Further evidence for causality violation

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Further evidence is presented for the violation of causality in the Compton scattering of light by protons for the energy region near the $\Delta(1232)$ resonance. A very simple precausal model has been found to reproduce both the photon absorption and scattering cross sections. Since dispersion relations are significantly violated in the same energy region, it is concluded that no causal model can fit the data as well as the precausal model.

INTRODUCTION

In the preceding paper,[1] we pointed out that the low-energy limit of the Kramers-Kronig dispersion relation for the scattering of light by protons was in substantial conflict with the experimental data on the proton electric and magnetic polarizabilities, which serve to determine the angular distribution of elastic photon scattering. We observed that the implication of this conflict is that microscopic causality is violated. We were able to reproduce the measured electric and magnetic polarizability of the proton with a theoretical model. We were able to show that this model explicitly violates local commutativity, in that the commutator of the charge density operator at two points did not always vanish for spacelike separations. In the present work we extend the results of Ref. 1 into the $\Delta(1232)$ resonance region, and demonstrate further evidence for causality violation.

We begin with a brief discussion of the existing problems in fitting the Compton scattering and photoabsorption data. We then consider a simple model of a damped, harmonically bound, radiating charge, except that we retain solutions which violate causality. We find that this simple model of "precausality" is able to account for the frequency spectrum of photoabsorption and scattering through the entire $\Delta(1232)$ resonance region. The γ decay branching ratio of the Δ is also well accounted for. These results from such a simple model are remarkable in that a conventional Breit-Wigner parametrization gives a very much inferior fit and dispersion relations are definitely violated in some regions for this same data. We feel the conclusion that precausal effects are manifest in nature to be strongly indicated based on these results.

THE PROBLEM

The earliest clear signal that there was a problem with the comparison of dispersion relations to photoproduction data can be seen in the analysis of Noelle, Pfeil, and Schwela.[2] These authors found a factor of 2 discrepancy between the amplitude $M_{1-}^{(3/2)}$ needed to fit pion photoproduction data, and the results from dispersion theory. This work was extended in Ref. 3 and included more recent and more accurate data, but the disagreement with dispersion theory persisted. A partial wave analysis of proton Compton scattering in the $\Delta(1232)$ energy region[4] showed substantial evidence for a discrepancy between the dominant f_{MM}^{1+} amplitude needed to fit data and the corresponding dispersion theory amplitude. The f_{MM}^{1+} amplitude describes the magnetic dipole excitation of the proton to the Δ resonance followed by magnetic dipole deexcitation, and is the dominant amplitude throughout the entire first resonance region. More recently, the systematic measurements of Genzel *et al.*[5] show very clearly that there is a discrepancy between the Compton scattering cross section and the results of dispersion theory, which is most readily discernible in their 90° data. Despite the evident violation of the dispersion relations, the possibility that causality might be violated seems to have been rejected out of hand. Numerous alternative explanations have been suggested over the years. Some typical examples selected from an extensive literature include such suggestion as erroneous data analysis,[6] neglect of certain pole terms in the dispersion relations,[7] or neglect of the annihilation channel.[8] The development[9,10] of *model-independent* lower bounds, however, seems to eliminate the dispersion-theoretic uncertainties. To quote from Ref. 9, "These violations clearly reveal that the results of the pion photoproduction multipole analyses used in the evaluation of the bounds and at least some of the experimental data for the Compton-scattering unpolarized differential cross section in the Δ resonance region are at variance. This conclusion, previously reached in a phenomenological way in [our Ref. 4] on the basis of a purely unitarity bound . . . is now reached rigorously." In Ref. 10, the model-independent lower bounds are improved (i.e., increased) so that the discrepancy with the experimental data is increased. Despite the evident violation of the dispersion relations, no claim of causality violation was made.

We see the failure of the data to conform to any dispersion relation as a serious problem, and since we will show that we are able to fit the data with an extremely simple precausal model, we do not believe that the problem of the discrepancy with dispersion relations lies with errors in the experimental measurements. Rather, we believe that the *a priori* assumption of the universal validity of causality must be abandoned.

Although our use of a classical model may be criticized on the grounds that only a fully relativistic, quantum-mechanical theory can possibly be a correct description, we will show in subsequent papers that our semiclassical treatment of the spectrum of Compton scattering is a reasonable approximation to a more complete, relativistic, quantum-mechanical theory which happens to violate causality. As the classical approach is extremely easy to apprehend and discuss, in the present article we confine our analysis entirely to the semiclassical treatment.

CLASSICAL THEORY OF PRECAUSALITY

We first consider the purely classical Abraham-Lorentz-Dirac model of a harmonically bound charge driven by an external electric field:

$$\ddot{\mathbf{x}}-\tau\dddot{\mathbf{x}}+\Gamma\dot{\mathbf{x}}+\omega_0^2\mathbf{x}=(e/m)\mathbf{E}(t)=\mathbf{f}(t)\ . \tag{1}$$

