Ionization of helium by highly charged ions at 1.4 MeV/amu

J. H. McGuire,* A. Müller, B. Schuch, W. Groh, and E. Salzborn
Institut für Kernphysik, Universität Giessen, D-6300 Giessen, West Germany
(Received 2 September 1986)

Experimental cross sections for ionization of helium by projectile ions with charge Z up to $Z=44$ are presented at a fixed velocity $v=7.48$ a.u. corresponding to 1.4 MeV/amu. Total cross sections, summed over projectile charge states, for single ionization, σ^+, are compared to Born and Glauber calculations for point projectiles of charge Z. The Glauber calculations give a better fit to the Z dependence of the single-ionization cross sections than do the Born calculations. For double ionization, the ratio of σ^{2+}/σ^+ increases as Z^2 for the smaller Z and less rapidly than Z^2 for larger Z projectiles, suggesting that the so called "direct" mechanism for double ionization is dominant, but that the first Born approximation is breaking down for the larger Z.

I. INTRODUCTION

Ionization in collisions of atoms with projectiles of low charge Z at high velocity v has been fairly well studied[1–10] so far as total cross sections are concerned. For small-Z projectiles at high velocities there is relatively little multiple ionization and the total cross sections are generally well described by the first Born approximation which varies as Z^2. In this paper we consider ionization of helium by large-Z projectiles at a fixed high velocity. At fixed v, as Z becomes large, the Born approximation breaks down, and in addition multiple ionization is not negligible.

Some previous studies have been done by Hvelplund and co-workers,[1,4,6] Shah and Gilbody,[2,3,7] and Rudd *et al.*[5] for total cross sections for ionization in atomic hydrogen[3] and helium[1,2,4–7] using high-velocity projectiles with charge Z up to 6 and 8, respectively. In 1978 ionization cross sections on H_2 were reported by Olson *et al.*[9] for Z up to 22 for velocities up to 1.14 MeV/amu and a universal curve (herein referred to as Schlachter-Olson scaling) for σ/Z versus E/Z supported by the classical Monte Carlo calculation was found. Other systems[10] at high energies with Z up to 54 have been observed, and similar dependences have been found. In the case of single ionization of atomic hydrogen by protons it is evident[11] that the Born approximation is not accurate to 2% until one reaches rather high collision velocities, i.e., 1 or more MeV/amu. For these systems the Glauber approximation[11,12] gives better agreement with data than the Born approximation. In helium there are observations of both single and double ionization, and two different mechanisms for double ionization have been used[13–15] to interpret the experimental data.

In this paper we present a joint experimental and theoretical study of the ionization of helium by incident ions of large Z. In our studies Z is as large as 44, i.e., a projectile charge 22 times larger than the target charge. Although for the larger Z these ions are not bare ions, the electrons are tightly bound, and in theory we regard the projectiles as point particles of charge Z.

II. EXPERIMENT

The experiments were conducted at the Gesellschaft für Schwerionenforschung Darmstadt m.b.H. (GSI) heavy-ion accelerator using one of the parasitic 1.4-MeV/amu beam lines. The charge states of ions could be varied by introducing a foil stripper or a gas target upstream from the charge-state-selecting magnet. Details of the technique used are described in Ref. 11.

In brief, a beam of momentum-analyzed ions A^{Z+} is collimated by 0.5-mm-diameter apertures to a maximum divergence of 1°. The projectile ions then cross a thermal beam of He atoms emerging from a hollow needle. The helium recoil ion produced are extracted perpendicularly to the projectile beam by the electric field between two parallel plates which are typically on potentials ± 600 V. After passing a 5-mm-diameter hole in the negatively biased plate and through a subsequent drift tube, the ions are post accelerated and detected by a multichannel-plate detector with a coaxial anode. The grid in front of the channel plates rejects secondary electrons released by the ions and thus enhanced the ion-detection efficiency. By variation of the ion energy it was shown that the relative detection efficiency is constant at energies greater than or equal to 3 keV and slowly decreases as the ion energy is reduced (10% less at 2 keV). All apertures passed by the recoil ions are provided with 95% transparency grids, thus maintaining a plane geometry. The length of the drift tube is chosen to accomplish time-of-flight focusing of recoil ions produced inside the condenser plates at different potentials because of the finite height of the projectile ion beam. The length of the drift tube is about equal to the distance between the condenser plates, i.e., 3 cm; final time focusing is reached after the acceleration to the channel-plate detector.

