Passive versus active interferometers: Why cavity losses make them equivalent

J. Gea-Banacloche
Max-Planck Institut für Quantenoptik, D-8046 Garching bei München, West Germany
and Center for Advanced Studies, Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131
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Active and passive interferometers (as used, for example, for rotation-rate sensing or gravitational-wave detection) are known to have essentially the same ultimate sensitivity, although they appear to work differently, have signals of different sizes, and be limited by different kinds of noise (shot noise for the passive case, spontaneous emission for the active case). This paper explains this remarkable coincidence. The underlying physics common to both systems is brought forth and the role of the losses in limiting the sensitivity is clarified. The possibility of squeezing the field is explicitly considered; it is shown when it can or cannot help, and why.

I. INTRODUCTION

In the use of interferometry for certain high-precision measurements (e.g., rotation sensors[1] or gravitational-wave detectors[2]) it is common to distinguish between passive devices (when light from an external source is injected into an empty cavity) and active devices (where the light is internally generated by a gain medium inside the cavity). In both cases what is being measured is the detuning of the cavity from resonance, which is proportional to the signal (rotation rate, for example) that one is really interested in. In a passive cavity the detuning causes a phase shift; in an active cavity it causes a frequency shift, that is, a phase shift which grows linearly with time. For this reason active cavities appear to be potentially more sensitive than passive cavities over a sufficiently long measurement time.

On the other hand, when the noise limiting the performance of both kinds of systems is taken into account, it is found that the sensitivity (signal-to-noise ratio) is essentially the same in both. The signal-to-noise ratios for *optimized* passive and active systems differ only by numerical factors depending on the experimental arrangement, but the order of magnitude and the dependence on the cavity losses and the power is the same. The limit for the passive device is usually derived by considering shot noise at the photodetector; for the active device, instead, it is given by fluctuations in the laser phase—the fluctuations which give rise to the laser linewidth and which arise from spontaneous emission in the gain medium.[3]

From a fundamental point of view, this is a profoundly unsatisfactory result. It is as if two different systems and two different noise sources somehow conspired to produce the same result. Even more amazing is the fact that essentially the same limit recurs in every conceivable detection scheme, in very different experimental arrangements, from laser gyroscopes[4] to gravitational-wave detectors.[5] Yet no explanation for this remarkable coincidence appears to have been presented in the literature.

One wonders, of course, whether there is a sort of fundamental limit lurking in the background. Yet neither shot noise nor spontaneous emission noise are ultimate limits to signal processing. Shot noise can be reduced by squeezing the vacuum,[6,7] and phase-sensitive amplifiers may be conceived which need not degrade appreciably the signal-to-noise ratio of one quadrature (the phase, for instance) of the signal they amplify (in the language of Caves's classic paper,[8] they may have negligible added noise for that quadrature, although they still have to amplify the signal's inherent noise along with the signal itself). Such an amplifier, operating with a squeezed-state input, would have negligible spontaneous-emission-induced phase fluctuations.

Where, then, does the ultimate limit come from? A careful study of the problem reveals that the cavity losses play a crucial role, and this note explains why. The coincidence of the limits for active and passive devices is not, as it could not be, a coincidence at all: the differences between the two kinds of devices are not, in a way, as deep as one might have expected; and, from a certain point of view, it is the fluctuation-dissipation theorem which lies at the heart of the matter. In the process of reaching this conclusion, just about every fundamental problem in quantum optics, from squeezing to vacuum fluctuations and the laser linewidth, makes at least a cameo appearance.

II. PASSIVE CAVITIES

The first point that needs to be established is what is common to the response of both active and passive cavities to a cavity detuning, and we begin by showing that one can look at the passive cavity in a way that makes it look very similar to an active one, and which shows exactly what it is that the active one does that makes it different. All the discussions that follow will concentrate on the field inside the cavity only, in a single mode, and inquire as to how well its phase is defined; the problems associated with extracting the light and actually performing the measurement will be ignored, since the fundamental limit may be found in the intracavity field already.