This equation accounts for normal damping through the Γ term, and radiative damping through the τ term. The radiative damping time constant in the Abraham-Lorentz-Dirac model is given by

$$\tau=\frac{2}{3}\frac{e^2}{m}\ . \tag{2}$$

The most general solution to this equation in the absence of a driving force is a linear superposition of terms of the form

$$x=x_0e^{-i\omega t}\ , \tag{3}$$

where

$$\omega^2+i\tau\omega^3+i\omega\Gamma-\omega_0^2=0\ . \tag{4}$$

Provided the appropriate cubic discriminant of this equation is positive, the roots of this equation are generally of the form

$$\begin{aligned}\omega_1&=\omega_0'-i\Gamma'/2\ ,\\ \omega_2&=-\omega_0'-i\Gamma'/2\ ,\\ \omega_3&=i/\tau'\ ,\end{aligned} \tag{5}$$

where the relation between the primed and unprimed quantities is

$$\begin{aligned}1/\tau&=1/\tau'-\Gamma'\ ,\\ \Gamma/\tau&=\Gamma'/\tau'-(\Gamma'^2/4+\omega_0'^2)\ ,\\ \omega_0^2/\tau&=(\omega_0'^2+\Gamma'^2/4)/\tau'\ .\end{aligned} \tag{6}$$

For small values of τ, the primed quantities become approximately equal to the unprimed quantities in (1).

The cubic secular Eq. (4) differs from the usual quadratic secular equation in that one of its roots has a positive imaginary part. This root leads to either runaway solutions or acausal solutions and thus is conventionally discarded. We now consider the consequences of retaining the positive imaginary root. The solution to (1) for a purely harmonic driving force

$$f(t)=F(\omega)e^{-i\omega t} \tag{7}$$

is

$$X(\omega)=\frac{F(\omega)e^{-i\omega t}}{-i\tau(\omega-\omega_1)(\omega-\omega_2)(\omega-\omega_3)}=G(\omega)F(\omega)e^{-i\omega t} \tag{8}$$

so that, making a Fourier decomposition of $f(t)$ we find the general solution

$$x(t)=\int_{-\infty}^{\infty}f(t')g(t-t')dt'\ , \tag{9}$$

where

$$g(t-t')=(1/2\pi)\int_{-\infty}^{\infty}G(\omega)e^{-i\omega(t-t')}d\omega \tag{10}$$

is the Green function for Eq. (1).

In the usual analysis,[11] there are no roots of the quadratic secular equation with positive imaginary parts, so that the Green function is guaranteed to vanish for $t<t'$, and therefore there is no effect of the future on the past. In the present case, because of the positive imaginary root, this Green function is generally precausal, in that effects may precede their cause.

We now consider an impulsive driving force

$$f(t)=f_0\delta(t) \tag{11}$$

and demand that both the position and velocity be continuous and finite for all times. The solution for this case is proportional to the Green function (10):

$$\begin{aligned}x(t)&\propto(\omega_3-\omega_2)e^{-i\omega_1 t}+(\omega_1-\omega_3)e^{-i\omega_2 t},\quad t>0\\ &\propto(\omega_1-\omega_2)e^{-i\omega_3 t},\quad t<0\ .\end{aligned} \tag{12}$$

From this we determine the acceleration

$$\begin{aligned}a(t)&=-\omega_1^2(\omega_3-\omega_2)e^{-i\omega_1 t}-\omega_2^2(\omega_1-\omega_3)e^{-i\omega_2 t},\quad t>0\\ &=-\omega_3^2(\omega_1-\omega_2)e^{-i\omega_3 t},\quad t<0\ ,\end{aligned} \tag{13}$$

with a frequency spectrum proportional to

$$a(\omega)=\int_{-\infty}^{\infty}a(t)e^{i\omega t}dt\ , \tag{14}$$

$$a(\omega)\propto\frac{\omega_1^2(\omega_2-\omega_3)}{\omega-\omega_1}+\frac{\omega_2^2(\omega_3-\omega_1)}{\omega-\omega_2}+\frac{\omega_3^2(\omega_1-\omega_2)}{\omega-\omega_3}\ . \tag{15}$$

We also get a velocity

$$\begin{aligned}v(t)&=-i\omega_1(\omega_3-\omega_2)e^{-i\omega_1 t}-i\omega_2(\omega_1-\omega_3)e^{-i\omega_2 t},\quad t>0\\ &=-i\omega_3(\omega_1-\omega_2)e^{-i\omega_3 t},\quad t<0\end{aligned} \tag{16}$$

with a frequency spectrum proportional to

$$v(\omega)\propto\frac{\omega_1(\omega_2-\omega_3)}{\omega-\omega_1}+\frac{\omega_2(\omega_3-\omega_1)}{\omega-\omega_2}+\frac{\omega_3(\omega_1-\omega_2)}{\omega-\omega_3}\ . \tag{17}$$

Since the impulsive force contains all frequencies with equal strength, we can find the entire frequency dependence of both the light scattering and absorption cross sections from the above solutions. The scattering cross section is determined by the component of the acceleration perpendicular to the observation direction $\mathbf{n}$, so that

$$d\sigma/d\Omega_{\text{scatt}}(\omega)=(e^2/8\pi)\,|\,\mathbf{n}\times\mathbf{a}(\omega)\,|^2\ . \tag{18}$$

By summing over polarizations, this purely classical expression yields an angular distribution

$$d\sigma/d\Omega_{\text{scatt}}(\omega)=(e^2/8\pi)\,|\,a(\omega)\,|^2(1+\cos^2\theta)\ . \quad (19)$$