Uniform extraction, transmission, and detection of He^+ and He^{2+} recoil ions was ensured by comparison of measured recoil-ion charge-state spectra with results obtained by using a different but well-tested spectrometer described previously.[12]

Downstream from the interaction region the projectile ions are charge-state analyzed by a magnet. Ions in the original charge state A^{Z+} are separated from projectiles which have changed their charge state. After passing through a thin Al foil, which is biased with -3.5 keV, these ions are collected on a metal plate. When passing the Al foil the ions produce a shower of electrons which are accelerated to the entrance funnel of a channel electron multiplier providing the start pulse for a time-to-amplitude converter (TAC). The pulses initiated by the slow helium recoil ions in the channel-plate detector stop the TAC. The He^{i+} ($i=1,2$) recoil-ion time of flight is proportional to the square root ($\sqrt{i}$) of its charge state. The projectiles however all have the same (short) time of flight to their detector. Therefore, the time-of-flight spectrum directly yields the charge-state distribution of the recoil ions. This spectrum is identified with pure ionization processes, i.e., collisions in which the projectile's charge state has not been charged. The time resolution in these spectra is up to 500.

By integrating the He^{1+} and He^{2+} peaks in the time-of-flight spectra we obtained relative fractions F_1 and F_2 of singly and doubly charged recoil ions. We then normalized these data to total net ionization cross sections σ_{tot} taken from work[16] of Schlachter *et al.* by using the classical trajectory Monte Carlo (CTMC) curve given for helium. We have used the CTMC curve to $E/Z=32$ keV/amu, i.e., beyond the observed data. Extrapolation of the observed helium data is consistent with this curve, so we do not expect errors larger than 30–50% from this normalization. These data can be represented as

$$\sigma_{tot}=\sigma^{+}+2\sigma^{2+} , \tag{1}$$

where σ^{+} and σ^{2+} are the cross sections for pure single and double ionization, respectively. Contributions from electron capture or stripping to σ_{tot} are below few percent in all cases. With

$$F_1=\sigma^{+}/(\sigma^{+}+\sigma^{2+}) \tag{2a}$$

and

$$F_2=\sigma^{2+}/(\sigma^{+}+\sigma^{2+}) , \tag{2b}$$

we obtain

$$\sigma^{+}=\sigma_{tot}F_1/(F_1+2F_2) \tag{2c}$$

and

$$\sigma^{2+}=\sigma_{tot}F_1/(F_1+2F_2) . \tag{2d}$$

The accuracy of the partial cross sections σ^{+} and σ^{2+} is mainly limited by the uncertainty of absolute σ_{tot} data, the ratio $R=F_1/F_2$ has an experimental uncertainty of less than 10%. Table I shows our measured fractions F_1,F_2 and the cross sections σ^{+},σ^{2+} determined from σ_{tot}.[16]

III. THEORY

Theoretical calculations of multiple ionization at the present time are more difficult than single ionization. Consequently, we shall use simpler methods for double ionization than for single ionization, which we consider first.

A. Single ionization

For weak perturbations of the atom by the projectile ion, one may expect the first Born approximation to be valid[17] for total cross sections for single ionization. A simple estimate of the error in the first Born approximation is given[18] by $(\int V dt)^2 \cong (Z/v)^2$ where V is the Coulomb interaction between the target electron and the projectile of charge Z with collision velocity v. Consequently, one may expect the first Born approximation to be adequate when Z is small or v is large, or equivalently when E/Z is greater than 100 keV/amu per Z on the Schlachter scaling[9] curve.

Since in this paper we consider a large range of Z at high v, we may expect to see a breakdown of the Born approximation at large Z when v (in atomic units) is not large compared to Z. Specifically, since the Born approximation varies exactly as Z^2 we may expect the data to deviate from a Z^2 scaling law at large Z^2, i.e., when $(Z/v)^2$ is not much smaller than unity.

While there are many excellent calculations[19–25] that include higher Born effects at low v, relatively little has been calculated[9] at high v and high Z, i.e., $Z \gg Z_{target}$, as considered in this paper. For comparison to our data presented in this region we use the Glauber approximation. For ionization of hydrogen there is evidence[11,12] that this Glauber approximation gives improved agreement with observations in comparison to the simpler Born approximation. Furthermore, at high velocities the Glauber approximation goes over to the simpler (and correct) Born approximation. As explained simply in a previous[11] paper, the Glauber approximation is a more complete solution to the Schrödinger equation than the simpler Born approximation. And for heavy-ion collisions the step between the intermediate eikonal and the

TABLE I. Charge-state fractions F_i ($i=1,2$) of He^{i+} recoil ions produced by 1.4 MeV/amu A^{z+} ion impact. The values for σ_{tot} are deduced from a compilation of scaled data by Schlachter *et al.* (Ref. 13).

