Objections to Handel's quantum theory of $1/f$ noise

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A quantum theory of quantum $1/f$ noise was proposed by Handel in 1975. It relates $1/f$ noise in currents to infrared divergences in cross sections due to, e.g., soft-photon emission. We state a number of objections to this theory. Many of the points we raise have been raised before, but have not appeared in the open literature. Our objections are partly of a practical nature, while some concern the mathematical framework of the theory.

INTRODUCTION

In 1975 Handel[1,2] proposed a relation between $1/f$ noise and infrared divergencies arising from soft photon emission (bremsstrahlung). In this picture, $1/f$ noise is due to interference ("quantum beats") between elastically and inelastically scattered waves, which emerge when a beam of particles is scattered under the influence of a potential. This mechanism has been termed "quantum $1/f$ noise." It predicts $1/f$ noise in any system whenever the cross section for scattering of particles exhibits an infrared divergence due to the generation of low-frequency excitations (photons, phonons, gravitons, etc.).

Attempts by Tremblay[3] to reproduce Handel's theoretical results[2,1] in the second-quantization language ran into severe difficulties of a fundamental nature. Tremblay did obtain the same result as Handel for the average current. However, he found that Handel's derivation of the current-fluctuation spectrum contains serious mistakes. The final conclusion in Ref. 3 is that the mechanism proposed by Handel cannot give rise to $1/f$ noise in current fluctuations. To our knowledge Tremblay's objections have not been answered in the subsequently published literature.[4] Nevertheless, Handel's theories are still extensively used by some, although by no means all, experimentalists. They even seem to be gaining in popularity.[5–8] It is, however, our conviction that the theoretical foundations of the quantum theory of $1/f$ noise are at present in as shaky a state as Tremblay[3] found them in 1978. Therefore, we wish to point out what are, in our opinion, errors or misconceptions in Handel's more recent publications on quantum $1/f$ noise.

Before proceeding we give a brief summary of Handel's quantum theory of $1/f$ noise in the version employed after 1978. In order to avoid misrepresentations of Handel's ideas we quote directly from one of his papers (Ref. 9, p. 750), thereby renumbering his Eq. (3.1) to become our Eq. (1).

"If the incoming beam of electrons is described by a wave function $\exp[(i/h)(\mathbf{p}_i\cdot\mathbf{r}-Et)]$, the scattered beam will contain a large nonbremsstrahlung part of amplitude a, and an (incoherent) mixture of waves of amplitude $ab_T(\epsilon)$ with bremsstrahlung energy loss ϵ ranging from some resolution threshold ϵ_0 to an upper limit $\Lambda<E$, of the order of the kinetic energy E of the electrons

$$\Psi_T=\exp[(i/h)(\mathbf{p}\cdot\mathbf{r}-Et)]a\left[1+\int_{\epsilon_0}^{\Lambda} b_T(\epsilon)e^{i\epsilon t/h}d\epsilon\right], \quad |t|<\tfrac{1}{2}T. \qquad (1)$$

Here $b_T(\epsilon)\equiv|b_T(\epsilon)|e^{i\gamma_\epsilon}$ has a random phase γ_ϵ which implies incoherence of all bremsstrahlung parts. This incoherence may be related to the undetermined character of the time of the photon emission. The threshold ϵ_0 is given by the lowest frequency f_0 measured ($\epsilon_0=hf_0$). The subscript T indicates that Eq. (1) represents only a sample of duration $T>f_0^{-1}$ of the Schrödinger field of the scattered wave. Since we are dealing with a stationary process, the Fourier transform $b_T(\epsilon)$ can be defined only for a finite duration sample, and $|b_T(\epsilon)|^2\sim T$ for large values of T."