The total scattering cross section is then

$$\sigma_{\text{scatt}}(\omega)=\tfrac{2}{3}e^2\,|\,a(\omega)\,|^2\ . \quad (20)$$

For the scattering of the spin-1 photon by a spin-$\frac{1}{2}$ nucleon through a spin-$\frac{3}{2}$ resonance, we instead use a differential scattering cross section which matches the classical value of expression (19) at forward angles and is given the correct quantum-mechanical angular distribution elsewhere,[12] as is dictated by the correspondence principle

$$d\sigma/d\Omega_{\text{scatt}}(\omega)=(e^2/8\pi)\,|\,a(\omega)\,|^2(7+3\cos^2\theta)/5\ . \quad (21)$$

This incorporation of the correct quantum-mechanical angular distribution is our only nonclassical effect. The total scattering cross section resulting from this expression is

$$\sigma_{\text{scatt}}(\omega)=\tfrac{4}{5}e^2\,|\,a(\omega)\,|^2\ . \quad (22)$$

The absorption cross section is determined by the power loss to the damping term which depends on the velocity of the charge, so that

$$\sigma_{\text{absorp}}(\omega)=m\Gamma\,|\,v(\omega)\,|^2\ . \quad (23)$$

The ratio of the scattering cross section to the absorption cross section is given by

$$\frac{\sigma_{\text{scatt}}(\omega)}{\sigma_{\text{absorp}}(\omega)}=\frac{\frac{4}{5}e^2\,|\,a(\omega)\,|^2}{m\Gamma\,|\,v(\omega)\,|^2}\ . \quad (24)$$

We can immediately confront this expression with the data for energies near the peak of the Δ resonance by observing that the expressions for the acceleration and velocity are dominated by the contribution from the nearby pole at ω_1. We thus find, using the accepted values for the energy and width of the Δ resonance to determine ω_1, the radiative branching ratio for the Δ resonance predicted by Eq. (24) is

$$\tfrac{4}{5}e^2\,|\,\omega_1\,|^2/(m\Gamma)=6.4\times10^{-3} \quad (25)$$

which is in reasonable agreement with the data. This is not a remarkable result, since almost the same expression results from a causal semiclassical model.

SIMPLE LIMITS

We note that the simple expression (20) contains five distinct and interesting regions. For example, consider the application of (20) to an electron bound in an atom, for which case the characteristic precausal frequency $1/\tau$ is very large compared to the atomic binding energy. For photon energies small compared to the binding energy, our expression reproduces the characteristic ω^4 dependence of Rayleigh scattering. As the frequency increases, we enter a "resonance" region where the cross section exhibits large enhancements. Well above the resonance region, but for frequencies small compared to $1/\tau$, the cross section becomes approximately constant, and equal to the classical Thompson scattering cross section, when properly normalized. As the product of $\omega\tau$ becomes significant, we find a region where the electron behaves as though it has a polarizability, in the sense of having a quadratically growing deviation from the constant Thompson cross section. Finally, at very high energy, the cross section vanishes inversely with the square of the energy. The absorption cross section behaves in a similar fashion, except that the extreme-low-energy limit grows only quadratically with energy, and there is no "Thompson"-like plateau region.

For the proton, we expect a similar behavior, and indeed the low-energy (below about 50 MeV) Compton scattering cross section for protons is given by the Thompson plateau limit of (20). The polarizability region above 50 MeV, however, shows clear evidence for substructure within the proton, and in fact is the first place we noticed evidence of causality-violating behavior.[1] In view of this we should expect to see a characteristic precausal time constant (2) determined by the quark charges and masses. For the up and down quarks, using masses derived empirically,[13] we find precausal time constants

$$\begin{aligned}\tau_{\text{up}}&=8e^2/27m_{\text{up}}\approx1/(2\ \text{GeV})\ ,\\ \tau_{\text{down}}&=2e^2/27m_{\text{down}}\approx1/(8\ \text{GeV})\ .\end{aligned} \quad (26)$$

Since the up-quark time constant is largest, it is likely to dominate in the proton, so that for two up quarks we expect an effective precausal time constant for the proton of

$$\tau_{\text{proton}}\approx1\ \text{GeV}^{-1}\ . \quad (27)$$

Since these values are so similar to the period of the Δ resonance, it is not a coincidence that the clearest example of acausal behavior is provided by light scattering near the Δ resonance frequency. Before we proceed to the comparison with data we consider the generalization of the above results to purely magnetic interactions, since we believe that the Δ resonance is primarily a magnetic excitation.

MAGNETIC PRECAUSAL MODEL

We now consider the appropriate equations for a magnetic dipole with radiative reaction effects explicitly included. Following Wheeler and Feynman,[14] we can calculate the radiative reaction fields for an absorbing universe explicitly.

