PHYSICAL REVIEW A VOLUME 35, NUMBER 6 MARCH 15, 1987

# Evidence for microscopic causality violation

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(Received 10 April 1986)

Evidence has been discovered for the violation of microscopic causality in the Compton scattering of light by protons. A calculation of the degree of acausality expected on the basis of a semiclassical model is found to be in quantitative agreement with the data.

It is well known that the classical model of the electron violates causality.[1] The Abraham-Lorentz-Dirac electron exposed to a sharp-edged pulse of electromagnetic radiation experiences a "preacceleration" which occurs before the wave front hits the electron. Such acausal behavior seems so philosophically distasteful that this feature of classical electrodynamics has usually been regarded as an artifact of the classical treatment, and merely symptomatic of an incomplete theory. We wish to emphasize that regardless of one's prejudices, the question of the possible existence of acausal physical phenomena should be answered based on experimental data, not on the basis of epistemology. It is a perfectly reasonable question to ask whether the acausality of classical electrodynamics has a corresponding description in quantum theory, and realization in nature. We maintain that classical preacceleration has a direct counterpart, both in quantum electrodynamics (QED) and in nature. We further claim that the existing data on proton Compton scattering strongly suggests causality-violating behavior.

In view of the apparently outrageous nature of our subject, we will first discuss precisely what we mean by "causality violation." We will explicitly contrast various types of causality violation with each other and discuss their connection to dispersion relations. We will motivate the search for causality violation specifically in "low-energy" proton Compton scattering. We then turn to the experimental data. In this paper we will discuss the low-energy limit, where a precise sum-rule test of the dispersion relations can be formulated. In the following article we consider the behavior of both proton Compton scattering and photomeson production cross sections throughout the region of the $\Delta(1232)$ resonance.

## MOTIVATION AND DEFINITIONS

In classical electromagnetism, the forward scattering amplitude for light of angular frequency $\omega$ scattered by a free nonrelativistic particle of mass $m$ and charge $e$ is given by

$$f(\omega)=-e^2/m(1+i\omega\tau)\ , \tag{1}$$

where

$$\tau=2e^2/3m\ . \tag{2}$$

(We use units such that $c=h/2\pi=1$.) This expression may easily be derived from the nonrelativistic limit of the Lorentz-Dirac equation of motion for a particle in a monochromatic electric field, including radiative reaction:

$$m(\ddot{\mathbf{x}}-\tau\dddot{\mathbf{x}})=e\mathbf{E}e^{-i\omega t}\ . \tag{3}$$

Heitler's quantum electromagnetic expression for the nonrelativistic, unpolarized, forward scattering amplitude of the electron is given by[2]

$$f(\omega)=-(e^2/m)[(1+i\omega\tau)^{-1}+h\ln(m/\omega)4e^2\omega^2/3\pi^2m^2] \tag{4}$$

which exactly agrees with the classical expression in the limit $h\to 0$, and exactly agrees with the perturbative QED expression[3]

$$f(\omega)=-(e^2/m)[1-i\omega\tau+h\ln(m/\omega)4e^2\omega^2/3\pi^2m^2] \tag{5}$$

to order $e^4$ (since $\tau$ is of order $e^2$). The first term in square brackets in (4) is the nonperturbative radiation damping term of Heitler, while the second term is the lowest-order radiative correction, which is a purely quantum-mechanical effect, as can be seen by the presence of $h$. Regarded as a power series in $\omega$, the quantum amplitude becomes

$$f(\omega)=-(e^2/m)[1-i\omega\tau-\omega^2\tau^2+h\ln(m/\omega)3\omega^2\tau^2/\pi^2e^2\cdots] \tag{6}$$

from which we see that radiation damping terms of order $\omega^2$ are insignificant compared to the purely quantum-mechanical terms at all real frequencies.

In general the phase shift $\delta(\omega)$ of the forward scattered light is determined by

$$\tan\delta(\omega)=\mathrm{Im}f(\omega)/\mathrm{Re}f(\omega) \tag{7}$$

which becomes $-\omega\tau$ in the nonrelativistic limit. From this phase shift the time delay in Compton scattering is given by

$$T_{\text{delay}}=d\delta/d\omega=-\tau\ \text{ for } \omega\ll m\ . \tag{8}$$

This negative time delay precisely corresponds to the phenomenon of preacceleration, except that rather than dismissing this effect as unphysical, we see that the same phenomenon is given by QED.

