Electromagnetic propagation and scattering in time-dependent moving media

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A new formalism is introduced for discussing propagation of electromagnetic waves in space- and time-varying media. The approach taken here is to supplement the velocity-independent solution by a correction factor. This factor involves a four-dimensional line integral and is therefore a WKB-type solution. The formalism is relativistically exact to the first order in v/c. Special cases are discussed, demonstrating how time-harmonic velocity fields modulate the propagating electromagnetic fields. Scattering problems involving time-varying surfaces have been considered in the past. The scatterers were situated in free space (vacuum), or if material media were involved, the mechanical interaction of the moving surfaces with the medium was usually ignored. The new theory presented here facilitates the analysis of electromagnetic scattering problems in the presence of combined surface and medium motion. Consequently, mass continuity at the moving surfaces can be preserved. This makes the modeling of such problems much more realistic. Simplified canonical problems involving interaction of electromagnetic and mechanical waves are discussed to highlight the effects resulting from the new boundary considerations. Surprisingly, for bounded surfaces the time-varying medium effects vanish in the far field. This means that previous results which simply ignored the medium motion are still valid as good approximations.

INTRODUCTION AND MOTIVATION

Electrodynamics in moving media and scattering by moving surfaces have captured the imagination of scientists for many decades. Problems of this kind contribute to our understanding of fundamental theories, e.g., see Pauli[1] and other early references cited by Censor.[2] High-precision sophisticated systems, e.g., the Global Positioning System, require relativistic electrodynamics considerations, which bring such problems into the domain of modern engineering. The general history can be traced from the historical remarks made by Sommerfeld,[3] who cites the early papers by Hertz,[4] Minkowski,[5] and Einstein,[6] and Pauli,[1] to a comprehensive review of the state of the art provided by Van Bladel.[7] See also Toman[8] for history of the Doppler effect.

Two distinct classes of problems are evident in the literature: (a) scattering by moving surfaces and (b) propagation in moving media. Both problems are based on Einstein's theory of special relativity[6] and Minkowski's subsequent results for electrodynamics in moving media.[5] Even for first-order v/c velocity effects one must refer to special relativistic theory in order to be able to carefully enumerate the heuristic aspects of his solution method. Thus, for example, it is not clear whether the use of the Lorentz transformation for constant velocities may be extended to space- and time-varying velocity fields $\mathbf{v}(\mathbf{r},t)$. The fact that practically all studies use this heuristic assumption, often without even declaring the heuristic nature of this approach, only serves to show how far we are from a satisfactory general theory. The effect of a space- and time-dependent velocity field on the Lorentz transformation and the ensuing transformations for the electromagnetic fields, and the associated boundary and transition conditions, is not known. The best argument for adopting the above assumption, which is also used below, is to say that the velocity field changes slowly, such that $\Delta v/v$ is small over distances comparable to wavelength and duration comparable to the period of the waves under consideration. This is a ray (as opposed to wave) approach, and is consistent with the WKB-type theory introduced below. Another salient feature of the existing literature is the fact that many studies are devoted to analyzing examples. This again demonstrates our limited knowledge. Here too the formalism is used to solve simple canonical problems and thus enhance our understanding of the general subject.

It is interesting to note that the above-mentioned two classes of problems evolved simultaneously without much interaction. Moving surfaces were considered in free space (vacuum), and when a material medium was considered, its interaction with the moving surfaces was ignored. For example, Censor[9,10] discusses moving media and surfaces at rest, ignoring the interaction. This, of course, violates mass continuity at the surface, which is a serious drawback. A later technique used by Censor[11,12] to treat time-independent nonuniform velocities can be traced back to Tai,[13] and Collier and Tai.[14] Essentially an exponential correction factor amounting to a WKB- or ray-type approximation was introduced, in order to obtain a solution of the relevant wave equation correct to the first order in v/c. The technique facilitated the analysis of scattering problems involving surfaces at rest and moving media, preserving mass continuity at the surface of the scatterer.[11,12] Presently this idea is extended to four-dimensional line integrals which enable us to discuss spa-

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tially and temporally dependent velocity fields.

By combining the motion of the surface and the surrounding medium, problems involving moving surfaces can be discussed, preserving mass continuity at the moving surface. This is a novel feature: Now for the first time we are able to analyze simple problems with and without the effect of the moving surface on the surrounding medium. Subject to all the restrictions enumerated below, the analysis of the present class of problems indicates that for bounded scatterers the additional effects produced by the surface setting in motion the surrounding medium decrease with distance. It means that in the far field these effects can be neglected. One should, however, be careful, because these conclusions are based on very restrictive assumptions. A similar situation occurred with respect to acoustical waves. Censor[15,16] analyzed scattering from time-varying surfaces, ignoring the interaction of the surface with the surrounding medium. Rogers[17] and later Piquette and Van Buren[18] argued that the analysis is invalid and that if the motion of the surrounding medium is taken into account, the effect will altogether disappear—this is an argument first raised by Petzval[8]—and that a nonlinear mechanism is necessary in order to adequately analyze this class of problems. Recently it has been shown,[19] using techniques similar to those used presently, that the same conclusions regarding the far field apply to acoustical waves as well.

PROPAGATION IN SPACE AND TIME-DEPENDENT MOVING MEDIA

The fields measured in the "laboratory" system of reference are governed by the Maxwell equations,[3] which for sourceless domains (i.e., zero current density $\mathbf{j}=\mathbf{0}$, zero charge density $\rho=0$) have the form

$$\begin{aligned} &\nabla\times\mathbf{E}=-\partial_t\mathbf{B} \ , \\ &\nabla\times\mathbf{H}=\partial_t\mathbf{D} \ , \\ &\nabla\cdot\mathbf{D}=0 \ , \\ &\nabla\cdot\mathbf{B}=0 \ , \end{aligned} \tag{1}$$

where $\partial_t\equiv\partial/\partial t$. In a "comoving" system of reference, in which the medium is at rest, the simple constitutive relations

$$\begin{aligned} &\mathbf{D}'=\epsilon\mathbf{E}' \ , \\ &\mathbf{B}'=\mu\mathbf{H}' \end{aligned} \tag{2}$$

are assumed. The primed fields $\mathbf{D}'(\mathbf{r}',t')$, etc., are functions of the comoving system coordinates $\mathbf{r}',t'$, which are related to the laboratory system by the Lorentz transformation.[3] The laboratory fields $\mathbf{D}(\mathbf{r},t)$, etc., are related to the comoving fields by the appropriate field-transformation formulas prescribed by the invariance of Maxwell's equations [i.e., prescribed by the postulate that in the comoving system (1) with primed coordinates and fields, etc., is valid]. Substituting the field-transformation formulas into (2) yields the Minkowski[1,3,5] constitutive relations, which to the first order in v/c have the form

$$\begin{aligned} &\mathbf{D}=\epsilon\mathbf{E}+\mathbf{\Lambda}\times\mathbf{H} \ , \\ &\mathbf{B}=\mu\mathbf{H}-\mathbf{\Lambda}\times\mathbf{E} \ , \\ &\mathbf{\Lambda}=\mathbf{v}(\mu\epsilon-\mu_0\epsilon_0) \ , \end{aligned} \tag{3}$$