Consider, then, first the response of a passive cavity to an elementary excitation of the field; specifically, consider the free decay of a mode of the electromagnetic field, of nominal frequency ω, inside a cavity which is slightly detuned (let the cavity resonant frequency be Ω and the detuning $\delta\Omega=\omega-\Omega$). Semiclassically, the boundary conditions result in a difference equation which, for small losses, may be approximated by a differential equation for the (slowly varying) complex amplitude $\mathcal{E}(t)$,

$$\dot{\mathcal{E}}=(i\delta\Omega-\gamma)\mathcal{E} \ , \tag{1}$$

where γ is the decay rate due to losses. Writing $\mathcal{E}(t)=E(t)e^{-i\phi(t)}$, one sees from Eq. (1) that the phase does grow linearly with time

$$\dot{\phi}(t)=\delta\Omega \ , \tag{2}$$

but the amplitude is damped,

$$\dot{E}(t)=-\gamma E(t) \ . \tag{3}$$

Equation (2) expresses the essential similarity between the active and passive cavities, Eq. (3) their only essential difference; namely, that in the passive case the field dies away in a time of the order of γ^{-1}. It is important to realize, in particular, that the linear growth of the phase (2), which is usually said to be characteristic of the active systems, is actually already present in the passive cavity. The decay of the field, however, prevents one from observing it for times much longer than γ^{-1} [compare the discussion below, in terms of X_2; in particular Eq. (6)]. The contribution of the active medium in an active system, therefore, is only to keep the field from decaying by amplifying it (in a phase-preserving way, that is, coherently), thus making the phase growth (2) observable.

Accordingly, Eq. (2) might be derived by simply taking the phase and amplitude evolution equations for an active system (for example, the ring gyro equations from Ref. 1) and formally removing the active medium by setting all the gain coefficients equal to zero; the result is nothing but Eq. (1), which shows that (2) may indeed be regarded as a property of the passive cavity alone. [Equation (1) may, of course, also be established directly for a passive cavity, as mentioned above; for instance, one may take the evolution equations for an ordinary Fabry-Perot (see, for example, Ref. 9) and just set the injected field equal to zero: then (1) gives the free decay of the field in the cavity.]

The main point of this discussion is that it is legitimate to consider an active system as just a passive cavity with an amplifying medium inside. The consequences of this will be discussed in Sec. III.

Some passive schemes do actually exhibit a "growing phase" in a sense; for instance, the "delay line" or Michelson-type interferometers in gravity-wave detection (the phase difference between the two arms grows with every round trip of the light between the mirrors), and single-pass, many-turn optical fibers for rotation-rate sensing (the phase difference between the counterpropagating beams grows with every turn). Passive cavities of the Fabry-Perot—type, instead, are used most often with an injected field to keep the intensity inside constant. Then the phase of the intracavity field does not grow beyond a maximum value $\phi_{\max}=\delta\Omega/\gamma$, because the field that has been in the cavity for a long time (accumulating a large phase shift) dies away, and "fresh" light, with a constant phase, is continually coming in to replace it. We shall regard this system as being roughly equivalent to a continuous repetition of an "elementary measurement," in which some intracavity field is allowed to evolve freely, sample the cavity, and eventually die away; then another fresh field is allowed to do the same, then another, etc. This point of view gives the correct result for the sensitivity of a Fabry-Perot—type passive device [Eq. (12) below] aside from numerical factors which depend on the experimental setup, and measurement strategy.

To proceed with the study of one of these "elementary measurements," it is convenient to replace the phase ϕ, which is not a good observable, by something more suitable. We introduce the quadratures X_1 and X_2 of the electric field by the equation

$$E(t)=e^{-i\phi_0}(X_1+iX_2) \ . \tag{4}$$

Here X_1 and X_2 are real (or, as quantum operators, Hermitian) and ϕ_0 is the initial value of the phase, so that initially $X_2=0$; then, as the phase grows, we may take X_2 to be our signal (X_2 is the phaselike quadrature, X_1 the amplitudelike quadrature). Equations for X_1 and X_2 follow immediately from (1):

$$\dot{X}_1=-\gamma X_1-\delta\Omega X_2 \ , \tag{5a}$$

$$\dot{X}_2=-\gamma X_2+\delta\Omega X_1 \ . \tag{5b}$$

These equations are easily integrated. We shall consider only the case when the signal is very small, so that $\delta\Omega t$, and therefore X_2, is always much smaller than 1; then the term in X_2 may be neglected in (5a). The solution for X_2 grows at first linearly, and then it is damped,

$$X_2(t)=e^{-\gamma t}\delta\Omega X_1(0)t \ ; \tag{6}$$

it is maximum precisely when $t=\gamma^{-1}$, so that the maximum signal equals $X_{2\max}=e^{-1}\delta\Omega X_1(0)/\gamma$. Note that $X_1(0)$ is just the electric field amplitude at $t=0$. We shall use units such that, quantum mechanically, $X_1^2+X_2^2=n+\frac{1}{2}$, where n is the photon number operator. Then $\langle X_1(0)\rangle \simeq \langle n\rangle^{1/2}$, if $\langle n\rangle$ is large.