From Eq. (1) Handel calculates the particle density $|\psi_T|^2$ and the particle current density $\mathbf{j}=(\hbar/2mi)(\psi_T^*\nabla\psi_T-\psi_T\nabla\psi_T^*)=(\mathbf{p}/m)|\psi_T|^2$. These quantities are averaged over a time T to yield their steady-state values. In order to calculate current correlations Handel considers the quantity (Ref. 9, p. 751)

$$\langle j(t)j(t+\tau)\rangle=(p^2/m^2)\langle\,|\psi_T|_t^2\,|\psi_T|_{t+\tau}^2\rangle, \qquad (2)$$

where the angular brackets denote time averaging over a period T (or statistical averaging over the random phases γ_ϵ). The result for the spectral density of density current fluctuations is [Ref. 9, Eq. (5.5)]

$$\langle|\psi|^2\rangle^{-2}S_{|\psi|^2}(f)\equiv\langle\mathbf{j}\rangle^{-2}S_j(f)\simeq 2\alpha A/f, \qquad (3)$$

where α is the fine-structure constant ($\alpha\simeq\frac{1}{137}$) and

$$A=(2/3\pi)(\Delta\mathbf{v})^2/c^2 \qquad (4)$$

with $\Delta \mathbf{v}$ being the velocity change of the particle being scattered and c the speed of light. Finally, if the total particle current is built up from contributions of N independent particles, the noise level in Eq. (3) has to be divided by N [Ref. 9, Eq. (5.6)]

$$S_I(f)=2\alpha A\langle I\rangle^2/(Nf) . \quad (5)$$

The latter expression is taken to be the prediction for $1/f$ noise in solids.[10]

Our objections to the quantum theory of $1/f$ noise described above are threefold. First of all we wish to comment on the applicability of the highly idealized theoretical model for the transport of electrical charges in dense systems. Second, we give a physical argument why the mechanism described by Handel cannot introduce any additional fluctuations in a beam of scattered particles in addition to the Poissonic fluctuations already present in the incoming beam. Third, we intend to show in a way as simple as possible that Handel's treatment of the above model makes use of theoretical tools and concepts that have no place in conventional quantum mechanics.

PHYSICAL OBJECTIONS

Let us accept the validity of the description of low-frequency electrical-current fluctuations in solids by the highly idealized picture described above: a beam of particles undergoes a single scattering event during which infrared quanta are generated. Two simple "practical" objections can be made. It was pointed out by Tremblay (Ref. 3, p. 183) that the fact that electrons have a finite inelastic mean free path (which cannot be larger than the sample) "prohibits the very-long-time coherence of the scattered electrons, which is necessary for the radiation of very-low-frequency photons."[11] Moreover, even if the latter objection did not apply, the theory could not explain the many experimental observations of $1/f$ noise in Faraday cages.[12] After all, in a cage of diameter L no photon with a wave vector less than π/L can be excited by the scattering process. As the frequency spectrum of quantum $1/f$ noise is directly related to the frequency of the emitted photons, no $1/f$ noise with a frequency less than c/L should be observed. For a typical cage size of $L\simeq 3m$ this leads to a lower-frequency cutoff of about 10^8 Hz. This frequency is some 14 orders of magnitude larger than the lowest frequency at which $1/f$ noise has been observed so far.

Before discussing a number of specific objections to Handel's theoretical derivation of $1/f$ noise, we wish to consider one of the most outstanding conclusions of the quantum $1/f$ theory, viz., the quantitative prediction of $1/f$ noise in the α decay of ${}^{95}\mathrm{Am}^{241}$.[5] We do not feel competent to suggest which factors may explain the rather startling experimental results reported in Ref. 5. However, it is easy to see that quantum $1/f$ noise cannot be responsible. To this end we need only remark that the theory does not take into account any interaction between different decay events. Hence, whatever mechanisms influence the α particles between the moment of emission and the moment of entering the counter, they can only cause a delay or a distribution of delay times. Since the decay itself is Poissonic, the convolution of decay times and delay times also is Poissonic. Hence, solely from the absence of interactions between different decay processes we conclude that the theory should predict Poisson behavior.