Z	F_1 (%)	F_2 (%)	σ_{tot} (cm^2)	σ^{+} (cm^2)	σ^{2+} (cm^2)
6	94.7	5.26	5.50×10^{-16}	4.96×10^{-16}	2.76×10^{-17}
15	86.0	14.0	2.37×10^{-15}	1.79×10^{-15}	2.91×10^{-16}
18	83.3	16.7	3.15×10^{-15}	2.24×10^{-15}	4.50×10^{-16}
20	82.8	17.2	3.68×10^{-15}	2.60×10^{-15}	5.41×10^{-16}
36	78.1	21.9	8.93×10^{-15}	5.72×10^{-15}	1.60×10^{-15}
37	78.1	22.0	9.29×10^{-15}	5.95×10^{-15}	1.68×10^{-15}
44	75.8	24.2	1.18×10^{-14}	7.21×10^{-15}	2.30×10^{-15}

final Glauber approximation has been[26] justified. Hence, we may expect the Glauber approximation to give a better Z dependence to total cross sections at large Z and large v than the Born approximation.

B. Double ionization

Although relatively few calculations for double ionization of helium exist, it has nevertheless been possible to understand something about the mechanisms[27–29] for double-ionization cross sections. Apparently two mechanisms for double ionization of helium at high collision velocities are possible, namely rearrangement and direct mechanisms. In the rearrangement mechanism,[27–29] double ionization occurs following single ionization by rearrangement of the electronic wave function in the final state. The rearrangement mechanism is often characterized by a ratio of double to single ionization that is independent of Z and v, i.e., dependent only on final-state properties of the target.

The direct mechanism[30] corresponds to direct Coulomb ionization of both electrons by the projectile. In this paper the direct mechanism is evaluated using $\sigma^{+}=4\pi\int P(b)db$ and $\sigma^{2+}=2\pi\int P^{2}(b)db$ where $P(b)$ is found from the semiclassical approximation (SCA) tables[31] of Hansteen, Johnsen, and Kocbach. In the SCA the ionization probability varies as Z^2. The curve SCA* uses a binding energy of $\frac{1}{2}(24.6+54.4)$ eV while SCA uses 24.6 eV. In the high-v Bethe limit the ratio of double to single ionization varies as $Z^2/(v^2\ln v)$. The direct mechanism is clearly dependent on both Z and v unlike the rearrangement mechanism.

For the data presented here v is fixed and Z is varied. As Z increases we may expect the direct mechanism to become more important than the rearrangement mechanism. If the direct mechanism dominates and if Z is not too large, the ratio of double to single ionization should vary like Z^2. For sufficiently large Z the Born approximation may fail and then deviations from the Z^2 scaling may become apparent. Hence in our double-ionization data we shall consider the Z dependence of the ratio of double to single ionization.

IV. RESULTS

Total cross sections observed for single and double ionization of helium in collisions of projectiles of various charge Z at a fixed velocity $v=7.48$ a.u. (1.4 MeV/amu) are presented in Fig. 1 and in the table. In Fig. 1 we have plotted the cross sections as a function of Z. Our measurements include observations for C^{6+}, Fe^{15+}, Kr^{18+}, Fe^{20+}, U^{36+}, Gd^{37+}, and U^{44+}. We have also included in Fig. 1 data from Knudsen *et al.*[6] for single and double ionization of helium by projectiles with charge states up to $Z=8$.