Considering Fourier components whose time dependence is given by $e^{-i\omega t}$, the full retarded field of a magnetic dipole $\mathbf{M}$ is given by

$$\mathbf{B}_{\text{retarded}}=\frac{k^2\mathbf{n}\times(\mathbf{M}\times\mathbf{n})e^{ikr}}{r}+\frac{3\mathbf{n}(\mathbf{n}\cdot\mathbf{M})-\mathbf{M}}{r^3}(1-ikr)e^{ikr}\ . \quad (28)$$

where $\mathbf{n}$ is a unit vector in the direction of the observer. The response of an absorbing universe to the radiation zone limit of this field is

$$\mathbf{B}_{\text{response}}=\frac{k^2\mathbf{n}\times(\mathbf{M}\times\mathbf{n})(e^{ikr}-e^{-ikr})}{2r}+\frac{3\mathbf{n}(\mathbf{n}\cdot\mathbf{M})-\mathbf{M}}{2r^3}\times[(1-ikr)e^{ikr}-(1+ikr)e^{-ikr}] \quad (29)$$

$$=(\mathbf{B}_{\text{retarded}}-\mathbf{B}_{\text{advanced}})/2\ . \quad (30)$$

In the direct particle interaction theory, the intrinsic field of the magnetic dipole would be

$$\mathbf{B}_{\text{intrinsic}} = (\mathbf{B}_{\text{retarded}} + \mathbf{B}_{\text{advanced}})/2 \tag{31}$$

so that the net field surrounding the magnetic dipole is just the full retarded field,

$$\mathbf{B}_{\text{net}} = \mathbf{B}_{\text{retarded}} . \tag{32}$$

From the detailed expression for the radiation response field, we find at the position of the dipole

$$\mathbf{B}_{\text{response}} = \tfrac{2}{3}\dddot{\mathbf{M}} . \tag{33}$$

This reaction field produces a torque on the magnetic dipole which causes a change in the angular momentum $\mathbf{J}$, and in terms of the gyromagnetic ratio γ, defined by

$$\mathbf{M} = \gamma \mathbf{J} , \tag{34}$$

we find a Bloch equation

$$\dot{\mathbf{M}} = \gamma(\mathbf{M}\times\mathbf{B}) . \tag{35}$$

If we include a transverse damping term as well, we find the equations of motion for a magnetic dipole exposed to a transverse electromagnetic wave traveling in the z direction:

$$\begin{aligned} \dot{M}_x &= \omega_0 M_y - \gamma M_z B_y - \tfrac{2}{3}\gamma M_z \dddot{M}_y - \tfrac{1}{2}\Gamma_t M_x , \\ \dot{M}_y &= -\omega_0 M_x + \gamma M_z B_x + \tfrac{2}{3}\gamma M_z \dddot{M}_x - \tfrac{1}{2}\Gamma_t M_y , \\ \dot{M}_z &= \tfrac{2}{3}\gamma M_x \dddot{M} - \tfrac{2}{3}\gamma M_y \dddot{M}_x - \tfrac{1}{2}\Gamma M_z . \end{aligned} \tag{36}$$

If we ignore the time variation of M_z we have only the two equations for x and y components to solve, which can be written in terms of

$$\begin{aligned} M_c &= M_x + iM_y , \\ B_c &= B_x + iB_y \end{aligned} \tag{37}$$

as

$$\dot{M}_c = -i\omega_0 M_c + i\gamma M_z B_c + \tfrac{2}{3}\gamma M_z \dddot{M}_c - \tfrac{1}{2}\Gamma M_c . \tag{38}$$

We thus have a magnetic counterpart to Eq. (1),

$$\tfrac{2}{3}\gamma M_z \dddot{M}_c - \dot{M}_c - i\omega_0 M_c - \tfrac{1}{2}\Gamma M_c = -i\gamma M_z B_c = f(t) . \tag{39}$$

Once more assuming a time dependence

$$M_c = M_0 e^{-i\omega t} \tag{40}$$

and defining

$$\tau^2 = \tfrac{2}{3}\gamma M_z , \tag{41}$$

we get a secular equation

$$\tau^2\omega^3 + \omega - \omega_0 + i\Gamma/2 = 0 \tag{42}$$

because we have no second time derivative term, the roots of this secular equation must sum to zero. If we did have a second derivative damping term in (39), there would be no constraint that the roots sum to zero, but there would still be only three roots. In any event, treating the case of magnetic radiation in the same way that we treated electric radiation above leads to exactly the same spectrum for magnetic scattering when expressed in terms of the three roots of the corresponding magnetic secular equation. In the more general case, we should expect to be able to describe a mixture of electric and magnetic modes in terms of three complex numbers, representing the roots of a general cubic secular equation.

THE EXPERIMENTAL EVIDENCE

We now consider the comparison of the above theory with the experimental Compton scattering data. We wish to maintain contact with previous analyses[1,15–17] of the low-energy Compton scattering data (which are generally done in terms of laboratory system energies and laboratory differential cross sections), and so we retain the "polarizability expansion" in powers of ω/M, and simply add our precausal scattering cross section incoherently to this low-energy form. It turns out that whether laboratory or center-of-mass quantities are used, the quality of fit and resulting parameters change only slightly. We thus rely on the low-energy polarizability expansion to account for the terms up to order ω^3. Since eventually the polarizability expansion becomes unreasonable (certainly by the time the quadratic correction terms produce a negative cross section), we cease to add in the polarizability expansion above a certain energy. We specifically cease adding in the polarizability expansion above 275 MeV, where its contribution has become generally less than the experimental uncertainties. The quality of the fit and the resulting best fit parameters are not very sensitive to this changeover point, as long as it does not occur at such high energy that the quadratic polarizability terms have become important. Very explicitly, the model we use for the cross section is that above 275 MeV we have only

$$d\sigma/d\Omega_{\text{scatt}}(\omega) = \mathcal{N}\,|\,a(\omega)\,|^2(7+3\cos^2\theta) , \tag{43}$$

where $\mathcal{N}$ is the normalization and below 275 MeV we also add in

$$\begin{aligned} d\sigma/d\Omega = e^4/(2M^2)\{ &[1-2\gamma(1-\cos\theta)+3\gamma^2(1-\cos\theta)^2-4\gamma^3(1-\cos\theta)^3](1+\cos^2\theta) \\ &+\gamma^2[(1-\cos\theta)^2+a_0+a_1\cos\theta+a_2\cos^2\theta][1-3\gamma(1-\cos\theta)] \\ &-\gamma^2M^3/e^2[2\alpha'(1+\cos^2\theta)+4\beta\cos\theta][1-3\gamma(1-\cos\theta)]\} . \end{aligned} \tag{44}$$