We need to enquire now about the precision with which the signal (6) can be known—that is, about the "noise." The quantum-mechanical operators for X_1 and X_2 have an intrinsic uncertainty expressed by the relation

$$\Delta X_1\Delta X_2\geq\tfrac{1}{4} \ , \tag{7}$$

but this by itself does not tell us how large or small ΔX_2 has to be. In particular, we might consider a squeezed state with negligible ΔX_2.

The crucial point, however, is that Eq. (6) has been obtained as the solution of a system of equations (5) for a *damped* field. Now, if this field is a quantum-mechanical one, the preservation of the commutation relations (ultimately, the uncertainty principle) requires that the damping mechanism (whatever it is) introduce noise, which will be represented by noise operators in the equations of motion. It is this noise that is going to determine the ultimate sensitivity.

The exact form of the noise operators is model dependent. Their correlations, which are all we need here, are determined by the fluctuation-dissipation theorem;[10] for definiteness, the reader may want to think of the classic model of damping by a bath of harmonic oscillators[11] (with the standard Markov approximation). At any rate, what we have to do is to rewrite Eqs. (5) as Langevin equations,

$$\dot{X}_1 = -\gamma X_1 + F_1(t) , \tag{8a}$$

$$\dot{X}_2 = -\gamma X_2 + \delta\Omega X_1 + F_2(t) \tag{8b}$$

(as explained before, we have neglected the term proportional to X_2 in the equation for X_1). The noise operators F_1 are F_2 and Hermitian. They are uncorrelated in the sense that their Hermitian correlation function $\langle F_1(t)F_2(t')+F_2(t')F_1(t)\rangle$ vanishes [the non-Hermitian correlation function $\langle F_1(t)F_2(t')\rangle$ is purely imaginary]. Most importantly, they satisfy

$$\langle F_1(t)F_1(t')\rangle = \langle F_2(t)F_2(t')\rangle = \tfrac{1}{2}\gamma\delta(t-t') . \tag{9}$$

The crucial point, apparent from Eq. (9), is really that the losses are *phase insensitive:* the damping bath puts the same amount of noise in each quadrature. It is for this reason that squeezing is destroyed by losses (as already pointed out by Caves[12]); it is from this fact that a fundamental limit arises.

When the system (8) is integrated, one finds for the noise in X_2

$$\Delta X_2^2(t) \simeq e^{-2\gamma t}(\Delta X_2^2)_0 + \tfrac{1}{4}(1-e^{-2\gamma t}) , \tag{10}$$

aside from terms which are smaller than those kept by a factor of $\delta\Omega/\gamma$ (which was assumed earlier to be very small). Equation (10) shows that, regardless of what the initial noise in the quadrature X_2 is, the noise associated with damping will (because of its phase-insensitive nature) tend to put in X_2 the noise associated with vacuum fluctuations—that is, $\Delta X_1 = \Delta X_2 = \frac{1}{2}$.

It is now a simple exercise to use Eqs. (6) and (10) to calculate the maximum signal-to-noise ratio. The result depends somewhat, of course, on the initial amount of squeezing that is present [that is, the value of $(\Delta X_2)_0$], but not in order of magnitude: the maximum signal-to-noise ratio is always reached after a time of the order of γ^{-1}; by that time, the noise in X_2 is already of the order of magnitude of that for the unsqueezed vacuum (i.e., $\frac{1}{2}$), and the minimum detectable signal $\delta\Omega$ (defined as the value of $\delta\Omega$ giving a signal-to-noise ratio of unity) is, therefore, of the order of

$$\delta\Omega_{\min} \sim \gamma / X_1(0) . \tag{11}$$

We must take into account now the possibility mentioned earlier of repeating the measurement a large number of times. Since each elementary measurement lasts for a time of the order of γ^{-1} over a total measurement time t_m, we may perform $N=\gamma t_m$ elementary measurements, and the signal-to-noise ratio will improve by a factor of $N^{1/2}$. Then the minimum detectable $\delta\Omega$ becomes

$$\delta\Omega_{\min} \sim \left[\frac{\gamma}{\bar{n} t_m} \right]^{1/2} , \tag{12}$$

where we have replaced the $X_1(0)$ of Eq. (11) by $\sqrt{\bar{n}}$, $\bar{n}$ being the average number of photons in the cavity. This is indeed the result obtained for passive interferometers in which one is constantly injecting fresh light, as was said above.