In Fig. 1 we have also plotted our calculations using the Born and the Glauber approximations for single ionization. In these calculations we have used hydrogenic wave functions to describe the ground state of helium. Since helium is not hydrogenic, this introduces an intrinsic error of about 40% in our calculations. Other Born calculations done using better wave functions give agreement

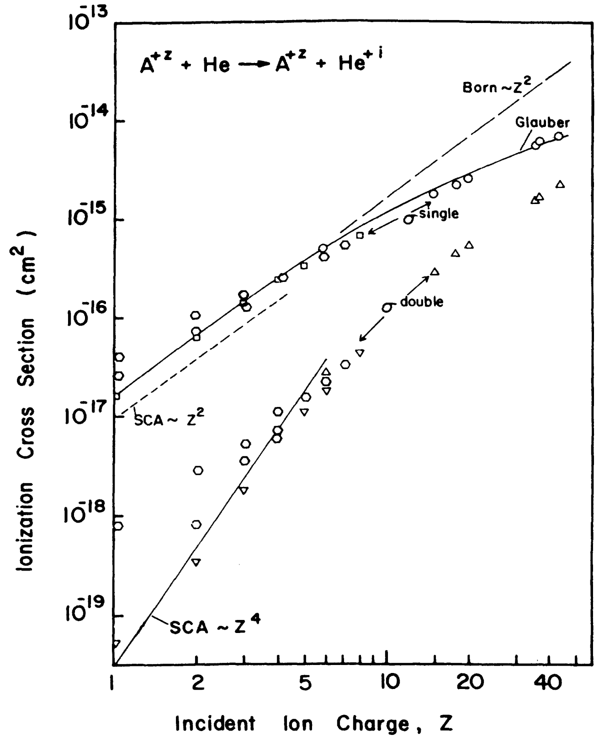

FIG. 1. Total cross sections versus projectile charge Z for single (σ^{+}) and double (σ^{2+}) ionization of helium summed over final states of the projectile. The symbol (○) represents our σ^{+}; △, our σ^{2+}; □, and **O**, σ^{+} from Knudsen *et al.* (Ref. 6); ▽ and **O**, σ^{2+} from Knudsen *et al.* (Ref. 6). Only □ and ▽ are for completely stripped ions.

within a few percent with the proton data at 1.4 MeV. Since we are interested in the dependence of the cross section on the charge Z of the projectile, we have normalized our Glauber and Born results to the $Z=1$ data by choosing a screened target charge in our calculations that fits the $Z=1$ data point. This corresponds to an effective screened charge Z_{target} of 1.3 for the helium target, somewhat smaller than[32] conventional values of about 1.6. Changing the Z_{target} value from 1.3 to 1.7 changes the overall normalization by about 50%, but changes the shape of the Z dependence considered here changed by less than 0.5%. The SCA calculations shown in Fig. 1 use $Z_{\text{target}}=1.7$ and for this reason are lower than our Born calculation using $Z_{\text{target}}=1.3$. It is not possible to fit all physical properties of our system by using a hydrogenic approximation for the ground state of helium. Using $Z_{\text{target}}=1.3$ gives a binding energy of 23 eV, close to the observed value of 24.6 eV, but a poor fit to numerically determined wave functions. Using $Z_{\text{target}}=1.6$ gives a good fit[9] to numerical wave functions, but gives a poor binding energy of 35 eV. To fully correct these problems, one must use nonhydrogenic wave functions for helium (and recalculate much algebra in the Glauber calculations). Since any electrons the projectile ions that are not fully stripped are tightly bound electrons, we regard all projectiles as an inert point charge of $Z=Z_p-n$ where Z_p is the nuclear charge of the projectile and n is the number of electrons on the projectile ion.

The Born approximation varies precisely as Z^2. This corresponds to a straight line on the log-log scale given in Fig. 1. The data falls below the Born calculation. The difference increases as Z increases, but not as much as the simple $(Z/v)^2$ estimate quoted above. For example, at $Z=44$ and $v=7.48$ a.u. (or 1.4 MeV/amu) one has $(Z/v)^2=35$, which is much larger than the difference between the data and the Born approximation. This is also evident from Fig. 2 where we present σ/Z^2 as a function of Z for single ionization. Our results are consistent with the earlier result of Haugen *et al.*[4] who also found deviation from Z^2 at fixed v. We note, however, that the difference between Born approximation and the data does vary approximately as $\frac{1}{10}(Z/v)^2$ consistent with a second Born correction. Nevertheless, in our opinion it is possible that effects beyond the second Born approximation are contributing to the highest Z systems considered here.

Also shown in Figs. 1 and 2 are our Glauber calculations for single ionization. When Z/v is small, Glauber and Born results are indistinguishable as expected. Although the Glauber approximation gives only the first Born amplitude exactly, it does contain contributions from all higher Born terms. Consequently, the Glauber results do not vary as Z^2. Our Glauber calculations are in reasonable agreement with observation until Z/v becomes large where the Glauber approximation may begin to fall a little beneath the trend of the data. This is consistent with previous comparisons to ionization of atomic hydrogen for various velocities v with $Z=1$, 2, and 3. It has been previously suggested[33] that for large Z/v some difference between Glauber calculations and data is possible due to electron capture to the continuum, not included in our Glauber calculations.

