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Multistability and confined traveling-wave patterns in a convecting binary mixture

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A novel localized structure of traveling waves (TW's) was observed at convection onset in ethanol-water mixtures. It is one of various possible flow patterns of TW's which were observed. We find that for almost-one-dimensional cell geometry unique dynamics of transient behavior (in the form of two counterpropagating TW's) can lead to various stable states.

We present an observation of a new nonlinear hydrodynamic structure which appears in pattern-forming nonequilibrium systems: a traveling-wave (TW) pattern localized in space. This self-organized confined structure exists in convecting binary mixtures as a steady state and persists along the finite-amplitude TW branch in spite of the fact that it is less efficient in heat transport than a fully convecting cell. This phenomenon occurs in the region of phase space where oscillatory convection in the form of TW's appears first through an inverse bifurcation from the conductive state. Our studies of the dynamics of transient behavior at the convective onset suggest that the origin of this localized state lies in the existence of waves reflected from lateral boundaries of the cell and of backflow due to mass conservation in a finite-geometry cell. Experimental evidence to this effect is presented.

The existence of confined TW patterns is accompanied by an additional new phenomenon of multistability in patterns selected by the nonlinear system, and in the corresponding effective heat transport. This phenomenon of multistability is intermediate between the simplicity of deterministic systems and the complexity of chaotic ones. A well-known example of such behavior in nonlinear dissipative systems is related to the wave-number selection problem. We report on various patterns which were observed for the same set of control parameters. Some of these patterns are unstable, and they were observed only as a transient, but others are stable. Our flow-visualization and light-intensity measurements reveal that the dynamics of transient behavior toward finite-amplitude oscillatory convection is universal. However, the same dynamics leads to different final states.

The experiment was conducted on ethanol-water mixtures at room temperatures. Our experimental setup together with the measuring techniques and their resolution have already been described elsewhere.[1,2] We operated at a weight concentration between of 24% and 28% ethanol, and with top-plate temperatures ranging from 22 to 35 °C. Variation of mean temperature and concentration gives us an opportunity to tune the separation ratio ψ, which is a measure of the coupling between the temperature and concentration gradients, and is a control parameter of the system. Another control parameter is the temperature gradient, as usual in Rayleigh-Benard convection. The oscillatory state that has been observed in the above-mentioned range of T and C is reached by an inverse bifurcation, and consists of rolls that move along the cell as TW's.[1-3]

We performed most of the experiments in a rectangular cell with height 0.297 cm and aspect ratio 1:4:12, and studied transient behavior also in a cell twice as long with height 0.293 cm and aspect ratio 1:4:24. Our experimental procedure consisted of raising the bottom-plate temperature by steps, applying to it a constant heat output. Typical changes in heat flux were between 0.1% and 2% of the critical value, which corresponds to temperature steps between about 2 and 40 mK at typical critical temperature difference across the cell on the order of 2 K. The system was monitored simultaneously by high-resolution heat-transfer measurements, a shadowgraph visualization technique, local light-intensity measurements at selected locations in the cell, and intensity scan on a line along the length of the cell which is averaged in time and space and computer enhanced.

In all measurements the first state that appears is a transitory one. These transients have been investigated already in 1974, in an impressive experiment by Caldwell,[4] and recently were revived by light-intensity measurements.[5] The main message from these studies is that over the long transient period where the amplitude is growing exponentially, the nonlinear effects are not important and the amplitude growth rate is proportional to ε ($\varepsilon \equiv \Delta T/\Delta T_c - 1$) with coefficient close to the theoretically predicted one, and the frequency is constant and fits pretty well to the theoretical value.[6] The most surprising observation which results from flow visualization in Ref. 5 is that the patterns observed in the transient regime can be represented reasonably accurately by two counterpropagating waves with characteristic length λ^{-1} on the order of the cell size. Our flow visualization suggests that the transient dynamics to TW's can be divided into three stages: First, a noisy signal appears along the cell in our light scan with characteristic length on the order of $2d$, with d the cell height; then two counterpropagating waves appear simultaneously emanating from the center of the cell. The interaction point of the two TW's is considered as a topological defect which simulates a standing wave of one wavelength independent of the cell size. In the final stage, competition between the two counterpropagating waves causes the defect to move. This motion strongly depends on slight asymmetry of the cell, imperfections, and location of the controlling parameters on the phase diagram. In the simple case the defect moves in one of the preferred directions and leaves the cell, and only one of the TW's remains in the cell. Thus, the behavior of the defect in the third stage defined the final spatial pattern.