The essential result that there be a time advance is dictated by the optical theorem

 

$$\mathrm{Im} f(\omega)=\omega\sigma_{\mathrm{tot}}(\omega)/4\pi>0 \quad (9)$$

and the fact that

$$\mathrm{Re} f(\omega)\approx -e^2/m<0 \text{ for all } \omega . \quad (10)$$

Using the fully relativistic Klein-Nishina[4] formula for Compton scattering in the optical theorem (9) leads to an expression for the time delay identical to (8) in the low-frequency limit, but which tends to

$$T_{\mathrm{delay}}\rightarrow -e^2/4\omega \text{ for } \omega \gg m \quad (11)$$

in the extreme relativistic limit. Numerically, the limit (11) is approximately reached already at $\omega\approx 10m$. For all frequencies, the Compton scattered waves are advanced. As the Klein-Nishina formula has been experimentally verified from low energy to extreme high ($\approx 5$ GeV) energy,[5] there is no doubt that the time advance, which varies smoothly between the limits (8) and (11), is a real phenomenon, and not some "unphysical artifact of classical electromagnetism."

Is it possible *in principle* to directly measure the time advance of electron Compton scattering? If we construct a photon wave packet which is characterized by a width $\Delta T$ in time, then we must have, by the uncertainty principle, a distribution of frequencies with width characterized by $\Delta\omega > 1/\Delta T$. The ratio of the time advance acquired by the scattered wave packet to the width of the incident wave packet is bounded by

$$T_{\mathrm{advance}}/\Delta T<\Delta\omega e^2/4\omega_{\mathrm{av}}<e^2/4 . \quad (12)$$

Thus the time advance is never as large as the width in time of the incident photon wave packet. However, the centroid of the scattered wave packet is advanced in time. Thus, in principle, by making repeated measurements of the transit times of identically produced photons, some would arrive ahead of the wave-packet centroid, some behind, but the statistical ensemble of arrival times would precisely trace out the scattered wave-packet shape, and its centroid would be found to be advanced. There is no restriction by the uncertainty principle on how precisely we may determine the centroid of a wave function. For example, the centroid of the spatial wave function for the ground state of the electron in the hydrogen atom is exactly at the nucleus, even though its width is approximately the Bohr radius.

Now consider the influence of this time advance on the propagation of light in a material medium. The index of refraction is approximately

$$n(\omega)=1+2\pi N f(\omega)/\omega^2 , \quad (13)$$

where $N$ is the electron density, and we have neglected nuclear scattering. For bound electrons $f(\omega)$ varies as $\omega^2$ for $\omega\rightarrow 0$, but for high enough frequency that our free-electron scattering amplitude is relevant, we find

$$n(\omega)\approx 1-2\pi N e^2/m\omega^2(1+i\omega\tau) . \quad (14)$$

This index of refraction leads to a phase velocity greater than $c$, but a group velocity less than $c$, which is the familiar case for light propagation above the plasma frequency in a plasma.

It is very instructive to consider how the group velocity less than $c$ arises microscopically. Consider a disk-shaped light pulse impinging on a planar, monolayer array of electrons. We again neglect the nuclei, but will discuss the effect of wave-packet spreading below. On either side of the monolayer, the light pulse propagates at the speed of light. As the pulse passes through the plane of electrons, the forward scattered pulse adds coherently to the incident pulse. However, the scattered pulse is *advanced in time* and *reversed in sign* with respect to the incident pulse. As a result, the scattered pulse interferes destructively more with the leading edge of the incident pulse than with its trailing edge, so that the total transmitted pulse is retarded in time. If a succession of monolayers of such scatterers are encountered, the net result is that the speed of light in the medium is less than $c$, and in fact is given, as expected, by the group velocity. Thus in order not to transmit energy at speeds exceeding $c$ it is absolutely essential that the negative scattered waves be *advanced* in time with respect to the incident waves. In the final analysis we conclude that the classical preacceleration of the electron has a counterpart both in quantum theoretical physics and nature, and is an essential feature that must be present in order that electromagnetic energy propagate at speeds less than $c$ in material media.

Furthermore, it turns out that a scattered photon, even though it may gain a time $\tau$ in being scattered, on average arrives at a remote detector *later* than an unscattered photon proceeding directly from source to detector. Consider a scattering center located approximately midway between, but slightly off the axis joining source and detector. For a source-detector separation $L$, the extra path length for a ray scattered through a small angle $\theta$ is $L\theta^2/2$. If the scattered wave packet is to arrive at the detector in a shorter time than a ray traveling along the direct path, then we must have

$$cT_{\mathrm{advance}}>L\theta^2/2 . \quad (15)$$

On the other hand, if we are to observe the scattered wave packet independently of the unscattered wave packet, whose transverse spread we assume to be characterized by $w$, we must have

$$\theta>w/L . \quad (16)$$

Finally, in order that the spreading of the scattering wave packet be negligible,[6] we must have

$$w^2/L>c/\omega . \quad (17)$$

This chain of inequalities leads to the requirement

$$T_{\mathrm{advance}}>1/2\omega . \quad (18)$$

But we have already found that

$$\omega T_{\mathrm{advance}}<e^2/4 , \quad (19)$$

so that (18) is never satisfied. In brief, the scattered wave packet, even though advanced in time by the scattering process, never beats out a wave packet taking a "shortcut" along the direct path from source to detector.