In general unitarity constraints may force the single-ionization cross sections to increase more slowly than Z^2. In the Born approximation the single ionization probability increases as Z^2 and therefore exceeds unity at large Z in violation of unitarity. The Glauber approximation satisfies unitarity approximately[34] at high velocities.

We note that our Glauber cross sections are in qualitative agreement with Schlachter scaling[9] and classical Monte Carlo calculations.[9] Glauber, Schlachter-Olson scaling, and classical Monte Carlo results all fall off more slowly than Born at low energies. However, Glauber results vary somewhat from Schlachter-Olson scaling where σ/Z is the same at a fixed E/Z for all Z. At 50 keV/amu Z, for example, our Glauber results give $\sigma/Z=0.62(\pi a_0^2/Z)$ at $Z=1$ and $1.06(\pi a_0^2/Z)$ at $Z=28$.

Now we consider double ionization. In Fig. 3 we plot the ratio of double to single-ionization cross sections σ^{2+}/σ^{+} as a function of Z. As discussed above, we have to consider both rearrangement (R) and direct (D) mechanism for analysis of the data. The total amplitude for both R and D using first Born approximation is

$$a=C_R Z+C_D Z^2 \ . \tag{3}$$

If the rearrangement mechanism dominates, then σ^{2+}/σ^{+} is independent of Z. At high velocities rearrangement dominates. The accepted value[35] for σ^{2+}/σ^{+} is given by the flat curve in Fig. 3 labeled R. This curve lies beneath all of our data, suggesting that rearrangement is not dominant here.

If the direct mechanism dominates, then σ^{2+}/σ^{+} varies with Z. For the smaller Z in Fig. 3 we see that σ^{2+}/σ^{+}

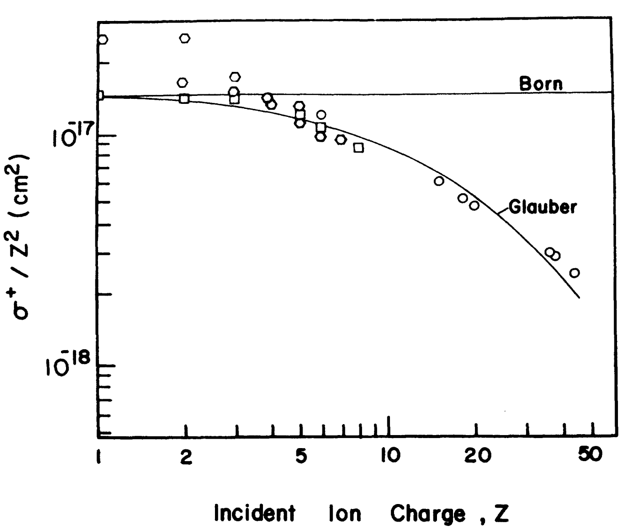

FIG. 2. Single ionization cross section divided by Z^2 versus projectile charge Z. Symbol ○ represents our σ^{+}, **O** and □, from Knudsen *et al.* (Ref. 6).