TABLE I. Precausal fit to Compton scattering cross section. All differential cross sections are in units of 10^{-32} cm^2/sr, laboratory photon energies are in units of MeV and angles are in degrees.

n	θ (deg)	ω_{lab} (MeV)	$\frac{d\sigma_{\text{expt}}}{d\Omega}$ (10^{-32} cm^2/sr)	$\Delta\sigma_{\text{stat}}$ (10^{-32} cm^2/sr)	$\Delta\sigma_{\text{syst}}$ (10^{-32} cm^2/sr)	$\frac{d\sigma_{\text{theor}}}{d\Omega}$ (10^{-32} cm^2/sr)	$\frac{\sigma_{\text{expt}}-\sigma_{\text{theor}}}{\Delta\sigma}$ (10^{-32} cm^2/sr)
1	70.0	60.0	1.06	0.08	0.08	1.28	−1.93
2	90.0	60.0	1.08	0.04	0.09	1.15	−0.69
3	120.0	60.0	1.18	0.05	0.09	1.34	−1.61
4	150.0	60.0	1.47	0.06	0.12	1.76	−2.12
5	90.0	55.0	1.08	0.18	0.11	1.17	−0.41
6	90.0	65.0	1.09	0.21	0.11	1.13	−0.17
7	90.0	75.0	1.08	0.16	0.11	1.10	−0.09
8	90.0	90.0	1.08	0.18	0.11	1.06	0.11
9	90.0	102.0	1.04	0.11	0.10	1.04	0.03
10	90.0	112.0	1.04	0.14	0.10	1.03	0.07
11	90.0	126.0	0.98	0.25	0.10	1.04	−0.23
12	90.0	137.0	0.93	0.53	0.90	1.08	−0.14
13	50.0	90.0	1.34	0.35	0.13	1.30	0.10
14	50.0	105.0	1.34	0.35	0.13	1.15	0.52
15	50.0	115.0	1.20	0.35	0.12	1.05	0.42
16	90.0	54.0	1.37	0.37	0.10	1.17	0.52
17	90.0	65.0	1.19	0.31	0.08	1.13	0.18
18	90.0	75.0	1.22	0.25	0.09	1.10	0.46
19	90.0	86.0	1.34	0.20	0.09	1.07	1.24
20	90.0	97.0	1.27	0.20	0.09	1.04	1.03
21	90.0	107.0	1.10	0.24	0.08	1.03	0.28
22	90.0	121.0	1.13	0.28	0.08	1.03	0.33
23	135.0	55.0	1.62	0.51	0.11	1.59	0.06
24	135.0	67.0	1.52	0.42	0.11	1.50	0.05
25	135.0	80.0	1.59	0.41	0.11	1.41	0.41
26	135.0	92.0	1.74	0.33	0.12	1.35	1.11
27	135.0	106.0	1.99	0.44	0.14	1.30	1.49
28	135.0	120.0	1.22	0.37	0.09	1.29	−0.20
29	135.0	130.0	1.67	0.87	0.11	1.32	0.40
30	75.0	60.0	1.12	0.08	0.07	1.23	−0.99
31	90.0	60.0	1.10	0.05	0.07	1.15	−0.56
32	120	60.0	1.34	0.08	0.08	1.35	−0.05
33	135.0	60.0	1.56	0.08	0.09	1.55	0.07
34	150.0	60.0	1.93	0.07	0.11	1.75	1.34
35	90.0	80.9	1.15	0.06	0.01	1.08	1.14
36	150.0	81.9	1.44	0.12	0.01	1.57	−1.06
37	90.0	85.4	1.09	0.04	0.01	1.07	0.52
38	150.0	86.3	1.37	0.10	0.01	1.54	−1.66
39	150.0	106.7	1.60	0.08	0.01	1.44	1.95
40	90.0	109.9	1.03	0.06	0.01	1.03	0.02
41	150.0	111.1	1.44	0.06	0.01	1.44	0.08
42	90.0	236.9	5.3	0.5	0.23	5.48	−0.34
43	110.0	238.6	5.5	0.90	0.24	6.16	−0.71
44	130.0	249.1	6.9	1.1	0.30	9.28	−2.09
45	69.9	265.6	7.1	1.0	0.31	10.14	−2.91
46	90.0	272.6	11.6	1.0	0.51	11.84	−0.21
47	109.9	279.7	11.7	0.8	0.51	13.34	−1.74
48	70.1	284.6	14.6	1.1	0.64	14.37	0.18
49	129.9	285.0	15.8	1.3	0.70	16.19	−0.27
50	90.2	289.4	14.1	1.0	0.62	14.57	−0.40
51	70.2	316.9	22.3	1.5	0.98	18.00	2.40
52	60.0	323.1	24.0	1.6	1.06	18.92	2.65
53	110.1	329.4	19.1	1.1	0.84	17.67	1.03
54	90.0	329.5	16.0	1.1	0.70	16.82	−0.63
55	130.0	334.8	19.5	1.3	0.86	19.39	0.07
56	49.8	367.4	16.1	0.80	0.71	15.74	0.34
57	69.8	370.8	15.7	1.1	0.69	13.68	1.55
58	89.7	380.6	12.1	0.80	0.53	12.08	0.02
59	109.7	382.4	10.6	0.80	0.47	12.50	−2.05
60	129.6	386.5	12.5	1.0	0.55	13.57	−0.94
61	50.0	406.7	11.1	1.4	0.49	11.73	−0.43
62	89.8	407.8	10.5	2.3	0.46	9.89	0.26
63	69.9	415.8	8.7	0.9	0.38	9.83	−1.16