We now turn to the relation between time advance and dispersion relations, essentially following the treatment of

Bjorken and Drell.[4] Consider a monochromatic plane-wave component for a given polarization propagating along the $x$ axis

$$a_{\mathrm{inc}}(\omega)e^{-i\omega(t-x)} . \tag{20}$$

A scattering center at the origin produces a corresponding frequency component given by

$$a_{\mathrm{scatt}}(\omega)=f(\omega)a_{\mathrm{inc}}(\omega) . \tag{21}$$

For a localized incident wave packet

$$A_{\mathrm{inc}}(x,t)=\int d\omega\, a_{\mathrm{inc}}(\omega)e^{-i\omega(t-x)} \tag{22}$$

the scattered wave packet tends asymptotically to

$$A_{\mathrm{scatt}}(x,t)\underset{x\to\infty}{\longrightarrow}(1/x)\int d\omega\, f(\omega)a_{\mathrm{inc}}(\omega)e^{-i\omega(t-x)} . \tag{23}$$

If the incident wave packet *exactly* vanishes for $x>t$, then the frequency components of the incident wave, given by

$$a_{\mathrm{inc}}(\omega)=(1/2\pi)\int_{-\infty}^{0} dx\, A_{\mathrm{inc}}(x,0)e^{-i\omega x} , \tag{24}$$

may be analytically continued into the upper half of the complex imaginary frequency plane, by simply replacing $\omega\to\mathrm{Re}\omega+i\,\mathrm{Im}\omega$, since the integral (24) has better convergence properties in this case. We will refer to the upper half of the complex frequency plane by $\Omega_+$ from now on. By demanding that there be no scattered wave before the incident wave reaches the scatterer, we impose the so-called "relativistic causality" condition:

$$A_{\mathrm{scatt}}(x,t)=0 \quad \text{for } x>t . \tag{25}$$

By the same reasoning used for the incident frequency components, the causality condition (25) implies that $f(\omega)a_{\mathrm{inc}}(\omega)$ may also be analytically continued into $\Omega_+$. Consequently, for any contour contained in $\Omega_+$, we may write a Cauchy relation for any frequency $\omega$ within such a contour

$$f(\omega)=(1/2\pi i)\int f(\omega')d\omega'/(\omega'-\omega) . \tag{26}$$

This identity leads to the dispersion relation

$$\mathrm{Re}f(\omega)=(1/\pi)\mathrm{P}\int_{-\infty}^{\infty} d\omega'\mathrm{Im}f(\omega')/(\omega'-\omega) \tag{27}$$

provided only that $f(\omega)$ vanishes sufficiently rapidly for $\omega\to\infty$. If $f(\omega)$ does not vanish sufficiently rapidly, we may write a similar Cauchy relation for $f(\omega)/\omega$, which leads, with the use of the optical theorem, to the famous Kramers-Kronig relation

$$\mathrm{Re}f(\omega)=\mathrm{Re}f(0)+\omega^2/2\pi^2\mathrm{P}\int_0^{\infty} d\omega'\sigma_{\mathrm{tot}}(\omega')/(\omega'^2-\omega^2) . \tag{28}$$

How does this relate to our discussion of the time advance in light scattering from electrons discussed above? The expressions for the scattering amplitude (1) and (4) have very different analytic structure compared to (5), even though (4) and (5) coincide exactly to order $e^4$. Both (1) and (4) have a pole in $\Omega_+$, while the perturbative expression (5) does not. Note that any finite number of terms of a series expansion of the classical function (1) in powers of $\omega\tau$ would still not produce a pole in $\Omega_+$. Such a power series expansion of (1) has a radius of convergence equal to the distance to the pole at $\omega=i/\tau$. The classical expression (1) will therefore not satisfy any dispersion relation which ignores the presence of this pole in $\Omega_+$. However, the corrections to the classical Kramers-Kronig relation for visible light will be at the level of $\omega^2\tau^2\approx10^{-16}$, and thus utterly unobservable in practice.