In the early stage of the transient the profiles show some sign of modulation along the whole cell (as suggested by the fit of Ref. 5 for the full range of the linear growth regime). However, at a later stage, but still deep in the linear growth regime, the profiles flatten out and become more like two distinct counterpropagating waves emanating at the meeting point. This structure corresponds to a nonlinear state which exists in a large part of the linear growth-rate regime. The interaction region is small relative to the cell length, consisting of two domain walls with thickness λ^{-1} of about $2d-4d$ for ψ between -0.06 and -0.15. Figure 1 presents a "typical" pattern in this regime. Moreover, the data we present in Fig. 1 are deeper in the linear growth rate regime than the data presented in Ref. 5. The possible explanation of the quantitative disagreement in the linear regime with the data of Ref. 5 and the theoretical prediction about the value of λ^{-1} of Ref. 7 are probably related to the existence of a backflow. It seems that this effect is more pronounced in our quasi-one-dimensional (1D) cells compared with the twice wider cell used in Ref. 5. The backflow could be a reason for smoothing out the large velocity amplitude close to the cell ends, and its coupling to the main flow can produce a nonlinear effect even in the regime of the linear growth.[8] This discussion indicates that the question of linear versus nonlinear behavior during the transients is actually rather tricky and has not yet been settled.

The first two stages of the transient show almost no noticeable change in the effective heat transport[9] and belong to the linear growth-rate regime. This insensitivity of a global measurement versus a local one may indicate some internal inconsistency of the one-mode-approximation approach in the transient region. The two counterpropagating waves by themselves are not stable. Their amplitude grows constantly, and we have never obtained such a configuration as a stable one with constant amplitude. In the third stage the topological defect spontaneously begins to migrate along the cell choosing one or another direction according to slight asymmetry of the apparatus. This migration in most cases is not smooth or regular and reflects competition between two counterpropagating waves and waves reflected from the lateral boundaries, which are complicated by backflow existing in finite geometry cells.[1,2] Such a process is demonstrated in Fig. 2, where sequential light-intensity profiles along the cell clearly show the existence and competition of two counterpropagating waves together with the reflecting wave and the region free from convection. Figure 2 provides experimental indication that development of a confined state from a fully convecting cell can be a consequence of interaction with the reflecting wave. This surprising mechanism plays a crucial role in a theoretical model proposed independently by Cross.[7] Thus, this three-stage transient is a general dynamics of the inverse bifurcation to the TW branch, which can lead to different final steady states. The most common state observed is that of TW's propagating in one direction,[1-3] filling the whole cell with no confinement.[10] A drastic change in frequency and effective heat transport occurs when the topological defect leaves the cell.

We observe, however, situations when the defect does not leave the cell. Instead, it gets "stuck" at a point midway along the cell, and states in which only a part of the cell convects are found. An example of such a spatial formation is presented in a sequence of patterns initially showing the evolution of two counterpropagating waves into a confined convecting state (Fig. 3). The TW in the right half of the cell first strengthens [Figs. 3(a) and 3(b)] then decays [Fig. 3(c)] (a "blinking" state). The wave in the left half then grows and dominates but does not progress into the nonactive region [Fig. 3(d)]. This leaves a TW confined to one half of the cell with little or no activity on the other half. Figure 4 shows profiles of light intensity along the length of the cell for a similar confined convec-

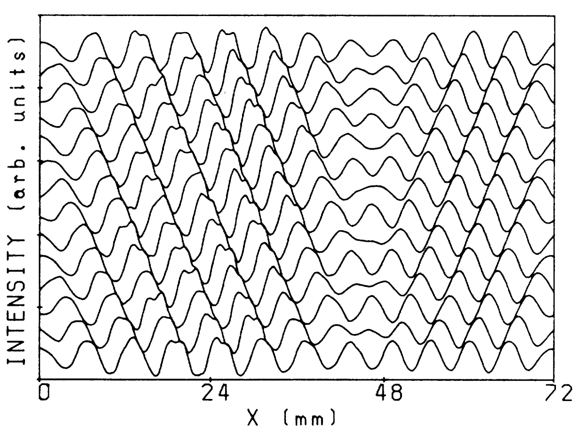

FIG. 1. Contour plot of intensity profiles along the cell show two counterpropagating waves for the sample of 24 wt.% of ethanol at 30°C (long cell with aspect ratio 1:4:24, time increases upwards, and the interval between successive profiles is 22 sec). This state is reached 105 min after changing the heat flux by about 1%.