where $\mathbf{v}$ is the velocity as observed in the laboratory system of reference and μ_0,ϵ_0 are the free-space (vacuum) values of the constitutive parameters. Note that special relativity assumes inertial systems, i.e., constant $\mathbf{v}$. In the following a space- and time-varying velocity field $\mathbf{v}(\mathbf{r},t)$ will be assumed, and the above equations will be heuristically assumed to hold. The validity of this conjecture has not been tested, as far as this author is aware. If spatial and temporal changes in $\mathbf{v}(\mathbf{r},t)$ are small, compared to wavelength and period, respectively, of the fields at hand, then the validity of (1)–(3) for space-time velocities seems to be plausible. The new equations governing the fields are now

$$\begin{aligned} &\nabla\times\mathbf{E}+\mu\partial_t\mathbf{H}-\partial_t(\mathbf{\Lambda}\times\mathbf{E})=\mathbf{0} \ , \\ &\nabla\times\mathbf{H}-\epsilon\partial_t\mathbf{E}-\partial_t(\mathbf{\Lambda}\times\mathbf{H})=\mathbf{0} \ . \end{aligned} \tag{4}$$

The solution of (4) is the subject of this study.

Extending a previous method, as explained above, we make the ansatz

$$\begin{aligned} &\mathbf{E}=\mathbf{E}_0(\mathbf{r})e^{-i\omega_0 t+i\beta} \\ &\mathbf{H}=\mathbf{H}_0(\mathbf{r})e^{-i\omega_0 t+i\beta} \ , \end{aligned} \tag{5}$$

where $e^{i\beta}$ is a correction factor and $\mathbf{E}_0(\mathbf{r})e^{-i\omega_0 t}$, $\mathbf{H}_0(\mathbf{r})e^{-i\omega_0 t}$ is the solution for the velocity-independent case $\mathbf{\Lambda}=\mathbf{0}$. Substituting (5) in (4) and keeping only first-order effects in v/c (higher-order effects can be retained for convenience, however, the approximation is valid only to within the first power of v/c), we obtain

$$\begin{aligned} &\nabla\times\mathbf{E}_0-i\omega_0\mu\mathbf{H}_0=\mathbf{0} \ , \\ &\nabla\times\mathbf{H}_0+i\omega_0\epsilon\mathbf{E}_0=\mathbf{0} \ , \\ &\nabla\cdot\mathbf{E}_0=0 \ , \\ &\nabla\cdot\mathbf{H}_0=0 \ , \end{aligned} \tag{6}$$

and

$$\begin{aligned} &i\nabla\beta\times\mathbf{E}_0+i\mu\partial_t\beta\mathbf{H}_0+i\omega_0\mathbf{\Lambda}\times\mathbf{E}_0-\partial_t\mathbf{\Lambda}\times\mathbf{E}_0=\mathbf{0} \ , \\ &i\nabla\beta\times\mathbf{H}_0-i\epsilon\partial_t\beta\mathbf{E}_0+i\omega_0\mathbf{\Lambda}\times\mathbf{H}_0-\partial_t\mathbf{\Lambda}\times\mathbf{H}_0=\mathbf{0} \ . \end{aligned} \tag{7}$$

Previously[12] time-independent velocities have been considered, for which (7) becomes

$$\begin{aligned} &(\nabla\beta+\omega_0\mathbf{\Lambda})\times\mathbf{E}_0=\mathbf{0} \ , \\ &(\nabla\beta+\omega_0\mathbf{\Lambda})\times\mathbf{H}_0=\mathbf{0} \ , \end{aligned} \tag{8}$$

whose solution is $\nabla\beta=-\omega_0\mathbf{\Lambda}$, hence $\beta=-\omega\int\mathbf{\Lambda}\cdot d\mathbf{r}$ is a spatial, three-dimensional line integral. This technique closely resembles the WKB method,[20] or what the physicist usually calls a ray approximation. The generalization to space-time rays involves four-dimensional line integrals.[21–24] Correspondingly, β is represented as

$$\beta(\mathbf{r},t)=\int_{\mathbf{r}_0,t_0}^{\mathbf{r},t}[\mathbf{K}(\bar{\mathbf{r}},\bar{t})\cdot d\bar{\mathbf{r}}-W(\bar{\mathbf{r}},\bar{t})d\bar{t}] \ , \tag{9}$$

a four-dimensional line integral extending from the fixed limit $\mathbf{r}_0,t_0$ to a variable $\mathbf{r},t$; here $\overline{\mathbf{r}},\overline{t}$ are integration (dummy) variables. The behavior of β at a point $\mathbf{r},t$ must be independent of how the integration is performed, i.e., the uniqueness of the physical fields $\mathbf{E},\mathbf{H}$ prescribes that the integral (9) depends on the limits only. In turn, this prescribes that the expression in square brackets in (9) be an exact differential. These conditions are satisfied subject to $\beta(\mathbf{r},t)$ being indifferent to order to differentation, i.e.,

$$\partial_t \nabla\beta = \nabla\partial_t\beta \tag{10}$$

which in turn prescribes

$$\begin{aligned} &\nabla\times\mathbf{K}=\mathbf{0}\ ,\\ &\partial_t\mathbf{K}+\nabla W=\mathbf{0}\ . \end{aligned} \tag{11}$$

Note that $\nabla\times\mathbf{K}$ is already included in the second equation (11) provided we are allowed to interchange $\nabla\times\partial_t\mathbf{K}=\partial_t\nabla\times\mathbf{K}$. Subject to (9)–(11) we now have (7) in the form

$$\begin{aligned} &\mathbf{F}\times\mathbf{E}_0-\mu W\mathbf{H}_0=\mathbf{0}\ ,\\ &\mathbf{F}\times\mathbf{H}_0+\epsilon W\mathbf{E}_0=\mathbf{0}\ ,\\ &\mathbf{F}=\mathbf{K}+(\omega_0+i\partial_t)\mathbf{\Lambda}\ . \end{aligned} \tag{12}$$