Even if the exact scattering amplitude has no poles in $\Omega_+$, there may still be a violation of the relativistic causality condition (25). For example, the function

$$f(\omega)=-(e^2/m)e^{-i\omega\tau} \tag{29}$$

coincides, to lowest order in $\omega\tau$, with both the classical scattering amplitude (1) and the QED amplitudes (4) and (5). It has the feature that it shifts the incident wave packet forward in time by $\tau$ completely undistorted. [We hasten to state that (29) is unphysical as it certainly violates the optical theorem whenever $\sin(\omega\tau)<0$.] This hypothetical scattering amplitude is analytic throughout $\Omega_+$. As a result, the relativistic causality condition (25) would be violated, yet the usual dispersion relations would be satisfied. Generalizing this example, we can readily see that if there is any finite time $\Delta T$, no matter how large, for which

$$A_{\mathrm{scatt}}(x,t)=0 \quad \text{for } x>t+\Delta T , \tag{30}$$

then the analyticity argument holds and the dispersion relations would be true, despite a potentially enormous violation of causality. Incidentally, retaining any finite number of terms in a power series expansion of (29) would still satisfy the relativistic causality condition (25), yet the limit of such a series disobeys (25).

For the order $e^4$ amplitude of QED, we find that the centroid of the scattered wave packet is advanced in time, but the leading edge is not, so that the causality condition (25) is satisfied. Consider

$$A_{\mathrm{scatt}}(x,t)\to(1/x)\int d\omega(-e^2/m)(1-i\omega\tau)\times a_{\mathrm{inc}}(\omega)e^{-i\omega(t-x)} \tag{31}$$

$$=(-e^2/mx)(1+\tau\partial/\partial t)A_{\mathrm{inc}}(x,t) \quad \text{for } x<t \tag{32}$$

$$\approx(-e^2/mx)A_{\mathrm{inc}}(x,t+\tau),\quad x<t \tag{33}$$

$$\equiv0,\quad x>t .$$

Similarly, each term in a power series expansion of (29) would lead to a corresponding term in a Taylor expansion of (33) about the time $t$, yet every individual term would vanish for $x>t$. The point is that safe conclusions about the analytic properties of a function cannot be drawn from only a finite number of terms in a perturbation expansion. For a general discussion of the dangers of analytic extrapolation in dispersion relation theory, see Ref. 7.

It is thus quite possible that the centroid of a scattered wave packet be advanced in time, and in fact is essential for Compton scattering in order that light not propagate macroscopically at speeds exceeding $c$ in material media, as we have shown. It is an open question at this point whether causality is violated, either in the manner of (30) or in the form resulting from the classical amplitude (1), for which there is no time $\Delta T$ such that (30) is true. It surely runs against intuition to have a wave which never exactly vanishes for any time in the past, no matter how remote. Yet the magnitude implied by the classical preacceleration is damped exponentially in the past, by $e^{t/\tau}$, so that even for a time corresponding to the light travel time across a Compton wavelength, we have a damping factor $e^{-206}$. Such small amplitudes are unobservable for all practical purposes. It is thus only our intuition which is offended, not our measurements. As our intuition has been seriously offended twice this century, first by relativity, and second by quantum mechanics, it should be left to the data to decide whether or not causality is violated, and whether or not dispersion relations are obeyed.

Having various examples and counter examples in hand, we now explicitly define our terms, essentially following Nussenzveig:[8] "Primitive causality" is the requirement that the "cause" precede the "effect." All of our forms of causality concern the time ordering of cause and effect, and we do not discuss the concept of effects without cause.

"Macroscopic relativistic causality" is the condition that energy not be transmitted at speeds exceeding $c$. We have shown that this condition is obeyed by Compton scattering even with preacceleration. "Microscopic relativistic causality" is the condition that a response strictly vanish at a time and place prior to a stimulus. We have seen that this condition is not true for classical Compton scattering, or for Heitler's expression, that it is true to order $e^4$, but that this is insufficient grounds for conclusions about nonperturbative effects. We have also seen that the existing data on Compton scattering from electrons is insufficient to decide whether this condition is violated at the level expected on the basis of the classical preacceleration. Henceforth, when we speak of generic causality violation, we mean, specifically, microscopic relativistic causality violation.

"Local commutativity," also known as "microscopic causality," is the condition that the commutator of two Heisenberg field operators taken at two spacelike separated points strictly vanish. It has been proven[9,10] that under very general, although not universal, circumstances this condition implies that corresponding dispersion relations be obeyed. The condition of local commutativity is a property of theories, and is thus, by itself, not directly related to experimental properties. It was noted in Ref. 9 that Heitler's radiation damping theory does not satisfy this condition. We quote, "The causality condition is not fulfilled for the scattering of light by a classical electron, as has been noted by Toll. The same criticism applies to the scattering formula obtained by Heitler's radiation damping theory. It turns out that in those cases the scattering amplitude has a pole in the upper half-plane. It gives a nonvanishing contribution to the commutator outside the light-cone which decays exponentially with a half-width of the order of the classical electron radius. This seems to be connected with the well known preacceleration of the classical electron."