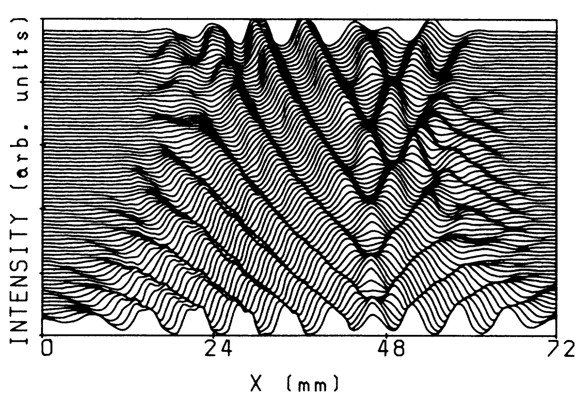

FIG. 2. Intensity profiles along the cell for the transient state during the third stage. Competition between two countrerpropagating waves and reflecting waves which leads to migration of the topological defect and confinement of convection is clearly demonstrated (the same sample and cell as that in Fig. 1, but 125 min after change in heat power). Time increases upwards, with intervals as in Fig. 1.

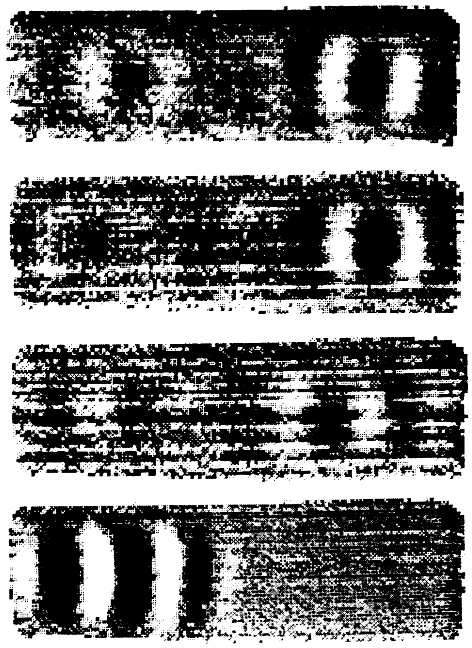

FIG. 3. Sequential pattern pictures for the low branch of convective heat transport for the sample of 27 wt.% of ethanol at 30.2°C (short cell with aspect ratio 1:4:12).

tive state in the long cell. One TW originating at the center of the cell is observed. The side that dominates depends on slight asymmetries, and we have obtained patterns with either side convecting for different runs. The frequency of oscillations is high (4–6 mHz), and less than but close to the neutral one. Figure 5 shows the Nusselt-number behavior for the sample of Fig. 3. The lower branch of the plot corresponds to a half convecting cell. This confined state is stable over many hours, and along a

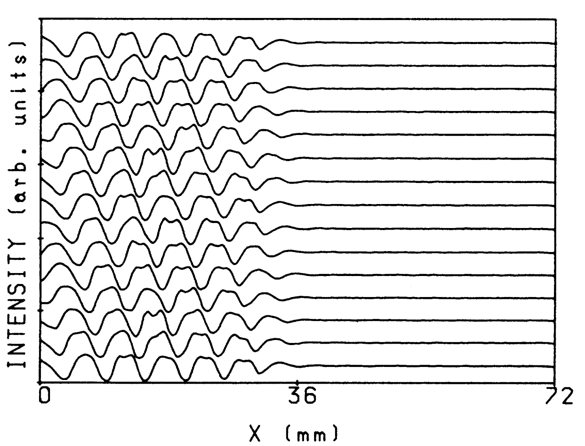

FIG. 4. Intensity profiles along the cell show right propagating rolls in the half convecting cell for the sample of 24% wt.% of ethanol at 30 °C (aspect ratio is 1:4:24, time increases upwards, with intervals as in Fig. 1).