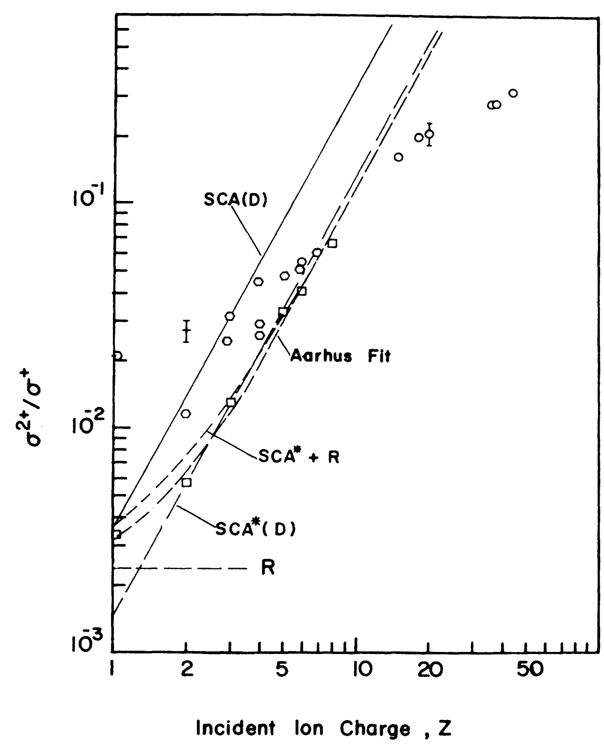

FIG. 3. Ratio of double to single ionization, σ^{2+}/σ^{+}, versus projectile charge Z. Symbol ○ represents our data; **O** and □, from Knudsen *et al.* (Ref. 6). The Aarhus fit is from Eq. (13) of Ref. 4, R corresponds to shakeoff in Ref. 10, the SCA and SCA* correspond to the SCA calculations of the D mechanism using a binding energy of 24.6 and 39.5 eV, respectively.

increases like Z^2 consistent with the direct mechanism in first Born or SCA approximation. In Fig. 1 we have included SCA calculations using the direct mechanism. The agreement given by this direct mechanism to the data is reasonable at the lower Z in both Figs. 1 and 3, as is the fit of Knudsen and co-workers[35] shown in Fig. 3. However, for the larger Z, the ratio σ^{2+}/σ^{+} increases more slowly than Z^2. We interpret this as a breakdown of the first Born approximation, (but not the direct mechanism). This interpretation is consistent with our previous discussion of single ionization at large Z.

V. SUMMARY

We have presented total cross sections for single and double ionization of helium by projectile ions of varying charge Z from $Z=1$ to 44 at a fixed collision velocity $v=7.48$ a.u. (1.4 MeV/amu). The single-ionization cross sections vary as Z^2 for $(Z/v)^2 \ll 1$ and increase less rapidly for larger $(Z/v)^2$. Glauber calculations reproduce the Z dependence well. In double ionization the direct mechanism appears to dominate over the rearrangement mechanism. At the smaller Z, the ratio σ^{2+}/σ^{+} increases as Z^2. At the larger Z, σ^{2+}/σ^{+} increases less rapidly than Z^2 consistent with a breakdown of the first Born or SCA approximation.

ACKNOWLEDGMENTS

We gratefully acknowledge discussion with H. Knudsen. This work was supported in part by the Division of Chemical Sciences, U.S. Department of Energy, and by Gesellschaft fur Schwerionenforschung mbH, Darmstadt, West Germany.

---

*Permanent address: Department of Physics, Kansas State University, Manhattan, KS 66506.

[1] P. Hvelplund, H. K. Haugen, and H. Knudsen, Phys. Rev. A **22**, 1930 (1980).

[2] M. B. Shah and H. B. Gilbody, J. Phys. B **14**, 2361 (1981).

[3] M. B. Shah and H. B. Gilbody, J. Phys. B **15**, 413 (1982).

[4] H. K. Haugen, L. H. Andersen, P. Hvelplund, and H. Knudsen, Phys. Rev. A **26**, 1950 (1982).

[5] M. E. Rudd, R. D. DuBois, L. H. Toburen, C. A. Ratcliffe, and T. V. Goffe, Phys. Rev. A **28**, 3244 (1983).

[6] H. Knudsen, L. H. Andersen, P. Hvelplund, G. Astner, H. Cederquist, H. Danared, L. Liljeby, and K. G. Rensfelt, J. Phys. **17**, 3545 (1984).

[7] M. B. Shah and H. B. Gilbody, J. Phys. B **18**, 899 (1985).

[8] R. E. Olson, Phys. Rev. **18**, 2464 (1978).

[9] R. E. Olson, K. H. Berkner, W. G. Graham, R. V. Pyle, A. S. Schlachter, and J. W. Sterns, Phys. Rev. Lett. **41**, 163 (1978). Also see A. S. Schlachter, K. H. Berkner, W. G. Graham, R. V. Pyle, P. J. Schneider, K. R. Stalder, J. W. Stearns, J. A. Tanis, and R. E. Olson, Phys. Rev. A **23**, 2331 (1981); A. S. Schlachter, K. H. Berkner, W. G. Grahman, R. V. Pyle, J. W. Stearns, and J. A. Tanis, *ibid.* **24**, 1110 (1981).