From our collection of examples, we can see that it is possible to obey the dispersion relations, yet violate macroscopic relativistic causality, as could be the case with expression (29) for a sufficiently large phase constant. It is possible to obey the dispersion relations, obey macroscopic relativistic causality, but disobey microscopic relativistic causality, as with (29) with a sufficiently small phase constant. It is also possible to violate the dispersion relations, obey macroscopic relativistic causality, but not obey microscopic relativistic causality, as is the case with the scattering amplitudes (1) and (4). Of course the conventional wisdom is that all causality conditions are obeyed, and that the corresponding dispersion relations are all obeyed. What possible motivation is there to look for a breakdown in this conventional picture, and what would it mean?

Here is a fascinating possibility. The scattering amplitude is closely related to the $S$ matrix, so that a function of the form (1) or (4) would imply that there is a pole in the $S$ matrix in $\Omega_+$. Poles of the $S$ matrix are known to correspond to particles,[11] but it is regarded as impossible, on the basis of causality, to have any poles in $\Omega_+$. We conjecture that not only does such a pole exist, but that it corresponds exactly to a particle, and that particle is the muon for the case of light interacting with electrons. The basis for this conjecture is the remarkable numerical coincidence between

$$1/\tau = 3m/2e^2 = 105.04 \text{ MeV} \tag{34}$$

and the muon mass, $m_\mu = 105.66$ MeV. Furthermore, there are tantalizing similarities between the pole at $\omega = i/\tau$, and other well-understood nonperturbative phenomena, specifically solitons and instantons.[12] In all three cases we are dealing with solutions to classical equations of motion corresponding to nonlinear quantum field theories. Just as the existence of the pole at $\omega = i/\tau$ cannot be detected by any finite number of terms of a perturbation expansion of (1) or (4), so too is the existence of a soliton inaccessible through perturbation theory alone. Also, just as the frequency $1/\tau$ is inversely proportional to the electromagnetic coupling constant, so too is a soliton's mass inversely proportional to the relevant coupling constant. A general discussion of the utility of classical theory for exposing nonperturbative aspects of quantum field theory is given in Ref. 13.

The classical solution corresponding to the pole $\omega = i/\tau$ is just the usual runaway solution of the Lorentz-Dirac equation. It is almost universally argued that this solution is completely unphysical, and must be discarded, since it diverges as $t \to +\infty$, and so is unacceptable. But consider the so-called acceptable solutions corresponding to poles of the form $\omega = \omega_0 - i\Gamma/2$. These solutions vary as $e^{-i\omega_0 t} e^{-\Gamma t/2}$, and have exactly the same divergent character for times in the distant past. Without imposing a boundary condition at some time in the past, these solutions are no better behaved than the "runaway" solutions. Of course our intuitive feeling, based on lifelong experience, wherein we remember the past, and cannot see the future, is that imposing boundary conditions "in the future" to determine the behavior of a particle "in the present" is unphysical, whereas imposing boundary conditions "in the present" to determine the behavior of a particle "in the future" is acceptable. There are nonetheless situations wherein the behavior of a system "in the present" is dependent upon the boundary conditions "in the future." Such a situation is most clearly apparent experimentally in the Aspect, Dalibard, and Roger experiment.[14] In order to determine just what the ensemble of results of such a "delayed choice" experiment will be, it is necessary to specify the entire time history of the experimental arrangement. Such a situation has the clearest theoretical expression in the "transactional interpretation" of quantum mechanics.[15] Still, we must confess that the precise physical meaning of a possible pole in $\Omega_+$ remains mysterious to us. In brief, we are suggesting that the muon is a "quantized runaway," but we do not have a theory to fully describe this suggestion. If we did, it would be clear whether or not a series of sequential leptons would be expected, but as it is this remains an intriguing further speculation. Experimentally our conjecture stands or falls based on the true behavior of the scattering amplitude for complex imaginary frequencies in the vicinity of $i/\tau$. According to our conjecture, macroscopic relativistic causality would hold, while microscopic relativistic causality would fail, as would dispersion relations which do not take into account the poles in $\Omega_+$. We take the liberty of defining such behavior as "precausal" as opposed to "acausal," to avoid suggesting effects without cause.