As far as the application to solids is concerned, we follow Handel (Ref. 9, p. 752) and assume that the incoming particles are Poisson distributed. Due to the close connection between the description of noise in a clean system such as α decay [cf. Eq. (3)] and the prediction of noise in solids [cf. Eq. (5)], the objection stated above carries over to the description of noise in solids as well; Handel's theory does not take into account correlations between different electrons and therefore should not predict noise other than Poisson noise.

In view of this observation we shall not repeat the calculations for Handel's experimental situation. The original version of the theory this was already done by Tremblay.[3] We shall rather restrict ourselves to pointing out some incorrect steps in the derivation.

MATHEMATICAL OBJECTIONS

Let us now look a little closer at the theoretical framework used by Handel to describe $1/f$ noise. We first note that Eq. (1) cannot be correct, since the wave functions of the emitted photons have been omitted from the second term. This is essential because no interference is possible between a state without photons[13] [the first term in Eq. (1)] and a state with at least one photon (the second term); they are orthogonal in the Hilbert space of the total system, which consists of the particle and the radiation field. The interference [quantum beats] (Ref. 13) is essential in Handel's derivation of $1/f$ noise. Handel has put forward the justification (Ref. 9, p. 753) that "in the case of any real $1/f$-noise measurement of the beam, the emitted bremsstrahlung has left the system or has been absorbed in the shielding." However, it would be in contradiction with the principles of quantum mechanics to think that the disappearance of the emitted photon restores the interference of the particle waves.

Another objection to Eq. (1) is that it is hard to understand how the wave function ψ_T can depend upon the duration time T, which "is of the same order as, or smaller than the duration of the $1/f$-noise measurement" (see Ref. 14, p. 109). When no measuring process is specified, wave functions cannot depend on such an arbitrary parameter. We shall come back to the discussion of Eq. (1) below.

Next we discuss the validity of Eq. (2). Tremblay (Ref. 3, p. 183) has already noted that "the quantity evaluated by Handel has neither a quantum-mechanical nor a classical meaning." Indeed, the quantum-mechanical analog of the current autocorrelation function is not given by (2) but by

$$\langle j(t)j(t+\tau)\rangle=\langle\psi(t)\,|\,JJ(\tau)\,|\,\psi(t)\rangle ,$$

where J is the current operator p/m and $J(\tau)=\exp(iH\tau/\hbar)J\exp(-iH\tau/\hbar)$. The parentheses represent the scalar product in Hilbert space. Another quantum analog is

$$\tfrac{1}{2}[\langle\psi(t)\,|\,JJ(\tau)\,|\,\psi(t)\rangle+\langle\psi(t)\,|\,J(\tau)J\,|\,\psi(t)\rangle]\ .$$

Depending on the context one has to use the one or the other, but in no circumstances does one encounter Eq. (2), or any other expression involving four factors ψ.[15] The quantity ψ is a vector in Hilbert space, not a water wave.

One of the implications of Eq. (2) is that for scattering of an electron at a charge Ze, the outcome of Eq. (2) is proportional to Z^4 (Ref. 16), whereas a proper definition of the current correlation function yields a proportionality to Z^2 (Ref. 3, p. 168).

In later publications (Refs. 17, abstract and 12, p. 100) Handel admits that the correlation function should be bilinear in the wave function. In a more recent paper Sherif and Handel[17] have attempted to derive an expression for the current correlation function which does not violate the basic principles of quantum mechanics. Assuming the validity of Eq. (1) they consider the description of scattering of two electrons by some potential. From the fact that the electrons do not interact (the wave function is assumed to be a simple product; nowhere is use made of the fact that electrons are fermions or that α particles are bosons), it should be clear from the start that the outcome should be related in a simple manner to the scattering of a single electron by the potential under consideration. But, as stated before, the latter would not yield $1/f$ noise. The reason why a $1/f$-noise spectrum could be derived [Ref. 17, Eqs. (3.6) and (4.21)] is that the random phases γ_ϵ have been taken the same for both electrons.[18] But the function γ_ϵ cannot be random [as it was assumed below Eq. (1)] and nonrandom at the same time. If uncorrelated phases γ_ϵ are taken, the density correlation function evaluated by Sherif and Handel [Ref. 17, Eq. (3.6)] becomes independent of time.