From (12) it follows that $\mathbf{E}_0,\mathbf{H}_0,\mathbf{F}$ are mutually perpendicular. While (1) and (2) are quite general, the ansatz (5) leads to (12) and perpendicular fields, which is a severe constraint. In the next section an example will be given for which this constraint can be waived. The pertinent velocity field is taken as an acoustical wave, and it is shown that to the first order in v/c it amounts to taking $\mathbf{F}=\mathbf{0}$, $W=0$ in (12), hence the orientation of $\mathbf{F}$ relative to $\mathbf{E}_0,\mathbf{H}_0$ is irrelevant. This is in fact a quasistatic approximation. In general the dependence on the orientation of the fields does not vanish. This means that $\mathbf{E}_0,\mathbf{H}_0,\mathbf{F}$ must be mutually perpendicular. If the total $\mathbf{E}_0\cdot\mathbf{H}_0$ is nonvanishing, the fields must be recast as a sum of partial fields satisfying the perpendicularity requirements. For example, the fields may be recast in terms of an integral of plane waves[25]

$$\begin{aligned} &\mathbf{E}_0(\mathbf{r})=\int \mathbf{e}(\mathbf{r})e^{i\mathbf{k}(\hat{\mathbf{r}})\cdot\mathbf{r}-i\omega_0 t}d\Omega\ ,\\ &\mathbf{H}_0(\mathbf{r})=\int \mathbf{h}(\mathbf{r})e^{i\mathbf{k}(\hat{\mathbf{r}})\cdot\mathbf{r}-i\omega_0 t}d\Omega\ , \end{aligned} \tag{13}$$

where $\hat{\mathbf{r}}$ denotes a unit vector depending on directions (e.g., θ and ψ, the polar and azimuthal angles, respectively, in a spherical coordinate system), $d\Omega=\sin\theta\, d\theta\, d\psi$ is an element of the surface on the unit sphere, and $\mathbf{e}\cdot\mathbf{h}=0$. In this case, for each direction $\hat{\mathbf{r}}$ there is an associated $\mathbf{F}(\hat{\mathbf{r}})$ vector parallel to $\mathbf{k}(\hat{\mathbf{r}})$.

As it stands, (12) is a homogeneous system of equations whose determinant must vanish for nonvanishing $\mathbf{E}_0,\mathbf{H}_0$ (or $\mathbf{e}_0,\mathbf{h}_0$ for plane waves), thus defining a relation

$$W=W(\mathbf{F})\ . \tag{14}$$

This scalar equation is the analog of the dispersion equation in ray theory.[21–24] The second equation of (11) provides another three scalar equations—altogether four scalar equations for the four scalar components of $\mathbf{K},W$. We have therefore shown that in principle the system is solvable. Examples will be given below. Taking the divergent of (12) yields

$$\nabla\times\mathbf{F}=\mathbf{0} \tag{15}$$

and since $\nabla\times\mathbf{K}=\mathbf{0}$ from (11), it follows that

$$\nabla\times\mathbf{\Lambda}=\mathbf{0}\ . \tag{16}$$

Hence, the present method works only for irrotational flows. For velocities $\mathbf{v}$ not satisfying (16), a different method must be employed.[26]

FURTHER CONSIDERATIONS AND ANALYSIS OF SIMPLE EXAMPLES

The interaction of a simple velocity field with a plane electromagnetic wave is relatively easy to compute. As shown below, this problem provides more insight into the implications of the above general formalism. Consider the electromagnetic field

$$\begin{aligned} &\mathbf{E}=\mathbf{E}_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega_0 t+i\beta}\ ,\\ &\mathbf{H}=\mathbf{H}_0 e^{i\mathbf{k}_0\cdot\mathbf{r}-i\omega_0 t+i\beta}\ , \end{aligned} \tag{17}$$

which for $\beta=0$ reduces to the simple plane harmonic wave, with $\mathbf{E}_0,\mathbf{H}_0$ constants. Let the velocity of the medium be associated with an acoustical wave, such that

$$\mathbf{\Lambda}=\mathbf{\Lambda}_0 e^{i\mathbf{Q}\cdot\mathbf{r}-i\Omega t}\ , \tag{18}$$

where $\mathbf{\Lambda}_0=$const and the wave propagates with a velocity $u=\Omega/Q$, where the phase velocity u and the displacement velocity $\mathbf{v}$ (3) are of similar magnitude. The frequency Ω is considerably smaller compared to ω_0, thus satisfying the requirements of ray theory. From (12) and (17), $\mathbf{F}$ is parallel to $\mathbf{k}_0$, hence

$$\mathbf{K}_\perp=-(\omega_0+\Omega)\mathbf{\Lambda}_\perp\ , \tag{19}$$

where $\perp$ denotes components perpendicular to $\mathbf{k}_0$. The component $\mathbf{K}_{\parallel}$ parallel to $\mathbf{k}_0$ is determined from (12) and (14), in the following way. Apply $\mathbf{F}\times$ to the first line (12) and substitute in the second one. Noting that $\mathbf{F}\cdot\mathbf{E}_0=\mathbf{F}\cdot\mathbf{H}_0=0$, this yields

$$F^2=F_{\parallel}^2+F_{\perp}^2=(W/C)^2\ , \tag{20}$$

where $C^2=1/\mu\epsilon$, and since $F_\perp=0$, we have

$$\mathbf{K}_{\parallel}+(\omega_0+\Omega)\Lambda_{\parallel}=(W/C)\hat{\mathbf{k}}_0\ . \tag{21}$$

Adding (19) and (21) yields a vector equation

$$\mathbf{K}+(\omega_0+\Omega)\mathbf{\Lambda}=(W/C)\hat{\mathbf{k}}_0\ . \tag{22}$$

For (22) to hold for all $\mathbf{r},t$, it follows that $\mathbf{K},W$ have as a factor the same exponential as in (18). From (11) and (18)

$$\nabla W=i\Omega\mathbf{K}=i\mathbf{Q}W\ , \tag{23}$$

hence $\mathbf{K},\mathbf{Q}$ are parallel vectors and

$$W=uK\ . \tag{24}$$

Consequently (22) can be recast as

$$\mathbf{K}-\hat{\mathbf{k}}_0 Ku/C+(\omega_0+\Omega)\mathbf{\Lambda}=0\ . \tag{25}$$

Inasmuch as $\mathbf{K}$ is already of first order in v/c, the second term in (25) is of second order and therefore negligible. Therefore, to the first order in v/c, (23) and (25) prescribe, for time-harmonic velocity fields,

$$\mathbf{K}=-(\omega_0+\Omega)\mathbf{\Lambda}\ ,\quad W(\mathbf{r},t)=i\Omega\int_0^{\mathbf{r}}\mathbf{K}(\bar{\mathbf{r}},t)\cdot d\bar{\mathbf{r}}\ . \tag{26}$$

This is a very interesting result. Except for Ω, which does not appear in (8), $\mathbf{K}$ is derived as in the time-independent velocity case. Moreover, since in (26) the dependence on directions relevant to the electromagnetic field (17) disappears, the condition $\mathbf{F}\cdot\mathbf{E}_0=\mathbf{F}\cdot\mathbf{H}_0=0$ is not important anymore.

The fact that $\mathbf{K}$ (26) has been computed from (22) by putting $W=0$ is far-reaching also for the computation of β, according to (9). We will now prove that the four-dimensional line integral degenerates into a three-dimensional spatial line integral, i.e., except for $\omega_0+\Omega$ instead of ω_0, we are led to a quasistatic approximation.