TABLE I. (*Continued*).

n	θ (deg)	ω_{lab} (MeV)	$\frac{d\sigma_{\text{expt}}}{d\Omega}$ (10^{-32} cm^2/sr)	$\Delta\sigma_{\text{stat}}$ (10^{-32} cm^2/sr)	$\Delta\sigma_{\text{syst}}$ (10^{-32} cm^2/sr)	$\frac{d\sigma_{\text{theor}}}{d\Omega}$ (10^{-32} cm^2/sr)	$\frac{\sigma_{\text{expt}}-\sigma_{\text{theor}}}{\Delta\sigma}$ (10^{-32} cm^2/sr)
64	89.8	420.8	9.5	0.7	0.42	9.05	0.55
65	109.9	427.4	8.4	0.5	0.37	9.10	−1.13
66	130.0	429.4	9.0	0.8	0.40	10.07	−1.20
67	140.0	120.0	1.77	0.24	0.18	1.34	1.42
68	139.8	139.0	2.28	0.24	0.23	1.43	2.55
69	139.7	163.0	2.17	0.24	0.22	1.80	1.14
70	139.6	184.0	2.99	0.24	0.30	2.50	1.28
71	92.1	193.0	1.93	0.20	0.19	2.10	−0.64
72	129.5	197.0	3.65	0.90	0.37	2.88	0.79
73	139.5	200.0	5.30	0.71	0.53	3.41	2.13
74	92.0	213.0	3.40	0.70	0.34	3.18	0.28
75	139.3	226.0	7.58	0.94	0.76	6.03	1.28
76	69.0	230.0	5.4	0.5	0.54	4.32	1.47
77	91.7	239.0	6.81	0.7	0.68	5.78	1.05
78	129.0	239.0	12.4	1.2	1.24	7.30	2.95
79	91.7	244.0	8.27	0.9	0.83	6.50	1.45
80	91.6	262.0	13.4	1.4	1.34	9.67	1.92
81	91.6	267.0	17.3	4.2	1.73	10.69	1.46
82	91.5	276.0	13.2	1.7	1.32	11.95	0.58
83	91.4	282.0	18.0	4.7	1.80	13.18	0.96
84	131.4	212.9	4.25	0.35	0.85	4.12	0.14
85	147.4	213.4	6.58	0.41	1.30	4.94	1.21
86	91.9	214.7	1.86	0.2	0.37	3.30	−3.43
87	107.2	215.2	2.49	0.26	0.50	3.54	−1.87
88	68.8	215.4	3.28	0.34	0.66	2.99	0.39
89	70.0	247.8	7.13	0.85	1.43	6.77	0.22
90	93.0	248.1	6.66	0.55	1.33	7.16	−0.35
91	131.9	248.5	9.59	0.63	1.92	9.33	0.13
92	108.4	248.8	8.54	0.65	1.71	7.71	0.45
93	147.7	249.2	13.0	0.7	2.60	11.08	0.71
94	142.0	210.0	4.4	0.8	0.0	4.33	0.08
95	138.0	260.0	15.2	1.6	0.0	12.71	1.55
96	92.0	272.5	15.5	3.1	0.0	11.85	1.18
97	90.0	310.0	14.5	1.6	0.0	16.96	−1.54
98	136.0	310.0	21.3	1.8	0.0	20.72	0.32
99	132.6	375.0	13.2	2.1	0.71	15.08	−0.85
100	132.8	400.0	11.1	1.1	0.60	12.53	−1.14
101	105.8	425.0	11.5	2.8	0.6	9.08	0.84
102	132.1	425.0	11.3	0.9	0.6	10.50	0.74
103	104.6	450.0	8.1	1.8	0.4	7.76	0.18
104	131.6	450.0	8.7	0.9	0.5	8.98	−0.27
105	104.4	475.0	9.2	1.2	0.5	6.77	1.87
106	132.2	475.0	8.6	0.7	0.5	7.87	0.85
107	103.7	500.0	8.3	1.0	0.5	5.99	2.07
108	131.8	500.0	7.3	0.6	0.4	6.96	0.47
109	90.6	450.0	6.3	1.2	0.0	7.55	−1.05
110	90.3	500.0	5.7	0.8	0.0	5.85	−0.19
111	90.0	287.5	15.8	1.1	4.0	14.23	0.38
112	120.0	307.5	20.6	2.1	5.0	18.61	0.37
113	75.0	312.5	18.0	1.6	4.5	17.55	0.09
114	90.0	312.5	14.3	0.9	3.5	17.06	−0.77
115	75.0	325.0	19.5	2.6	5.0	17.51	0.35
116	90.0	362.5	13.3	1.3	3.5	13.85	−0.15
117	90.0	412.5	12.0	1.7	3.0	9.57	0.70
118	80.0	450.0	10.0	0.75	1.00	7.65	1.88