The above-mentioned objections apply as well to a large number of papers written by Handel or by Handel and co-workers on the subject of $1/f$ noise.[5,9,14,17,19–23] The version of the theory employed before 1978 (Refs. 1,2,24–26) contains the wave function for the electron and creation operators for the photon field. Therefore our objections to Eq. (1) of the present paper do not apply to this older version. However, the current correlation function is also taken to be of fourth order in the electron wave functions, like Eq. (2) of the present paper (see also Ref. 3). Thus our objection to Eq. (2) applies to all the papers cited here, except for Ref. 17. But we have just argued that no $1/f$-noise should emerge from that calculation also.

We finally discuss some aspects of the publication[12] titled *Any particle represented by a coherent state exhibits 1/f noise*. The author describes the interaction of a bare electron with the interaction Hamiltonian $H'=A_\mu j^\mu = -(e/c)\mathbf{v}\cdot\mathbf{A}+e\Phi$ where A_μ is the photon field operator and j^μ the electron current operator. We copy Eq. (3.2) of Ref. 12,

$$|\,\mathbf{k},0\rangle^{(1)}=|\,\mathbf{k},0\rangle^{(0)}+\sum_{\mathbf{q}}|\,\mathbf{k}-\mathbf{q},q\,\rangle^{(0)}\times\frac{^{(0)}\langle\mathbf{k}-\mathbf{q},q\,|\,H'\,|\,\mathbf{k},0\,\rangle^{(0)}}{\epsilon_{\mathbf{k}}-\epsilon_{\mathbf{k}-\mathbf{q}}-\omega_q}\ . \tag{6}$$

It expresses the first-order perturbation expansion of the dressed electron state with wave number $\mathbf{k}$ and no photon [indicated by the superscript (1)] as a sum of the unperturbed states [indicated by the superscript (0)] having no photon or one photon with wave number $\mathbf{q}$. We have copied this equation because we think that Eq. (1) of the present paper should also have this form; photon wave functions are properly included and the duration time T of the measurement does not enter. Also the mysterious random phase γ_ϵ does not appear in Eq. (6).

Unfortunately, we have to object to other points in the paper. First, in Eq. (2.7) of Ref. 12 the quantity

$$P_q(t,\tau,x)=|\,\psi_q^2\,|_t\,|\,\psi_q^2\,|_{t+\tau} \tag{7}$$

is evaluated. Here $\psi_q(x,t)$ is the wave function of a coherent photon state with wave vector q as a function of a field variable x. The same objection can be raised to P_q in Eq. (7) as to the expression in the right-hand side of Eq. (2); it is proportional to the fourth power of a wave function. In particular, it is not a fourth-order combination of field operators, which would be quantum-mechanically meaningful.[15] Hence P_q should play no role in a theory aiming to describe experiments. But let us continue the discussion of Ref. 12. Handel (Ref. 12, p. 98) calculates the correlation function of $|\,\psi_q\,|^2$ by integrating (7) over x from $-\infty$ to $+\infty$ and averaging over the time t. Next, using Eq. (6), the parameter z_q, characterizing the coherent photon state $\psi_q(x)$, is fixed.[27] Then the correlation function is combined for modes with different wave vectors q [Ref. 12, Eqs. (4.1) and (4.2)] to yield [(Ref. 12), Eq. (4.3)] the spectral density of fluctuations

$$\frac{S_{|\psi|^2}}{\langle\,|\,\psi\,|^2\rangle^2}=\frac{2\alpha}{\pi f}\simeq 0.0046/f\ . \tag{8}$$