Since (9) is independent of the path of integration, we choose

$$\int_{0,0}^{\mathbf{r},t}=\int_{0,0}^{\mathbf{r},0}+\int_{r,0}^{\mathbf{r},t}\ . \tag{27}$$

Hence for complex β_c

$$\begin{aligned}\beta_c&=\int_{0,0}^{\mathbf{r},0}\mathbf{K}\cdot d\bar{\mathbf{r}}-\int_{\mathbf{r},0}^{\mathbf{r},t}W\,d\bar{t}\\&=\int_{0,0}^{\mathbf{r},0}\mathbf{K}\cdot d\bar{\mathbf{r}}+\left[\int_{0,\bar{t}}^{\bar{\mathbf{r}},\bar{t}}\mathbf{K}\cdot d\bar{\mathbf{r}}\right]_{\mathbf{r},0}^{\mathbf{r},t}\\&=\int_{0,t}^{\mathbf{r},t}\mathbf{K}\cdot d\bar{\mathbf{r}}=\int_0^{\mathbf{r}}\mathbf{K}(\bar{\mathbf{r}},t)\cdot d\bar{\mathbf{r}}\\&=K_0e^{-i\Omega t}\int_0^{\mathbf{r}}e^{i\mathbf{Q}\cdot\bar{\mathbf{r}}}\hat{\mathbf{\Lambda}}_0\cdot d\bar{\mathbf{r}}\ ,\end{aligned} \tag{28}$$

i.e., the time integration of W simply yields a factor $1/i\Omega$, and according to (26) the integral in large parentheses is obtained. The time t appearing on both limits of the integral $\int_{0,t}^{\mathbf{r},t}\mathbf{K}\cdot d\bar{\mathbf{r}}$ simply means that $\mathbf{K}$ is evaluated at this time, and the factor $e^{-i\Omega t}$ can be taken outside the integral. The remaining integral is integrated with respect to the direction of the velocity, say $\hat{\mathbf{\Lambda}}_0\cdot d\bar{\mathbf{r}}=d\bar{x}$, and $\mathbf{Q}$ is also parallel to $\mathbf{\Lambda}$, hence

$$\beta_c=(K_0/iQ)e^{i\mathbf{Q}\cdot\mathbf{r}-i\Omega t}\ . \tag{29}$$

At this point it must be realized that β in (9) is real, hence we have to take $\beta=\mathrm{Re}\beta_c$,

$$\beta=-\frac{\omega_0+\Omega}{\Omega/u}\Lambda_0\sin\left[\frac{\Omega}{u}\hat{\mathbf{\Lambda}}\cdot\mathbf{r}-\Omega t\right]\ . \tag{30}$$

Substituting (30) in (17) yields the desired result. It is now clear how the velocity modulates the phase of the plane wave. In order for β to be consistent with the fundamental assumptions of the above theory, it must be of order v/c, hence for $\omega_0>>\Omega$, we must have $(u/c)(\omega_0/\Omega)$ of order 1 or less.

For $\omega_0>>\Omega$ such that (30) simply becomes a quasistatic approximation, it is legitimate to ask whether the whole theory of the preceding section and the following argument of (28), leading to (30), is really necessary. To answer this question, an example will be given for which the quasistatic approximation is inadequate. Consider a velocity field harmonic in time and uniform in space,

$$\mathbf{\Lambda}=\mathbf{\Lambda}_0e^{-i\Omega t}\ , \tag{31}$$

where $\mathbf{\Lambda}_0=$const. As a limiting case of (18) it prescribes that $Q\rightarrow 0$, hence $u\rightarrow\infty$. However, this violates the assumption of small u/c which led from (25) to (26). Consequently, a different type of solution is to be found. Combining (22) and $\nabla W=i\Omega\mathbf{K}$, we now have

$$\nabla W+i\Omega(\omega_0+\Omega)\mathbf{\Lambda}-\hat{\mathbf{k}}_0(i\Omega/C)W=0 \tag{32}$$

as the equation governing W. For $\mathbf{\Lambda}=\Lambda\hat{\mathbf{k}}_0$ parallel to $\hat{\mathbf{k}}_0$ we obtain (say $\hat{\mathbf{k}}_0$ is in the x direction)

$$\partial_x W+i\Omega(\omega_0+\Omega)\Lambda-(i\Omega/C)W=0 \tag{33}$$

whose solution is

$$W=W_0e^{(i\Omega/C)x-i\Omega t}+\Lambda C(\omega_0+\Omega)\ ,\quad \mathbf{K}=K\hat{\mathbf{x}}=\hat{\mathbf{x}}\partial_x W/i\Omega=\hat{\mathbf{x}}(W_0/C)e^{(i\Omega/C)x-i\Omega t}\ , \tag{34}$$

and W_0 is arbitrary. If $W=0$ for $x=0$, $t=0$, then

$$W_0=-C\Lambda_0(\omega_0+\Omega)\ . \tag{35}$$

Integrating (9) subject to (34) and (35) along the contour $\int_{0,0}^{x,0}dx+\int_{x,0}^{x,t}dt$ yields

$$\beta=\mathrm{Re}[(C/i\Omega)(\omega_0+\Omega)\Lambda_0e^{-i\Omega t}(1-e^{i(\Omega/C)x})]\ . \tag{36}$$

Clearly (36) is different from (30), and depends on $\mathbf{E}_0,\mathbf{H}_0$ field directions. For arbitrary directions of $\mathbf{\Lambda}_0$ the solution of (31) is not readily available, but obviously depends on field directions.

Subsequently, only interaction of electromagnetic and mechanical waves of the kind leading to (30) will be considered. The reasons for that are the simplicity of the result, and the fact that in the case of scattering by moving surfaces, acoustical waves of the kind (18) are generated by these moving surfaces. Consider, for example, spherical waves defined by

$$\mathbf{\Lambda}=\hat{\mathbf{r}}\frac{\Lambda_0}{r}e^{iQr-i\Omega t}\ , \tag{37}$$

where $\Lambda_0=$const and absorbs the extra lenght-unit involved. For this case, and subject to the approximation (26), we have

$$\beta=-\mathrm{Re}\left[(\omega_0+\Omega)\Lambda_0e^{-i\Omega t}\int_{r_0}^{r}(e^{iQ\bar{r}}/\bar{r})d\bar{r}\right]\ . \tag{38}$$

For distances small compared to the acoustical wavelength, $Qr\ll 1$,

$$\beta\approx-(\omega_0+\Omega)\Lambda_0\ln(r/r_0)\cos(\Omega t)\ . \tag{39}$$

Note that the reference distance r_0 must be finite for the present case. Also note that $Qr\ll 1$ means that the distance is small compared to the acoustical wavelength, but the same distance may be large compared to the electromagnetic wavelength $2\pi/k_0$. For the opposite case $Qr>>1$, (38) becomes

$$\beta \approx -(\omega_0+\Omega)(\Lambda_0/r)\sin(Qr-\Omega t) , \quad (40)$$

where the reference distance is taken as $r_0 \to \infty$. Clearly as r increases, β decreases, hence the velocity effect due to a spherical velocity wave vanishes in the far field (at a distance which is many acoustical wavelengths). For cylindrical waves

$$\mathbf{\Lambda}=\hat{\mathbf{r}}\Lambda_0 H_0(Qr)e^{-i\Omega t} , \quad (41)$$

where $\hat{\mathbf{r}}$ is perpendicular to the cylindrical axis and H_0 is the zero-order Hankel function of the first kind. For $Qr \ll 1$, we have