In the above expression, $\gamma=\omega/M$, and the values

$$a_0=42.9,\quad a_1=-34.6,\quad a_2=-3.1 \tag{45}$$

are determined independently, and very precisely, by the magnetic moment of the proton. We show in Table I a detailed comparison of the above model with the available data on the Compton scattering cross section for energies below 500 MeV. We stop at 500 MeV, since the second baryon resonance begins to become important at energies somewhat above this. Table II contains a key to which

TABLE II. Summary of results in Table I.

Data points	Ref.	χ^2	χ^2/n	χ	$\chi/\sqrt{n}$
1–4	18	11.31	2.83	−6.36	−3.18
5–15	19	0.75	0.07	0.20	0.06
16–29	20	7.15	0.51	7.37	1.97
30–34	15	3.10	0.62	−0.19	−0.08
35–41	16	9.24	1.32	0.99	0.37
42–66	5	43.23	1.73	−7.18	−1.44
67–83	21	39.92	2.35	22.08	5.36
84–93	22	17.76	1.78	−2.41	−0.76
94–98	23	6.28	1.26	1.59	0.71
99–108	24	12.08	1.21	4.75	1.50
109–110	25	1.13	0.57	−1.23	−0.87
111–117	26	1.51	0.22	0.98	0.37
118–118	27	3.53	3.53	1.88	1.88

data points are measured by which experimental groups, as well as a breakdown of the individual contribution of each group's data to the total sum of squared residuals, χ^2. Because of the great diversity in the experimental data, we have added in quadrature the quoted systematic errors, when available, to the listed statistical errors in order to determine the estimated standard errors of the individual points. The total χ^2 is 157 for 111 degrees of freedom, which would not be very likely if the errors were purely statistical. However, by examining the largest deviations in detail, it is clear that the data contains some outlyers. For example, the very worst point, number 86, is at an angle and energy nearly identical to that of point number 74 taken by a different group, and the alternative measurement does agree with our theory. The second worst point, number 78, is also apparently an outlyer, in that it is in conflict with other points at similar angles and energies. Furthermore, it is at one extreme end of the energy range covered by the experiment, and the observation conditions may have been marginal. Since some of the other large residuals are apparent outlyers, we explored the effects of pruning some of the most poorly fit points. If the four worst points, for example, are removed from the data set, the resulting parameters are only slightly changed while the χ^2 becomes 117 for 107 degrees of freedom.

In contrast to our precausal model, we note that the resonance data is very poorly fit by a simple Breit-Wigner parametrization without invoking rather arbitrary background amplitudes. In the Bernardini reference,[21] for ex-

TABLE III. Theoretical photoabsorption cross sections compared to Bloom data.

ω (MeV)	$\sigma_{\rm expt}$ (μb)	$\Delta\sigma$ (μb)	$\sigma_{\rm theor}$[a]	$\frac{\sigma_{\rm expt}-\sigma_{\rm theor}}{\Delta\sigma}$	$\sigma_{\rm theor}$[b]	$\frac{\sigma_{\rm expt}-\sigma_{\rm theor}}{\Delta\sigma}$
188	78.8	41	86.5	−0.19	58.2	0.50
205	118.9	38	117.3	0.04	86.5	0.85
223	168.2	34	161.5	0.19	131.4	1.08
242	202.4	31	224.8	−0.73	202.3	0.01
260	323.4	32	302.8	0.64	296.4	0.85
279	387.1	34	399.3	−0.36	415.5	−0.83
298	504.2	37	490.8	0.36	518.0	−0.37
318	532.6	37	545.5	−0.35	557.2	−0.66
337	542.3	32	536.8	0.17	524.6	0.56
357	480.8	30	478.3	0.08	454.9	0.87
377	411.0	31	401.3	0.31	382.8	0.91
397	311.9	33	328.6	−0.51	321.7	−0.29
418	249.6	31	265.6	−0.52	271.3	−0.70
438	210.9	26	218.6	−0.30	234.2	−0.90
459	174.2	27	180.5	−0.24	204.1	−1.11
481	188.8	26	149.9	1.49	179.6	0.36
			Total $\chi^2=4.44$ 12 degrees of freedom		Total $\chi^2=8.75$ 12 degrees of freedom	

[a]Acausal model fit with parameters: $\omega_1=315-i\,80$ MeV, $\omega_3=+i\,5503$ MeV.