Considering the above analysis, we view the testing of dispersion relations as of great importance. For the case of Compton scattering from the electron we can say no more than that the Klein-Nishina formula is entirely consistent with the data. As a result, as we have discussed above, it is impossible to determine on the basis of the existing data whether the dispersion relations are violated to the extent expected on the basis of our conjecture of a "muon pole" near $\omega=i/\tau$. Furthermore, it will be extremely difficult to ever see the effects of the precausal pole at $i/\tau$ in electron Compton scattering, since the quantum-mechanical effects are important at $\omega \approx m \ll 1/\tau$. It would be much easier to see precausal effects if the preacceleration frequency were not so large compared to the mass $m$. Fortunately, there appears to be a situation for which this is the case.

Consider the influence of radiative reaction on the electromagnetic interactions of the quarks in a proton. When probed at a scale of about 1 GeV, the up-quark mass is estimated[16] to be about 5 MeV. From this mass we can estimate a preacceleration frequency of $1/\tau_{\text{proton}} \approx 1$ GeV. This is self-consistently of the same order of magnitude as the energy scale for which the scale-dependent quark mass was estimated. We regard this as a strong hint that one might expect trouble with the dispersion relations for proton Compton scattering at relatively low energy. We now turn to the experimental data, which should be the final arbiter of the validity of such ideas.

## THE EXPERIMENTAL EVIDENCE

Our claim for causality violation is specifically founded on the observation that the Kramers-Kronig dispersion relation for Compton scattering of photons by protons is apparently violated. As a framework for understanding this we consider a semiclassical version of the Skyrme-Witten model of the proton, and we find quantitative agreement with the degree of violation apparently observed.

The first application of causality conditions to quantum field theory was the work of Gell-Mann, Goldberger, and Thirring.[9] They derived the Kramers-Kronig dispersion relation for the scattering of photons by a quantized matter field. Their formal expression of the microscopic causality condition is the demand that the commutator of two field operators vanish for spacelike separations. This requirement seems so axiomatic that the validity of the Kramers-Kronig relation for light scattering by protons has been regarded as sacrosanct. As a result, rather than subjecting this most basic dispersion relation to a test, in most subsequent analyses of Compton scattering data, its validity has been assumed.

As written explicitly by Gell-Mann and Goldberger,[17] microscopic causality demands that the low-frequency forward Compton scattering differential cross section obey the relation

$$(d\sigma/d\Omega)_{0^\circ}=e^4/m^2+\omega^2\left[4\mu_A^4-(e^2/m\pi^2)\times\int_0^\infty \sigma_T(\omega')d\omega'/\omega'^2\right]+(\text{higher-order terms in }\omega)\ , \tag{35}$$

where $\mu_A$ is the anomalous part of the magnetic moment of the proton, $\sigma_T$ is the total absorption cross section, and $\omega$ is the frequency of the photon in the laboratory.

The low-energy limit of elastic photon scattering is closely related to the static polarizability of the proton, since a low-frequency photon field is essentially equivalent to a static superposition of perpendicular $E$ and $B$ fields. Specifically, if the second-order electromagnetic energy change in static electric and magnetic fields is given by

$$\Delta U=-\alpha\mathbf{E}^2/2-\beta\mathbf{B}^2/2\ , \tag{36}$$

then the dispersion relation implies a relation between the electrostatic and magnetostatic polarizabilities and the $\omega^2$ term in the forward scattering cross section:

$$\alpha+\beta+e^2\langle r^2\rangle/3m=(1/2\pi^2)\int_0^\infty \sigma_T(\omega')d\omega'/\omega'^2\ . \tag{37}$$

Gell-Mann and Goldberger,[17] Goldhaber,[18] and Ericson and Hufner[19] discuss very clearly the relation between the low-energy photon scattering cross section and the polarizability in both quantum and classical mechanics. In view of the importance of relation (37) to our analysis we display a brief classical derivation of it. We first note the low-energy theorem of Goldhaber[18] to the effect that "the unpolarized cross section is completely classical through order $\omega^2$, with the corollary that for spinless targets this result applies to the amplitude itself, and extends to much higher order." We are thus justified in following the classical arguments of Ericson and Hufner.[19] For simplicity we assume a spherically symmetric spinless charge distribution of total charge $e$ and mass $m$, and thus neglect the anomalous magnetic moment term in equation (35), which in any case makes only a small contribution (comparable to the experimental errors in the measurements). Goldhaber gives a more detailed and enlightening discussion of the origin of the anomalous magnetic moment term through Thomas precession effects. In the presence of an electric field

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0\cos(\mathbf{k}\cdot\mathbf{r}-\omega t) , \tag{38}$$

the net force on the proton center of mass is

$$\begin{aligned}\mathbf{F}(t)&=e\mathbf{E}_0\int\rho(r)\cos(\mathbf{k}\cdot\mathbf{r}-\omega t)d^3r\\&=e\mathbf{E}_0G_E(k^2)\cos(\omega t)\\&\approx e\mathbf{E}_0(1-k^2\langle r^2\rangle/6+\cdots)\cos(\omega t) .\end{aligned} \tag{39}$$