$$H_0 \approx -i\frac{2}{\pi}\ln\frac{2}{\gamma r} , \quad (42)$$

where $\gamma=1.78\ldots$ is a constant. It follows that

$$\beta=-2(\omega_0+\Omega)(\Lambda_0/\pi)r[\ln(\gamma r/2)-\gamma/2]\sin(\Omega t) , \quad (43)$$

where the reference distance is taken $r_0=0$ (note $\lim_{r\to 0} r\ln r=0$). On the other hand, for $Qr \gg 1$,

$$H_0 \sim (2/i\pi Qr)^{1/2}e^{iQr} , \quad (44)$$

hence

$$\beta=\mathrm{Re}[(\omega_0+\Omega)\Lambda_0(-2/i\pi Q^3 r)^{1/2}e^{iQr-i\Omega t}]$$
$$=(\omega_0+\Omega)\Lambda_0(\pi Q^3 r)^{-1/2}$$
$$\times[\cos(Qr-\Omega t)-\sin(Qr-\Omega t)] \quad (45)$$

and again $\beta \to 0$ as $r \to \infty$, so that in the far field the velocity effect vanishes. The above examples will be subsequently exploited to solve scattering problems in the presence of space-time—dependent moving media.

THREE TYPES OF SCATTERING PROBLEMS

The main objective of the present study is to gain more understanding regarding scattering problems in the presence of moving media. This happens, for example, when the moving scatterer sets in motion the surrounding medium. Models discussed previously involved moving media and scattering surfaces at rest and moving scatterers in unperturbed media. Both models violate mass continuity at the surface, hence it is desirable to solve problems in which mass continuity is preserved and compare the results. Accordingly, subsequently three cases will be considered.

Case (a). Medium moving and scatterers at rest. Boundary conditions are satisfied at the static surface.

Case (b). Medium at rest and moving surfaces. Boundary conditions are satisfied at the surface in the comoving frame of reference.

Case (c). Medium and scatterer are in motion as to preserve mass continuity. Boundary conditions are satisfied at the surface in the comoving frame.

This program is carried out below for plane, cylindrical, and spherical scatterers. The procedure is similar to that used for acoustical wave propagation.[19]

PLANE SCATTERERS

Case (a). The incident wave is given in (17). For simplicity let $\mathbf{E}_0=\hat{\mathbf{z}}E_0$ be polarized in the $\hat{\mathbf{z}}$ direction, and $\mathbf{k}_0$ be in the xy plane. The scatterer is a plane defined by $x=0$. For reasons relevant to case (c) below, the velocity is defined as

$$\mathbf{\Lambda}=\hat{\mathbf{x}}\Lambda_0 e^{iQx+i\Omega t} , \quad (46)$$

i.e., an acoustical wave propagating in the $-\hat{\mathbf{x}}$ direction. Similarly to (29),

$$\beta=-\frac{\omega_0-\Omega}{Q}\Lambda_0\sin(Qx+\Omega t) \quad (47)$$

which is considered to be of first order in v/c. At the boundary $x=0$ the incident wave can be represented as

$$\mathbf{E}=\hat{\mathbf{z}}E_0\left[e^{-i\omega_0 t}-\frac{\omega_0-\Omega}{2Q}\Lambda_0 e^{-i(\omega_0-\Omega)t}+\frac{\omega_0-\Omega}{2Q}\Lambda_0 e^{-i(\omega_0+\Omega)t}\right] \quad (48)$$

consistent with (17) and (47) to the first order in v/c. The effect of the moving medium is therefore to develop two sidebands at frequencies $\omega_0 \pm \Omega$. Let the boundary condition be

$$\mathbf{E}+\mathbf{E}_s=\mathbf{0}\,|_{\text{at } x=0} \quad (49)$$

prescribing that the scattered field $\mathbf{E}_s$ at the boundary is given by (48) with the opposite sign. It follows that the scattered wave is given by

$$\mathbf{E}_s=-\hat{\mathbf{z}}E_0 e^{i\mathbf{k}_{0s}\cdot\mathbf{r}-i\omega_0 t+i\beta} ,$$
$$\mathbf{k}_{0s}=-\hat{\mathbf{x}}\mathbf{k}_0\cdot\hat{\mathbf{x}}+\hat{\mathbf{y}}\mathbf{k}_0\cdot\hat{\mathbf{y}} , \quad (50)$$

and the associated $\mathbf{H}_s$ field.

Case (b). This problem is discussed by Van Bladel[7] and De Zutter.[27] Presently only first-order effects are discussed, which makes the argument simpler; however, the results are still adequate for the purpose of comparing the three cases, as defined above, and for all practical purposes, first-order approximations are all that will be needed. Now the incident wave is given by (17) with $\beta=0$. The boundary is vibrating according to

$$\mathbf{v}=\hat{\mathbf{x}}v_0\cos(\Omega t) , \quad (51)$$

i.e., the boundary is moving with a displacement

$$x=\xi_0\sin(\Omega t) \quad (52)$$

and the amplitude $\Omega\xi_0$ must be such that our assumptions on v/c are valid. The boundary conditions must be satisfied in the comoving system, i.e., $\hat{\mathbf{x}}\times\mathbf{E}'=\mathbf{0}$ for a perfect conducting metallic boundary. To the first order in the velocity the relativistic transformations for the fields[3] prescribe $\mathbf{E}'=\mathbf{E}+\mu\mathbf{v}\times\mathbf{H}$, which after some manipulation becomes for the present case [(17) with $\beta=0$ and $\mathbf{E}=\mathbf{E}_z$]

$$\mathbf{E}'=\hat{\mathbf{z}}E(1-\hat{\mathbf{k}}_0\cdot\hat{\mathbf{x}}v/C) . \quad (53)$$

Taking the reflected wave as in (50), with $\beta=0$, yields the same transformation (53) for the reflected wave

$$\mathbf{E}_s'=\hat{\mathbf{z}}E_s(1-\hat{\mathbf{k}}_{0s}\cdot\hat{\mathbf{x}}v/C)=\hat{\mathbf{z}}E_s(1+\hat{\mathbf{k}}_0\cdot\hat{\mathbf{x}}v/C) . \quad (54)$$

At the boundary we must therefore satisfy, to the first order in v/c,

$$E_0[1-2\hat{\mathbf{k}}_0\cdot\hat{\mathbf{x}}(v_0/C)\cos(\Omega t)]e^{i\mathbf{k}_0\cdot\hat{\mathbf{x}}(v_0/\Omega)\sin(\Omega t)+i\mathbf{k}_0\cdot\hat{\mathbf{y}}y-i\omega_0 t}+E_s=0\ . \tag{55}$$