[b]Austern resonance model fit with Breit-Wigner resonance parameters: $\omega_0=301$ MeV, $\Gamma/2=70.5$ MeV.

ample, a one-level resonance model using the Walker[28] parametrization gives a terrible fit, as can be seen in their Fig. 16, for example. Furthermore, because of the significant violation of dispersion relations, most evident in the energy distribution for a scattering angle of 90° displayed by Genzel *et al.*,[5] we conclude that no causal model will fit the data as well as our simple precausal model. The parameters of our fit, and their standard deviations obtained from the diagonal elements of the error matrix, are (for the case with the four worst points removed)

$$\begin{aligned}
&\alpha'=(17.1\pm0.9)\times10^{-4}\ \text{fm}^3\ ,\\
&\beta=(-3.0\pm1.3)\times10^{-4}\ \text{fm}^3\ ,\\
&\omega_1=292.9\pm2.7-i\,71.6\pm3.1\ \text{MeV}\quad(\omega_2=-\omega_1^*)\ ,\\
&\omega_3=+i\,719\pm212\ \text{MeV}\ .
\end{aligned}\tag{46}$$

We note the polarizabilities we find are consistent with those found by Akhmedov and Fil'kov when they corrected the low-energy Compton scattering data for ω^4 terms. This result vindicates their analysis, and strengthens our conclusions of Ref. 1. We also note that the precausal frequency ω_3 is remarkably close to the crude estimate given in expression (27) above.

We now turn to the absorption cross section. Since in this case we have no other interfering processes, the absorption cross section is given simply by

$$\sigma_{\text{absorption}}(\omega)=\mathcal{N}\,|\,v(\omega)\,|^2\ .\tag{47}$$

We have already discussed the relative normalization of the scattering and absorption cross sections in (24), so that it only remains to compare the energy dependence of (47) with the data. Despite the great significance of the total photoabsorption cross section of the proton, this important quantity has not been carefully measured in the region of the $\Delta(1232)$ resonance. The apparent best direct measurement of the total absorption cross section by Armstrong *et al.*[29] appears to be fraught with systematic errors well outside the claimed values. By summing the measurements of Fischer *et al.*[30] of the neutral pion photoproduction to those of Fujii *et al.*[31] for charged pion production, we find substantial discrepancies with the measurements of Armstrong.[29] The most glaring discrepancy is the Armstrong cross section at 265 MeV of 424.5 μb, which appears to be high by about 100 μb compared to the summed cross section of 338 μb for single pion photoproduction, whereas the error quoted is only 8 μb. On the other hand, the indirect measurements of Bloom *et al.*,[32] based on extrapolation to $q^2=0$ of inelastic electron scattering, appear to agree very well with the integrated pion photoproduction data. For this reason, we consider only the Bloom data.

From the extremely small value of χ^2 (4.44 for 12 degrees of freedom, which occurs statistically only 2.6% of the time if the estimated errors are statistical) that we are able to obtain in our fit to the Bloom data, as exhibited in Table III, we are led to consider the possibility that the systematic errors have been overestimated. In the Bloom work, systematic errors were incorporated at several stages of the data analysis. First in the measurements of the electron scattering cross sections, second in the subtraction of "multiple photon" tails in the data, third in the estimate of radiative corrections, and finally an arbitrary overall systematic error was added in. We feel it is very plausible that they have been conservative in their data analysis. One of the implications of an overestimation of the errors would be that the discrepancy with dispersion relations discussed in Ref. 4 would become even more significant. Also the rough agreement with phenomenological analyses would disappear. We illustrate this by comparing the Bloom data to a simple one-level resonance model of Austern, where an apparently reasonable fit, with $\chi^2=8.75$ for 12 degrees of freedom, becomes a highly unlikely $\chi^2=23.7$ (which is exceeded only 2.3% of the time) if the assumed experimental errors are arbitrarily scaled to make our precausal model fit have a χ^2 per degree of freedom of unity. On the other hand, the reason the Bloom data remains unpublished[33] is that there was not a satisfactory conclusion as to what the best estimate of the systematic errors should have been, and the possibility that the agreement with our precausal model is merely fortuitous cannot be overlooked. If nothing else comes of our analysis of this data in terms of a causality-violating model, we at least hope that the total photon absorption cross sections will be better measured.

An important test of the consistency of our model is to demand that the same parameters fit both the absorption cross section and the scattering cross section. The photoabsorption data appears to need a somewhat higher value for the precausal frequency, but with the value of ω_3 constrained to be $922i$ MeV, which is one estimated standard error above the value of our best fit result in the analysis of the Compton scattering data, we obtain a χ^2 of 5.44 for the fit to the Bloom data, with the resonance parameter:

$$\omega_1=317.9\pm2.9-i\,83.0\pm4.6\ \text{MeV}$$

so that after accounting for the covariance of the fit parameters, we conclude that the absorption and scattering cross sections are consistent with a single set of precausal model parameters.

We must emphasize that, although arbitrary phenomenological parametrizations may well fit the data on scattering and absorption as well as our simple precausal model, no purely causal model can violate the dispersion relations, so the fact that our model not only fits the data, but by construction violates the dispersion relations, indicates that the most natural explanation of our successful fits is not that there is a conspiracy of experimental errors, but that nature, in fact, behaves in a precausal way.

SUMMARY

We have presented a simple semiclassical model of precausality, which fits the proton Compton scattering and photabsorption data better than any dispersion theory. We also suggest that because of a possible overestimate of systematic errors in the best existing measurement of the total photoabsorption cross section, previous agreements with phenomenological models and mild disagreements with dispersion relations may become significant disagreements. We conclude that there is significant evidence for the existence of precausal behavior in nature.

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