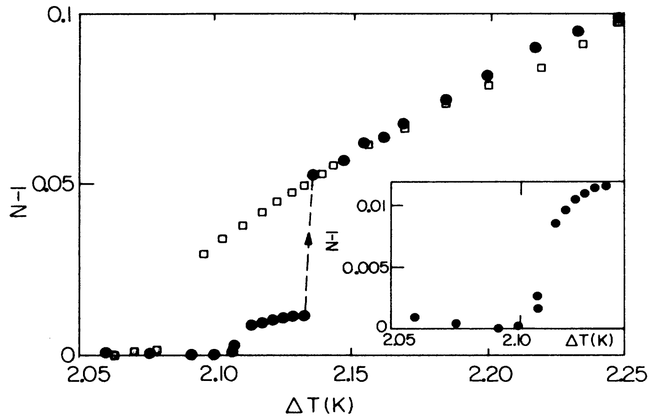

FIG. 5. Convective contribution $N-1$ to heat transport as a function of ΔT for the same sample as in Fig. 3 (circles: on heating; squares: on cooling).

range of 40–50 mK in ΔT. Then it evolves directly to a fully convecting cell with a jump in N of about 4% which is accompanied by a corresponding change in the frequency.

One of the natural reasons for the reported pattern behavior could be geometrical imperfection of the cell. Since the convection threshold is strongly dependent on the cell height, a confined convective structure can be obtained in a cell which is nonuniform in height. Nevertheless, we observed a perfect parallel roll structure along the whole cell at the onset of stationary convection in pure water. We also checked the parallelity of our cell by interferometric methods, and obtained less than 3 μm on the whole length. Finally, at the onset we see convection in a binary mixture in all parts of the cell, and only then does convection decay in part of it. This, together with the fact that we have obtained, in consecutive runs at unchanged conditions, convection confined to each of the two opposite sides of the cell while the other half remained inactive, assures us that geometrical imperfections in the cell are below the level of sensitivity of the convective structure.

Recently, two theoretical groups[7,8] considered the effect of finite geometry on spatial structure of TW's in a weakly nonlinear regime. The three possible solutions of an amplitude equation in a finite-1D-geometry cell with reflection effect obtained by Cross[7] are very similar to the spatial patterns observed in our experiment. He suggests a selection criterion for appearance of a confined structure (group velocity of TW's in nondimensional units $\tilde{s} > 2$).[7] At this point we would like to comment on the values of the control parameters at which the confined TW pattern is observed. In the small cell (aspect ratio 1:4:12) this phenomenon was attained in a pretty narrow region of the phase diagram, at ψ between -0.06 and -0.1, and steps in heat flux $q/q_c - 1 < 2\%$. However, in the large cell the confined TW state was always observed in the range of ψ considered in the experiment (between about -0.06 and -0.15) and $q/q_c - 1 < 3\%$. Preliminary studies show agreement with the criterion given by Cross in the small cell, at the transition from a TW that occupies half the cell to a fully convecting cell. At the moment our analysis of the full data shows a similar trend, but gives no conclusive results on this criterion. It is possible that a generalized criterion can be obtained by taking into account mass conservation (backflow) which probably plays a crucial role in the almost-1D cell geometry.

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[6]The value of $r_{c0}\partial\gamma/\partial r$ (in the terminology of Ref. 5) calculated from the data of Fig. 10 of Ref. 4 gives 0.025 instead of the theoretical value 0.03. Since the early experiment by Caldwell has been done for an inverse bifurcation with only 5–6% jump in heat transfer at the transition instead of about 50% in the experiment reported in Ref. 5, a one-mode approximation would be expected to describe the former data in a much more reliable way.

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[9]This observation was also clearly demonstrated in Fig. 8 of Ref. 4.

[10]In the case of large aspect ratio we observed additional final spatial patterns that we did not find in the short cell, and the corresponding dynamics of the third stage was more complicated. These results will be published elsewhere.