We have seen now the two ways in which losses affect adversely the performance of a (passive) interferometer. First, because the field is damped, the "phaselike quadrature" which carries the information about the signal does not grow past a certain maximum value (reached after a time of the order of γ^{-1}). Second, the losses introduce some noise whose effect is to ensure that, after a time of the order of γ^{-1} again, the noise in that quadrature is the same as for unsqueezed vacuum fluctuations, regardless of whether one started with a squeezed state or not. It is precisely this latter effect which ensures that the "shot-noise limit" calculation gives the same order of magnitude as Eq. (12), since shot-noise may be related in various ways (depending on the experimental arrangement) to vacuum fluctuations at the photodetector.[7,13]

III. ACTIVE SYSTEMS

Since we have identified what limits the sensitivity of a passive interferometer, we might think of doing something about it in the following way (which, as discussed earlier, leads essentially to an "active" scheme): to counteract the damping of the field due to the losses, introduce a gain medium in the cavity which coherently regenerates the signal. Then, with the losses effectively gone from Eq. (8b) (and X_1, the "amplitudelike" quadrature, locked to some saturation value), X_2 would be free to grow linearly with time instead of eventually decaying as in Eq. (6).

When the operation of an active device is understood in this way, the reason why it does not work (better than the passive system, that is) is actually almost obvious: the gain medium cannot but amplify the signal *and* the noise *together.* The active system could not, therefore, have a larger signal-to-noise ratio than the underlying passive system.

This may be formally shown without much difficulty. Assume that the evolution of X_2 is given by Eq. (8b), without the losses, and with a constant $X_1 = X_1(0)$. We are neglecting any "added" noise (in the terminology of Ref. 8) introduced by the amplifier which might, therefore, be a totally classical device, or a phase-sensitive amplifier[8] with negligible added noise for the quadrature X_2. It might seem at first sight that we are restricting ourselves to linear amplifiers only, but this is not so. We are only requiring that the amplifier's treatment *of the quadrature* X_2 be, to a good approximation, linear. This had better be the case, at any rate, since otherwise the relating of the output of the device to the signal of interest is not a trivial task; in any event, the validity of this assumption is practically guaranteed in all the cases of interest here (namely the detection of very weak signals, where $\delta\Omega t_m \ll 1$, so that $X_2 \ll 1$). The amplifier may (and, in the case of an ordinary laser medium, will) treat the quadrature X_1 nonlinearly, but the linear approximation will describe its processing of X_2 quite well.

The solution for $X_2(t)$ is then

$$X_2(t)=\delta\Omega X_1(0)t+\int_0^t F_2(t')dt' \ . \tag{13}$$

We see that the noise in X_2 does indeed add up, undamped (unlike in the passive case), just like the variable X_2 itself. Using again Eq. (9), we find for the magnitude of this noise

$$\Delta X_2^2=(\Delta X_2^2)_0+\tfrac{1}{2}\gamma t \ . \tag{14}$$

Again Eqs. (13) and (14) may be used to investigate the signal-to-noise ratio, and again, when the measurement time is long enough ($t_m > \gamma^{-1}$), the initial amount of squeezing in X_2 is found to make very little difference. Over a total measurement time t_m, the minimum detectable $\delta\Omega$ (with the signal-to-noise ratio equal to 1, as before) is

$$\delta\Omega_{\min}\sim\left[\frac{2\gamma}{\bar{n}t_m}\right]^{1/2} , \tag{15}$$

which is essentially the same as Eq. (12). The only advantage over Eq. (11) is the one arising from a large number of independent measurements, which we might say the active cavity performs automatically for us, not surprisingly, since we are sustaining the field inside: the passive cavity with a constant injected field did the same. One might say that the only difference between the two systems is that the active cavity is an "integrator," in that we might think of it as adding up the results of all the elementary measurements [which accounts for the linear growth of $X_2(t)$]; each one, of course, with its corresponding noise. The result is, of course, neither more nor less precise (save, perhaps, for a numerical factor of the order of unity) than the "average" $X_{2\max}$ calculated by the passive device.

This continuous adding up of noise results in the diffusion process of Eq. (14), familiar indeed from discussions of the laser linewidth.[14] Note that, semiclassically, $X_2^2(t)\simeq n[\phi(t)-\phi_0]^2$, so that Eq. (14) does describe a phase-diffusion process.