Let us guess $\mathbf{E}_s=\hat{\mathbf{z}}E_s$ in the form

$$E_s=-E_0(e^{i\mathbf{k}_{0s}\cdot\mathbf{r}-i\omega_0 t}+A_+e^{i\mathbf{k}_+\cdot\mathbf{r}-i\omega_+ t}+A_-e^{i\mathbf{k}_-\cdot\mathbf{r}-i\omega_- t})\ , \tag{56}$$

where $\mathbf{k}_{0s}$ is given in (50) and A_+,A_- are of order v/c. We already assume the sideband frequencies

$$\omega_\pm=\omega_0\pm\Omega \tag{57}$$

and try to satisfy (55) and (57) simultaneously at the boundary defined by (52). The result is

$$A_\pm=-(v_0/C)\hat{\mathbf{k}}_0\cdot\hat{\mathbf{x}}(1\pm\omega_0/\Omega) \tag{58}$$

provided we meet the condition that $(v_0/C)(\omega_0/\Omega)\ll 1$. If this condition cannot be met, more sidebands will be necessary to satisfy the boundary condition, as done by Van Bladel[7] and De Zutter.[27] In this study it is attempted to compare the three cases described above; therefore, in order to keep the examples as simple as possible, only the first pair of sidebands is derived. Using the expansion of Bessel's function near the origin, the corresponding expressions given by Van Bladel[7] and De Zutter[27] [e.g., see Eq. (10.129) in Van Bladel[7]] are seen to be identical. The propagation vectors are determined from

$$\mathbf{k}_\pm=-\hat{\mathbf{x}}k_{x\pm}+\hat{\mathbf{y}}k_{0y}\ ,\quad k_\pm^2=\omega_\pm^2/C^2\ , \tag{59}$$

which includes the aberration effect.

Case c. For this case we combine cases (a) and (b), i.e., β [Eq. (47)] is now included in the incident and scattered wave. Inasmuch as the same term appears in the exponential of both the incident and scattered fields, it has no effect at the boundary. The effect appears in the scattered wave by adding $i\beta$ in the exponents of (56). To the first order in $(\omega_0-\Omega)\Lambda_0/Q$ we obtain

$$E_s=-E_0(e^{i\mathbf{k}_{0s}\cdot\mathbf{r}-i\omega_0 t}+E_+e^{-i\omega_+ t}+E_-e^{-i\omega_- t})$$
$$E_\pm=A_\pm e^{i\mathbf{k}_\pm\cdot\mathbf{r}}\pm\frac{\omega_0-\Omega}{Q}\frac{\Lambda_0}{2}e^{i\mathbf{k}_{0s}\cdot\mathbf{r}\mp iQx}\ . \tag{60}$$

CYLINDRICAL SCATTERERS

As an example for problems of this kind, we consider a circular cylinder of radius a, whose surface is a perfect conductor such that the tangential $\mathbf{E}$ field vanishes at the surface. For simplicity only one polarization will be considered, that of the $\mathbf{E}=\hat{\mathbf{z}}E$ field parallel to the cylindrical axis. The interacting acoustical wave will be taken as a cylindrical wave leading to (43) or (45).

Case (a). The incident wave is a plane wave as in (17), polarized along the z axis and propagating in the $\hat{\mathbf{x}}$ direction, with β given by (43) or (45), depending on the distance in acoustical wavelengths from the center of the cylinder defined by $r=a$. The incident wave is recast in terms of cylindrical waves[25]

$$\mathbf{E}=\hat{\mathbf{z}}E_0e^{ik_0x-i\omega_0 t+i\beta}=\hat{\mathbf{z}}E_0e^{-i\omega_0 t+i\beta}\sum_{n=-\infty}^{\infty}i^nJ_n(k_0r)e^{in\psi}\ , \tag{61}$$

where J_n are the nonsingular Bessel functions and ψ is the azimuthal angle; r is the distance perpendicular to the axis of the cylinder. In view of (61), the scattered wave is chosen as

$$E_s=E_0e^{-i\omega_0 t+i\beta}\sum_n i^na_nH_n(k_0r)e^{in\psi}\ , \tag{62}$$

where a_n are coefficients and H_n denotes the Hankel functions of the first kind of order n. The term β is identical for the incident and reflected waves, hence the application of the boundary condition

$$E+E_s=0\,|_{\text{at } r=a} \tag{63}$$

simply yields

$$a_n=-J_n(k_0a)/H_n(k_0a) \tag{64}$$

as in the static case. The velocity effect is therefore confined to the effect of β in (62), according to (43) or (45). According to (45) it is obvious that for $r\to\infty$ the velocity effect vanishes.

Case (b). Here we assume $\beta=0$ for a medium at rest, and consider the cylinder to pulsate according to

$$r=a+\xi_0\sin(\Omega t)\ , \tag{65}$$

i.e., the associated velocity is $\hat{\mathbf{r}}\xi_0\Omega\cos(\Omega t)=\hat{\mathbf{r}}v_0\cos(\Omega t)$ in the radial direction. At the moving boundary according to (53) and (65) and to the first order, the incident wave becomes

$$\mathbf{E}'\cong\hat{\mathbf{z}}E_0[i-\hat{\mathbf{k}}_0\cdot\hat{\mathbf{r}}(v_0/C)\cos(\Omega t)]e^{-i\omega_0 t}\sum_n i^nJ_n[k_0a+(k_0v_0/\Omega)\sin(\Omega t)]e^{in\psi}$$
$$\cong\hat{\mathbf{z}}E_0e^{-i\omega_0 t}\sum_n i^ne^{in\psi}\{J_n(k_0a)-J_n(k_0a)\mathbf{k}_0\cdot\mathbf{r}\cos(\Omega t)+[J_n'(k_0a)k_0v_0/\Omega]\sin(\Omega t)\}\ , \tag{66}$$

where J_n' denotes the derivative with respect to the argument and only the first correction term in the Taylor expansion is retained. Subject to the present approximation, (66) displays a dependence on the original frequency ω_0 and the sidebands $\omega_0\pm\Omega$; therefore, these frequencies must also be present in the scattered wave. According to the (first order in v/c) relativistic transformation $\mathbf{E}'=\mathbf{E}+\mu\mathbf{v}\times\mathbf{H}$, taking $\mathbf{H}$ to the zero-order approximation, we have in the $\hat{\mathbf{z}}$ direction

$$E_s' = E_s + (v_0/C)\cos(\Omega t)e^{-i\omega_0 t}E_0 \times \sum_n i^{n+1} a_n H_n'(k_0 a) e^{in\psi} , \tag{67}$$

where a_n is given by (64). The boundary condition to be satisfied is

$$\mathbf{E}' + \mathbf{E}_s' = \mathbf{0} \,|_{\text{at } r = a + \xi_0 \sin(\Omega t)} . \tag{68}$$

We therefore choose E_s as

$$E_s = E_0 \sum_n i^n e^{in\psi} [a_n e^{-i\omega_0 t} H_n(k_0 r) + a_n^+ e^{-i\omega_+ t} H_n(k_+ r) + a_n^- e^{-i\omega_- t} H_n(k_- r)] ,$$
$$\omega_\pm = \omega_0 \pm \Omega , \tag{69}$$
$$k_\pm = \omega_\pm / C ,$$

and derive a_n^+, a_n^-, which are already of first order, subject to (65)—(69). This yields

$$a_n^\pm = \frac{-i}{2H_n(k_\pm a)} [\pm a_n H_n'(k_0 a)(k_0 \xi_0 \pm v_0/C) \pm k_0 \xi_0 J_n'(k_0 a) + i J_n(k_0 a) \hat{\mathbf{k}}_0 \cdot \hat{\mathbf{r}} v_0/C] . \tag{70}$$

Therefore, the problem is solved, subject to the abovementioned approximations.