In the above expression $G_E(k^2)$ is the proton electric form factor for momentum transfer $k^2$. The center of mass undergoes periodic oscillation

$$m\ddot{\mathbf{R}}(t)=\mathbf{F}(t)=e\mathbf{E}_0G_E(k^2)\cos(\omega t) \tag{40}$$

producing an oscillating electric dipole moment $e\mathbf{R}$. In addition, because of the polarizability there is an induced electric dipole moment

$$\mathbf{d}(t)=\alpha\mathbf{E}_0G_E(k^2)\cos(\omega t) . \tag{41}$$

Similarly, the response to an incident magnetic field is an induced magnetic dipole moment

$$\mathbf{M}(t)=\beta\mathbf{B}_0G_M(k^2)\cos(\omega t) . \tag{42}$$

In the lowest order in $\omega^2$ the form factors appearing in the induced dipole moments can be replaced by unity. The resulting magnetic field at a distance $r$ in direction $\mathbf{n}$ in the radiation zone resulting from the oscillating electric and magnetic dipole moments is

$$\mathbf{B}(t)=[\ddot{\mathbf{D}}(t)\times\mathbf{n}+(\ddot{\mathbf{M}}\times\mathbf{n})\times\mathbf{n}]/r . \tag{43}$$

The total electric dipole moment $\mathbf{D}(t)$ is the sum of the induced dipole moment and the dipole moment resulting from the motion of the center of mass, which again suffers a retardation correction

$$\begin{aligned}\ddot{\mathbf{D}}(t)&=\ddot{\mathbf{d}}(t)+e\ddot{\mathbf{R}}(t)G_E(k^2)\\&\approx[(e^2/m)(1-\omega^2\langle r^2\rangle/3)-\omega^2\alpha]\mathbf{E}_0\cos(\omega t) .\end{aligned} \tag{44}$$

The power radiated into the solid angle $d\Omega$ is

$$dP=\mathbf{B}^2(t)r^2d\Omega/4\pi \tag{45}$$

which, after dividing by the incident power flux, results in a differential cross section

$$\begin{aligned}d\sigma/d\Omega=&[(e^2/m)(1-k^2\langle r^2\rangle/3)-\alpha\omega^2]^2(1+\cos^2\theta)/2\\&-2(e^2/m)\omega^2\beta\cos\theta .\end{aligned} \tag{46}$$

By comparing with the low-frequency expansion of Eq. (35) for forward scattering, and ignoring the anomalous magnetic moment term which has been left out of our simple derivation, we find the relation (37). We thus see that the term involving the mean square charge radius of the proton is a simple dynamical effect which would be present even if $\alpha$ and $\beta$ vanished, resulting from the expansion of the proton electric form factor in powers of momentum transfer

$$G_E(k^2)=1-k^2\langle r^2\rangle/6+\cdots \tag{47}$$

so that the value of

$$e^2\langle r^2\rangle/3m=3.3\times10^{-4}\text{ fm}^3 \tag{48}$$

is very well determined. Various authors lump this term together with the electrostatic polarizability, and call the sum a "dynamic" polarizability. We will always explicitly keep these terms separate for clarity of discussion.

The numerical value of the dispersion integral,

$$(1/2\pi^2)\int_0^\infty\sigma_T(\omega')d\omega'/\omega'^2=(14.2\pm0.3)\times10^{-4}\text{ fm}^3 , \tag{49}$$

is well determined by detailed measurements[20] of the total photoproduction cross sections through the resonance region, and by assuming that the cross section does not become so large at some high energy that the integral is significantly altered.

The experimental situation for $\alpha$ and $\beta$, summarized in Table I, is quite different. In principle, from measurements of the angular distribution for Compton scattering [now including the effects of the magnetic moment neglected in expression (46) above],

$$\begin{aligned}d\sigma/d\Omega=e^4/(2m^2)\{&[1-2\gamma(1-\cos\theta)+3\gamma^2(1-\cos\theta)^2](1+\cos^2\theta)\\&+\gamma^2[(1-\cos\theta)^2+a_0+a_1\cos\theta+a_2\cos^2\theta]\\&-\gamma^2m^3/e^2[2(\alpha+e^2\langle r^2\rangle/3m)(1+\cos^2\theta)+4\beta\cos\theta]\}+(\text{higher-order terms in }\gamma) ,\end{aligned} \tag{50}$$

the values of $\alpha$ and $\beta$ can be determined. In the above expression, $\gamma=\omega/m$, and the values

$$a_0=42.9,\quad a_1=-34.6,\quad a_2=-3.1 \tag{51}$$

are determined independently, and very precisely, by the magnetic moment of the proton. In practice, the existence of forward peaked backgrounds makes it difficult to observe low-energy Compton scattering at angles much forward of 90°. In the first measurements,[21] for example, a point at 45° was excluded on the basis of contamination by such backgrounds. From the expression for the differential cross section, it can be seen that the forward