The density $|\,\psi\,|^2$ is not defined explicitly, but it should be some integral over $|\,\psi_q\,|^2$. The electronic current $\mathbf{j}$ is defined by (Ref. 12, p. 99) $\mathbf{j}=e(\mathbf{k}/m)\,|\,\psi\,|^2$. The spectral density of current correlations is stated to be related to (8), namely [(Ref. 12), Eq. (4.3)],

$$\frac{S_j(f)}{\langle j\rangle^2}=\frac{2\alpha}{\pi f}\ . \tag{9}$$

This result has become known as the "coherent (state) quantum $1/f$ noise," as opposed to the "incoherent (state) quantum $1/f$ noise" given by Eqs. (3) and (5).

However, apart from the objection mentioned above, we fail to see how the *photon* density $|\,\psi\,|^2$ can enter the definition of the *electron* current. This definition becomes even more puzzling if we recall that the variable x, entering Eq. (7), is a field variable and not a variable in coordinate space.

CONCLUSION

We have discussed several fundamental objections to Handel's quantum theory of incoherent [Eqs. (3) and (5)] and coherent [Eq. (9)] (state) $1/f$ noise. Some of these objections concern the applicability of the theory to typical "experimental" situations. In particular we have argued that the theory does not take the finite mean free path of the electrons into account. Nor does it deal satisfactorily with measurements in Faraday cages. Subsequently we have indicated that the absence of interactions between different particles (electron or α-particles) should imply Poisson noise and, consequently, absence of $1/f$ noise. Finally we have considered some aspects of the mathematical derivation. Here we focus our criticism on the fact that two specific steps [concerning Eqs. (1) and (2)] made in a large number of Handel's publications contradict the basic principles of quantum mechanics. Reference 12 has been discussed in some more detail. In all situations considered, we conclude that the $1/f$ noise has not been derived properly, and that a proper derivation does not yield $1/f$ noise.

It seems to us that if the theory is to be saved, *all* our objections should be answered. We think that it is highly improbable that this can be done within the framework of conventional quantum mechanics. We feel that, in spite of some recent cosmetic improvements, the fundamental objections raised by Tremblay[3] almost a decade ago still stand.

As the theoretical basis for Handel's quantum theory of $1/f$ noise appears to be lacking, we must conclude that the agreement with experiments[5-8] is fortuitous.

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[1]P. H. Handel, Phys. Rev. Lett. **34**, 1492 (1975).

[2]P. H. Handel, in *Linear and Nonlinear Electron Transport in Solids,* Proceedings of the Seventeenth NATO Advanced Study Institute Antwerp, 1975, edited by J. T. Devreese and V. van Doren (Plenum, New York, 1976), p. 515.

[3]A.-M. Tremblay, thesis, Massachusetts Institute of Technology, 1978.

[4]In fact, the only reference to Tremblay's work that we are aware of is by M. B. Weissman, in *Proceedings of the Sixth International Conference on Noise in Physical Systems,* edited by P. H. E. Meijer, R. D. Mountain, and R. J. Soulen (Dept. of Commerce, Washington D.C., 1981), p. 133. Weissman also objected to the theory of quantum $1/f$ noise. He argues that it is a number-fluctuation theory and that it does not exhibit mobility fluctuations. For nonexperts we note that the paper by Handel and Sherif (Ref. 14) is not a calculation within the method of second quantization. This is seen most directly from the prediction for the $1/f$ noise level [Ref. 14, Eq. (5.12)], where the factor A depends on the amplitude $\mathbf{a}$ of the vector potential.

[5]J. Gong, C. M. van Vliet, W. H. Ellis, G. Bosman, and P. H. Handel, in *Noise in Physical Systems and 1/f Noise,* edited by M. Savelli, G. Lecoy, and J.-P. Nougier (North-Holland, Amsterdam, 1983), p. 381.