Case (c). Including the fluid motion, caused by the motion of the surface at frequency Ω, prescribes a factor $e^{i\beta}$ in both the incident wave, as given in (61), and in the scattered wave, given in (69). This factor is identical for both fields and therefore has no effect when (68) is satisfied. Depending on the distance, expressed in wavelengths of the acoustical wave, β is given by (43) or (45). However, it is clear that in the far acoustical field the effects diminishes. This means that for cases of this kind the medium's motion can be altogether ignored and case (b), although it violates mass continuity, yields a satisfactory result.

SPHERICAL SCATTERERS

The case of scattering by a static sphere is a classical problem, e.g., see Stratton.[25] Here we consider the first-order effect produced by a pulsating perfectly conducting sphere. The vector spherical waves and harmonics are defined here as in Stratton,[25] see also Twersky[28] and Censor and Le Vine.[26]

Case (a). The incident wave is a plane wave as in (17). In order to conform with the analysis of Stratton[25] (pp. 563ff), we take $\mathbf{k}_0 = \hat{\mathbf{z}} k_0$, $\mathbf{E} = \hat{\mathbf{x}} E$, hence

$$\mathbf{E} = \hat{\mathbf{x}} E_0 e^{ik_0 z - i\omega_0 t + i\beta} = \hat{\mathbf{x}} E_0 e^{-i\omega_0 t + i\beta} \sum_{n=1}^{\infty} d_n (\mathbf{M}_{on}^{(1)} - i\mathbf{N}_{en}^{(1)}) ,$$
$$d_n = i^n \frac{2n+1}{n(n+1)} ,$$
$$\mathbf{M}_{\substack{o\\e}n}^{(1)} = \pm \frac{\hat{\boldsymbol{\theta}}}{\sin\theta} j_n(k_0 r) P_n^1(\cos\theta) \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} \psi - \hat{\boldsymbol{\psi}} j_n(k_0 r) \partial_\theta P_n^1(\cos\theta) \begin{Bmatrix}\sin\\ \cos\end{Bmatrix} \psi , \tag{71}$$
$$\mathbf{N}_{\substack{o\\e}n}^{(1)} = \hat{\mathbf{r}} \frac{n(n+1)}{k_0 r} j_n(k_0 r) P_n^1 \begin{Bmatrix}\sin\\ \cos\end{Bmatrix} \psi + \hat{\boldsymbol{\theta}} f(k_0 r) \partial_\theta P_n^1 \begin{Bmatrix}\sin\\ \cos\end{Bmatrix} \psi + \hat{\boldsymbol{\psi}} \frac{f(k_0 r)}{\sin\theta} f(k_0 r) P_n^1 \begin{Bmatrix}\cos\\ \sin\end{Bmatrix} \psi ,$$
$$f(\rho) = [\rho j_n(\rho)]'\rho, \quad \rho = k_0 r ,$$

where the prime denotes differentiation ∂_ρ.

In the present form (71) follows Stratton's Eqs. (1)—(3), (Ref. 25, p. 564), in a slightly compacted manner. As in Stratton,[25] the superscript (1) denotes the nonsingular spherical Bessel functions. P_n^1 is the associated Legendre polynomial and o,e stand for odd, even functions, respectively. In terms of vector spherical harmonics (71) can be written as

$$\mathbf{M}_{\substack{o\\e}n}^{(1)} = j_n(k_0 r) \mathbf{C}_{\substack{o\\e}n} ,$$
$$\mathbf{N}_{\substack{o\\e}n}^{(1)} = [n(n+1) j_n(k_0 r)/kr] \mathbf{P}_{\substack{o\\e}n} + f(k_0 r) \mathbf{B}_{\substack{o\\e}n} , \tag{72}$$

and we shall use the orthogonality properties of the vector spherical harmonics $\mathbf{C},\mathbf{P},\mathbf{B}$ as needed, below. The $\mathbf{H}$ field associated with $\mathbf{E}$ (71) is given by Stratton,[25] to which we add a factor $e^{i\beta}$. The scattered field is again according to Stratton

$$\mathbf{E}_s = E_0 e^{-i\omega_0 t + i\beta} \sum_{n=1}^{\infty} d_n (a_n \mathbf{M}_{on}^{(3)} - i b_n \mathbf{N}_{en}^{(3)}) , \tag{73}$$

where the superscript (3) denotes the dependence on the spherical Hankel functions of the first kind $h_n(k_0 r)$, replacing $j_n(k_0 r)$ in (71) and (72). For scattering by a perfectly conducting sphere of radius $r = a$, the tangential $\mathbf{E}$ field at the surface must vanish

$$\hat{\mathbf{r}}\times(\mathbf{E}+\mathbf{E}_s)=\mathbf{0}\,|_{\text{at } r=a} \ . \tag{74}$$

Note that in (71)–(74) $\hat{\mathbf{r}}\times\mathbf{N}$ eliminates the longitudinal vector spherical harmonics, i.e., $\hat{\mathbf{r}}\times\mathbf{P}=\mathbf{0}$, and the result of (74) is[25]

$$\begin{aligned} a_n &= -j_n(k_0a)/h_n(k_0a) \ , \\ b_n &= -[k_0aj_n(k_0a)]'/[k_0ah_n(k_0a)]' \ . \end{aligned} \tag{75}$$

Thus the problem is solved and the only difference between the present case of a moving medium and the static case is the factor $e^{i\beta}$, where β is given by (39) or (40), depending on the distance. From (40) it is clear that as r increases, the effect of the moving medium decreases. This will also be relevant for case (c) discussed below.

Case (b). Here we deal with scattering by a pulsating sphere immersed in a medium at rest. The surface's motion is described by (65), where r now stands for the distance from the origin (as opposed to the distance from the cylindrical axis for the cylindrical scatterer case). The boundary condition has to be satisfied at the boundary in the comoving frame of reference. Accordingly, to the first order, we have to consider $\mathbf{E}'=\mathbf{E}+\mu v\hat{\mathbf{r}}\times\mathbf{H}$, where $\mathbf{H}$ is taken as the zero-velocity approximation at $r=a$. Hence, at the boundary

$$\mathbf{E}'=\mathbf{E}\,|_{r=a+\xi_0\sin(\Omega t)}-(v_0/C)\cos(\Omega t)E_0e^{-i\omega_0 t}\times\sum_n d_n\hat{\mathbf{r}}\times(\mathbf{M}_{en}^{(1)}+i\mathbf{N}_{on}^{(1)}) \ , \tag{76}$$

where the expression in square brackets follows from Stratton's Eq. (1) (Ref. 25, p. 564). Once more we use a Taylor expansion and retain the leading terms

$$\mathbf{E}\,|_{r=a+\xi_0\sin(\Omega t)}=\mathbf{E}\,|_{r=a}+\xi_0\sin(\Omega t)\partial_r\mathbf{E}\,|_{r=a} \ . \tag{77}$$