What is the origin of this noise, when we have ignored the "added noise" introduced by the amplifier? Formally it comes from the noise operator F_2 associated with the damping of the field. But all that these operators did, in the passive case, was to restore the normal vacuum fluctuations. Thus the noise in (14) is, roughly speaking, amplified vacuum fluctuations. *Unsqueezed* vacuum—the phase-insensitive nature of the losses sees to that. Physically, one might think of the losses as letting unsqueezed vacuum "leak into" the cavity (just as they let the inside field "leak out"), with quotation marks to indicate that we are not in general thinking of transmission losses (which are essentially reversible, and can be counteracted in various ways) but of irreversible absorption (maybe also diffraction, etc.) losses.

The process (14) accounts for *one-half* of the phase diffusion in a laser, which is usually attributed entirely to spontaneous emission. It is somewhat odd to see the losses take half of the credit for it here, although this is the way it comes naturally from a Langevin approach (compare the discussion in Ref. 14, and the work of Lax in Ref. 15). In this context, it is well known that different orderings of the operators lead to different interpretations. In fact, with the choice we have made here of working consistently with *Hermitian* operators, it is not surprising to find one-half of the spontaneous emission to come from amplified vacuum fluctuations (the missing half would come from the amplifier's own added noise, which we have neglected); compare this with the results in Ref. 16.

One final comment may be made. It seems reasonable to assume, as we have done, that by neglecting the amplifier's added noise we are indeed looking at the most favorable scenario, from the point of view of keeping the signal-to-noise ratio as large as possible. We might, however, wonder about the possibility of the amplifier introducing some noise which might be *anticorrelated* with F_2 in the equation of motion for X_2. But, since F_2 is uncorrelated with any other noise in the problem (including, in the sense mentioned earlier, F_1), this could only happen through some kind of feedback of X_2 upon itself, that is, some nonlinearity in the amplifier's treatment of X_2, which we have already discarded as being negligible. The amplifier's added noise could therefore only make matters worse, as it does indeed in the case of ordinary laser media (by the factor of 2 mentioned above).

IV. CONCLUSIONS

All the foregoing, either as contained in the mathematics or in the simpler statement: "The active device sustains (against the cavity losses, i.e., by amplifying it) and adds up both the signal *and* the noise of the passive device," explains how "shot noise" and "spontaneous emission," apparently conspired to make active and passive systems equivalent. In reality, "passive" and "active" systems are only different ways to process a single elementary measurement—one whose maximum duration and associated noise is determined solely by the cavity losses.

The limit encountered here is "fundamental" only in as much as the losses are unavoidable. It would seem from what we have presented here that one always has to gain from increasing the measurement time t_m, even as to make $t_m \gg \gamma^{-1}$; if that were the case then all the systems would be "loss limited", as the ones discussed here. There are, however, cases where t_m cannot be increased beyond certain limits (in a gravity-wave detector, for instance, it should not be chosen larger than half the expected period of the wave; in ring laser gyros, there are other sources of error which degrade the performance for very large integration times). If the losses can be reduced to the point when γ^{-1} is greater than the allowed measurement time, the system is no longer loss limited. In this case ($t_m < \gamma^{-1}$), the active and passive devices are still equivalent [expand the exponentials in (6) and (7), and compare with (13) and (14)] but *now the initial amount of squeezing becomes relevant,* and can indeed increase the sensitivity substantially, as explained, e.g., in Ref. 12, for gravity-wave detectors.

Aside from this, of course, in a practical apparatus the passive and active devices will not in general be equivalent from an experimentalist's point of view, each one having other merits and problems of its own (in different contexts, these have been discussed in Refs. 1, 2, and 5, among many other places). It is in this context that all those "numerical factors of the order of unity" that we might afford to ignore in this paper will, of course, become relevant.

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[3]It is perhaps worth pointing out that we are not talking here about the so-called "standard quantum limit" (see, e.g., Ref. 12 below) which is independent of the cavity losses and therefore only obtainable when these are negligible. We have in mind a loss-limited operation, as will be explained later.

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[13]Since, by the fluctuation-dissipation theorem, the losses (at zero temperature as assumed here) only introduce the noise necessary to ensure that the attenuated signal still has its undiminished quantum fluctuations, and since it is these quantum fluctuations (together with the photodetector's partition noise) which give rise to the shot-noise limit (see Ref. 7), it is, in this author's opinion, misleading to treat the loss-related noise as a noise that would be present *in addition* to shot noise in a passive system, as was done recently by A. Abramovici, Opt. Commun. **57**, 1 (1986).

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