[6]A. van der Ziel, P. H. Handel, X. Zhu, and K. H. Duh, IEEE Trans. Electron Devices **TED-32**, 667 (1985).

[7]J. Kilmer, C. M. van Vliet, G. Bosman, A. van der Ziel, and P. H. Handel, Physica Status Solidi **6**, 429 (1984).

[8]A. van der Ziel, C. J. Hsieh, P. H. Handel, C. M. van Vliet, and G. Bosman, Physica B **124**, 299 (1984).

[9]P. H. Handel, Phys. Rev. A **22**, 745 (1980).

[10]This seems an oversimplification. When N electrons are discussed, we expect a (total) positive charge Ne.

[11]In Ref. 19, p. 50 this objection has been answered as follows: "The $1/f$-noise will be present already in the basis functions in terms of which the individual wave packets are expressed; the basis functions, however, are defined over the entire duration of the experiment." However, the mathematical process of expanding in infinite plane waves cannot have the physical effect of introducing a long correlation time.

[12]Problems related to screening are also mentined on page 99 of the paper by P. H. Handel in *Noise in Physical Systems and 1/f Noise,* edited by M. Savelli, G. Lecoy, and J.-P. Nougier (North-Holland, Amsterdam, 1983), p. 97. See also Ref. 9, p. 753.

[13]Even when one agrees that the first term in Eq. (1) should include photons with energy less than ϵ_0 (see Ref. 9, p. 750 or Ref. 19, p. 46) the one-photon term of the second term of Eq. (1) remains orthogonal to the first term in Eq. (1). The reason is that at least one of the quantum numbers (the energy) of the photons is different.

[14]P. H. Handel and T. S. Sherif, in *Noise in Physical Systems and 1/f Noise,* edited by M. Savelli, G. Lecoy, and J.-P. Nougier (North-Holland, Amsterdam, 1983), p. 109.

[15]In the method of second quantization, ψ is an operator rather than a state vector. Observables are bilinear in the state vector, usually the vacuum $|0\rangle$. Operators may occur to any power.

[16]The reason is that the scattering amplitude a, entering Eq. (1), is proportional to Z, see Ref. 3, p. 168.

[17]T. S. Sherif and P. H. Handel, Phys. Rev. A **26**, 596 (1982).

[18]We note that the possibility of different phases $\gamma_j(\epsilon)$ $(j=1,2)$ has been left open in Eq. (4.2) of Ref. 17, where $b_{Tj}(\epsilon)$ $(j=1,2)$ has been introduced. Without discussion the phases have been taken equal in other places of the same paper.

[19]P. H. Handel, in Proceedings of the Second International Symposium on $1/f$ Noise, Orlando-Gainesville, Florida, 1980 (unpublished), page 42.

[20]P. H. Handel, Ref. 19, p. 96.

[21]P. H. Handel, C. M. van Vliet, and A. van der Ziel, Ref. 14, p. 93.

[22]P. H. Handel and T. Musha, Ref. 14, p. 101.

[23]A. van der Ziel and P. H. Handel, Physica B **125**, 286 (1984).

[24]P. H. Handel, Phys. Rev. Lett. **34**, 1495 (1975).

[25]P. H. Handel and C. Eftimiu, in Proceedings of the Symposium on $1/f$ Fluctuations, Tokyo, 1977 (unpublished), p. 183.

[26]P. H. Handel and D. Wolf, in *Noise in Physical Systems,* edited by D. Wolf (Springer, Heidelberg, 1978), p. 169.

[27]When we try to rederive this result in the Coulomb gauge, we find a different result for $|z_q|^2$. In the nonrelativistic limit $(v \equiv \hbar k/m \ll c)$ we finally find that Handel's prediction for the noise level [Ref. 12, Eq. (4.3); our Eqs. (8) and (9)] should include a factor $(v/c)^2$. This is surprisingly close to Eq. (3), where a *velocity difference* enters.