The boundary condition is

$$\hat{\mathbf{r}}\times(\mathbf{E}'+\mathbf{E}_s')=\mathbf{0}\,|_{r=a+\xi_0\sin(\Omega t)} \ , \tag{78}$$

hence we need $\hat{\mathbf{r}}\times\mathbf{E}'$. Using the properties of the vector spherical harmonics

$$\begin{aligned} &\hat{\mathbf{r}}\times\mathbf{P}=\mathbf{0} \ , \\ &\hat{\mathbf{r}}\times\mathbf{C}=\mathbf{B} \ , \\ &\hat{\mathbf{r}}\times\mathbf{B}=-\mathbf{C} \ , \end{aligned} \tag{79}$$

we find from (76) and (77) $\hat{\mathbf{r}}\times\mathbf{E}'$ at the boundary

$$E_0e^{-i\omega_0 t}\sum_n d_n(j_n\mathbf{B}_{on}+if_n\mathbf{C}_{en}+\xi_0\sin(\Omega t)(\partial_a j_n\mathbf{B}_{on}+i\partial_a f_n\mathbf{C}_{en})+(v_0/C)\cos(\Omega t)(j_n\mathbf{C}_{on}-if_n\mathbf{B}_{en})] \ , \tag{80}$$

where $j_n, f_n, \partial_a j_n, \partial_a f_n$ have the argument k_0a. It is clear from (80) that in addition to the original frequency ω_0, we have also the sidebands $\omega_\pm=\omega_0\pm\Omega$ present. Furthermore, the velocity-dependent terms multiplied by v_0/C have $\mathbf{C},\mathbf{B}$ factors of opposite parity. Consequently, the scattered wave is chosen as

$$\begin{aligned} \mathbf{E}_s &= E_0e^{-i\omega_0 t}\sum_n d_n(a_n\mathbf{M}_{on}^{(3)}-ib_n\mathbf{N}_{en}^{(3)}+\mathbf{G}_n) \ , \\ \mathbf{G}_n &= a_+^o e^{-i\Omega t}\mathbf{M}_{on}^{(3)(+)}+a_+^e e^{-i\Omega t}\mathbf{M}_{en}^{(3)(+)}+a_-^o e^{i\Omega t}\mathbf{M}_{on}^{(3)(-)}+a_-^e e^{i\Omega t}\mathbf{M}_{en}^{(3)(-)} \\ &\quad +b_+^o e^{-i\Omega t}N_{on}^{(3)(+)}+b_+^e e^{-i\Omega t}\mathbf{N}_{en}^{(3)(+)}+b_-^o e^{i\Omega t}\mathbf{N}_{on}^{(3)(-)}+b_-^e e^{i\Omega t}\mathbf{N}_{en}^{(3)(-)} \end{aligned} \tag{81}$$

where superscripts $(+),(-)$ indicate dependence on k_+,k_-, respectively, as defined in (69). All the correction terms involving $a_{+,-}^{o,e}, b_{+,-}^{o,e}$ in (81) constitute legitimate vector wave functions and solutions of the Maxwell equations. It is assumed that all these terms, i.e., G, are already of first order. By inspection of (80) and (81), and replacing j_n by h_n, which will be symbolized by replacing f_n by g_n, we obtain $\hat{\mathbf{r}}\times\mathbf{E}_s'$ at the boundary

$$\begin{aligned} &E_0e^{-i\omega_0 t}\sum_n d_n\{[a_nh_n\mathbf{B}_{on}+ib_ng_n\mathbf{C}_{en}+\xi_0\sin(\Omega t)(a_n\partial_a h_n\mathbf{B}_{on}+ib_n\partial_a g_n\mathbf{C}_{en}) \\ &\qquad +(v_0/C)\cos(\Omega t)(b_nh_n\mathbf{C}_{on}-ia_ng_n\mathbf{B}_{en})]+\hat{\mathbf{r}}\times\mathbf{G}_n\} \ , \\ &\hat{\mathbf{r}}\times\mathbf{G}_n=a_+^o e^{-i\Omega t}h_n^+\mathbf{B}_{on}+a_+^e e^{-i\Omega t}h_n^+\mathbf{B}_{en}+a_-^o e^{i\Omega t}h_n^-\mathbf{B}_{on}+a_-^e e^{i\Omega t}h_n^-\mathbf{B}_{en} \\ &\qquad -b_+^o e^{-i\Omega t}g_n^+\mathbf{C}_{on}-b_+^e e^{-i\Omega t}g_n^+\mathbf{C}_{en}-b_-^o e^{i\Omega t}g_n^-\mathbf{C}_{on}-b_-^e e^{i\Omega t}g_n^-\mathbf{C}_{en} \ , \end{aligned} \tag{82}$$

where $h_n^\pm, g_n^\pm$ indicate the dependence on $k_\pm$. The sum of (80) and (82) vanishes, and from the fact that terms are orthogonal with respect to frequency, e,o parity, and orthogonality of vector spherical harmonics, the coefficients are computed according to

$$\begin{aligned} &\xi_0(\partial_a j_n + a_n \partial_a h_n) + 2ia_-^o h_n^- = 0 \,, \\ &\xi_0(\partial_a j_n + a_n \partial_a h_n) - 2ia_+^o h_n^+ = 0 \,, \\ &\xi_0(\partial_a f_n + b_n \partial_a g_n) - 2b_-^e g_n^- = 0 \,, \\ &\xi_0(\partial_a f_n + b_n \partial_a g_n) + 2b_+^e g_n^+ = 0 \,, \\ &(v_0/C)(j_n + b_n h_n) - 2b_-^o g_n^- = 0 \,, \\ &(v_0/C)(j_n + b_n h_n) - 2b_+^o g_n^+ = 0 \,, \\ &(v_0/C)(f_n + a_n g_n) + 2ia_-^e h_n^- = 0 \,, \\ &(v_0/C)(f_n + a_n g_n) + 2ia_+^o h_n^+ = 0 \,, \end{aligned} \tag{83}$$

and therefore the problem is considered solved.

Case (c). By juxtaposition of cases (a) and (b) it is possible to account for the effect produced by the motion imparted to the surrounding medium. It is clear from the above discussion of the cylinder that the moving medium will not affect the evaluation of the boundary conditions and the effect will vanish as r becomes very large.

CONCLUDING REMARKS

In the past, scattering of electromagnetic waves by moving surfaces was considered mainly in free space (vacuum). For objects immersed in a material medium, there existed no theory that could take into account the motion produced by the moving surfaces. Therefore, the motion of the surrounding medium was heuristically neglected. The present study provides a formalism for dealing with this class of problems. Simple cases discussed above show that for bounded objects, the effect of the medium set in motion by the moving surfaces can actually be neglected at large distances from the scatterer. This conclusion vindicates the heuristic approximation of neglecting the medium's motion. However, the present results show that this approximation is valid only far away from the objects.

It must be stressed that the present theory also relies on certain restrictions and heuristic assumptions, as mentioned above. In order to derive more general conclusions, this class of problems must be further investigated.

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