field, equilibrium is acquired at $t \approx 6000$ ns; however, when the magnetic field is applied, the time of approach to thermal equilibrium is $t \approx 14\,000$ ns (curve d). Thus, electric and magnetic fields have opposite effects on the lifetime of positrons. Another important observation of the magnetic effect is that the minimum which occurs in the presence of electric fields disappears when the magnetic field is applied. We are unable to explain this phenomenon at this juncture. However, this is an interesting observation of the present calculations and should be studied experimentally.[13] Figure 4 shows the dependence of equilibrium annihilation decay rate $\overline{Z}_{\rm eq}$ on electric fields. For fields ≥ 8 $\rm V\,cm^{-1}$, $\overline{Z}_{\rm eq}$ becomes almost independent of electric field. At lower fields, there is a fall in $\overline{Z}_{\rm eq}$, but at higher fields a gradual increase, though very small, is present.

The average energy, $\overline{\varepsilon}$ $(t \to \infty)$, of the positrons at zero electric and magnetic fields is also computed using Eq. (3), which comes out to be 0.042 eV, whereas the calculated equilibrium value is 0.039 eV. The agreement between the two values is again found to be quite good.

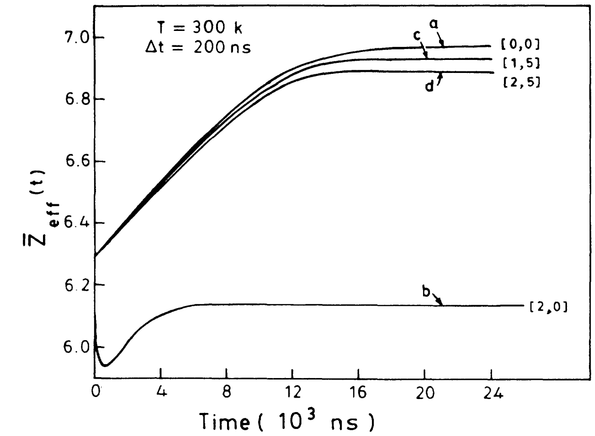

FIG. 3. Variation of annihilation decay rate $[\overline{Z}_{\rm eff}(t)]$ with time at various electric and magnetic fields, as marked (E,H) on the curves: the first value is that of the electric field in $\rm V\,cm^{-1}$ and the second is that of the magnetic field in kG.

TABLE I. Comparison of experimental and theoretical values of $\overline{Z}_{\rm eff}$ $(t \to \infty) = \overline{Z}_{\rm eff}$ at $E=H=0$ and $T=300$ K.

Experiment	
Coleman *et al.*[a]	5.99±0.06
Mao and Paul[b]	6.02±0.16
Canter and Roellig[c]	5.96±0.15
Theory	
Srivastava and Grover[d]	7.15
Schrader and Svetic[e]	7.0±0.3
Campeanu[f]	6.85
Our previous work[g]	6.99
Present work	6.97

[a]Reference 5.
[b]Reference 6.
[c]Reference 7.
[d]Reference 1.
[e]Reference 4.
[f]Reference 8.
[g]Reference 12.

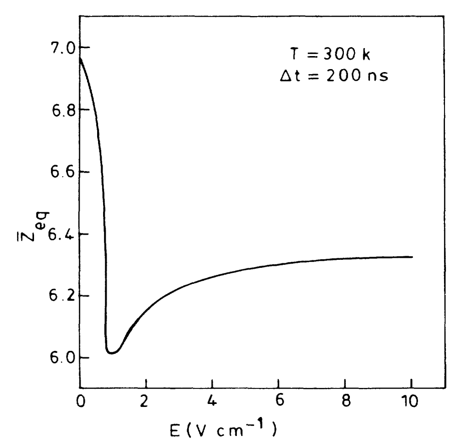

FIG. 4. Variation of equilibrium decay rate $(\overline{Z}_{\rm eq})$ with electric fields at room temperature 300 K.

IV. CONCLUSIONS

The occurrence of minima in the annihilation decay rate (Fig. 3) has been attributed to the presence of a "shoulder" in the positron-lifetime spectra.[8] If this is so, then it should be possible to verify this by performing the experiment under the external electric and magnetic fields, as our calculations suggest. The values obtained for various quantities are based on the model of McEachran *et al.*[9] however, if some other models were tried, the results are expected to be different. The accuracy of any model can be judged by performing an experiment and measuring the values, as suggested by this paper.

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