[10] J. Ullrich, C. L. Cocke, S. Kelbch, R. Mann, P. Richard, and H. Schmidt-Bocking, J. Phys. B **17**, L785 (1984); S. Kelbch, J. Ullrich, R. Mann, P. Richard, and H. Schmidt-Bocking, *ibid.* **18**, 323 (1985); S. Kelbch, J. Ullrich, W. Rauch, H. Schmidt-Bocking, M. Horbatsch, R. M. Dreizzler, S. Hagmann, R. Anholt, A. S. Schlachter, A. Müller, P. Richard, Ch. Stoller, C. L. Cocke, R. Mann, W. E. Meyerhof, and J. D. Rasmussen, J. Phys. B **19**, L47 (1986).

[11] J. H. McGuire, Phys. Rev. A **26**, 143 (1982).

[12] J. E. Golden and J. H. McGuire, Phys. Rev. A **15**, 499 (1977); **12**, 80 (1975).

[13] J. H. McGuire, Phys. Rev. Lett. **49**, 1153 (1982); Proceedings of the Second U.S.-Mexico Atomic Physics Symposium on Two Electron Phenomenon, Cocoyoc, Mexico, 1986 (unpublished).

[14] A. Müller, B. Schuch, W. Groh, E. Salzborn, H. F. Beyer, P. H. Mokler, and R. E. Olson, Phys. Rev. A **33**, 3010 (1986).

[15] W. Groh, A. Muller, A. S. Schlachter, and E. Salzborn, J. Phys. B **16**, 1843 (1983).

[16] A. S. Schlachter, K. H. Berkner, W. G. Graham, R. V. Pyle, P. J. Schneider, K. R. Stalder, J. W. Stearns, J. A. Tanis, and R. E. Olson, Phys. Rev. A **23**, 2331 (1981).

[17] D. H. Madison and E. Merzbacher, in *Atomic Inner Shell Processes,* edited by B. Crasemenn (Academic, New York, 1972), Chap. 1.

[18] This was first suggested to J. H. McGuire by M. Day (private communication).

[19] R. E. Olson and A. Salop, Phys. Rev. A **16**, 531 (1977); R. E. Olson, R. E. Berkner, K. H. Graham, W. G. Pyle, R. V. Schlachter, and J. W. Stearns, Phys. Rev. Lett. **41**, 163 (1978).

[20] D. Belkic, J. Phys. B **13**, L589 (1980).

[21] H. Ryufuku, Phys. Rev. A **25**, 720 (1982).

[22] R. K. Janev and L. P. Presnyakov, J. Phys. B **13**, 4233 (1980).

[23] R. Shakeshaft, Phys. Rev. A **18**, 1930 (1980).

[24] A. L. Ford, J. F. Reading, and R. L. Becker, Phys. Rev. A **23**, 510 (1981).

[25] C. Bottcher, Phys. Rev. Lett. **48**, 85 (1982).

[26] J. H. McGuire, J. Phys. B **16**, 3805 (1983).

[27] M. H. Mittleman, Phys. Rev. Lett. **16**, 498 (1966).

[28] J. M. Peek, Phys. Rev. **160**, 124 (1967).

[29] F. W. Bryon and E. J. Joachain, Phys. Rev. Lett. **16**, 1139 (1966).

[30] V. A. Sidorovitch, V. S. Nikolaev, and J. H. McGuire, Phys. Rev. A **31**, 2193 (1985).

[31] J. M. Hansteen, O. M. Johnsen, and L. Kocbach, At. Data Nucl. Data Tables **15**, 305 (1975).

[32] Good agreement with observation for $p+\mathrm{He}$ using a conventional value of $Z_{\mathrm{target}}=1.618$ was obtained in Ref. 9 by setting $Z_{\mathrm{target}}=1$ for the $l=0$ partial wave and introducing the binding energy separately into the kinematics, as is also done in the better Born calculations. This is not important to the Z dependence considered here.

[33] J. T. Park, J. E. Aldag, J. M. George, J. L. Preacher, and J. H. McGuire, Phys. Rev. A **15**, 508 (1977).

[34] R. J. Glauber, in *Lectures in Theoretical Physics,* edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. I, p. 315.

[35] Cf. Eq. (13) of Ref. 6. The fit of Eq. (13) of Ref. 6 may change somewhat when recent antiproton data is included [L. H. Andersen, P. Hvelplund, H. Knudsen, H. P. Moller, K. Elsner, K. G. Rensfelt, and U. Uggerhoj, Phys. Rev. Lett. **57**, 2147 (1986)].