TABLE I. Measurements of electric and magnetic polarizability. All quantities in this table have units of $10^{-4}$ fm$^3$.

| Reference | $\alpha+e^2\langle r^2\rangle/3m$ | $\beta$ | Sum | Difference |
|---|---|---|---|---|
| Goldansky (Ref. 21) | 9.0±2.0 | 2.0±2.0 | fixed | 7.0±4.0 |
| Bernabeu (Ref. 22) | 10±2±5 | 4±2±5 | fixed | 6.0±10.0? |
| Baranov (Ref. 23) | 10.7±1.1 | −0.7±1.6 | 10.0 | 11.4±1.9 |
| Akhmedov (Ref. 25) | 20.0±1.1 | −6.0±1.6 | fixed | 26.0±1.9 |

scattering cross section depends only on the sum of $\alpha$ and $\beta$, the 90° cross section depends only on $\alpha$, while the 180° cross section depends only on the difference of $\alpha$ and $\beta$. As a result, the experimental data is far more sensitive to the difference of $\alpha$ and $\beta$ rather than the sum. In analyses of the Compton scattering, the sum $\alpha+\beta+e^2\langle r^2\rangle/3m$ has, with only one exception, been constrained to equal the value obtained from the dispersion relation. Because of this constraint on the data analysis and the lack of data for forward angles, it is not reasonable to believe either the $\alpha$ or the $\beta$ values alone; only their difference has been well determined by the fit to the data. As a conservative estimate of the errors in this difference, we quote in the table the linear sum of the errors quoted for the individual terms derived from the Goldansky data. The original Goldansky best fit values indicate a positive value for the difference

$$\alpha+e^2\langle r^2\rangle/3m-\beta\ . \tag{52}$$

This is a surprising result. One reason for surprise is the implication that the electric polarizability is so large, whereas the single most important feature of the nucleon excitation spectrum is the almost pure magnetic dipole $\Delta$ resonance. With this motivation, Bernabeu, Ericson, and Fontan[22] reanalyzed the Goldansky data, attempting to estimate the effects of systematic errors, but still found that the electric polarizability was dominant, although within the limits of their assumed errors they could make (52) negative. Since they held the sum of the polarizabilities fixed at the sum rule value, this analysis sheds no light on the validity of the forward dispersion relation. New data[23] by Baranov *et al.* were analyzed without demanding a fit to the dispersion relation, and in fact they found a very significant difference. These data were thought to be questionable,[24] since they "seem to violate forward dispersion relations very significantly at certain energies." Since this data was fit without regard to the dispersion relation, the errors in $\alpha$ and $\beta$ are less correlated, so that we have conservatively added the errors in quadrature in the difference column of our tabulation. Since the data is most directly sensitive to the difference of $\alpha$ and $\beta$, true uncertainties are probably smaller than this estimate. Finally, in the most recent analysis[25] of the Baranov data by Akhmedov and Fil'kov, the neglect of higher powers of $\omega$ in the Baranov data analysis was corrected for, but the sum of polarizabilities was constrained to fit the dispersion relation.

The interpretation of these results has been controversial for decades. Our point of view is that the data clearly indicates that the value of $\alpha$ is substantially larger than $\beta$. Including the Akhmedov and Fil'kov corrections for the neglect by Baranov of higher powers of $\omega$, this difference is at least a ten standard deviation effect. We will show below that this nonvanishing difference is sufficient to demonstrate microscopic causality violation.

Let us now reconsider the dispersion relation (37) without the assumption of microscopic causality. To be concrete, we consider the Skyrme-Witten model of nucleons, although our main conclusion is model independent. The Skyrme-Witten Lagrangian, including anomaly terms, is[26,27]

$$\begin{aligned} L=&(F_\pi^2/16)\mathrm{Tr}[(D_\mu U)(D^\mu U^{-1})]-\tfrac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)(\partial^\mu A^\nu-\partial^\nu A^\mu)\\ &+(e/16\pi^2)\epsilon^{\mu\nu\alpha\beta}A_\mu \mathrm{Tr}Q(\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U U^{-1}+U^{-1}\partial_\nu U U^{-1}\partial_\alpha U U^{-1}\partial_\beta U) \qquad (53)\\ &+i(e^2/8\pi^2)\epsilon^{\mu\nu\alpha\beta}(\partial_\mu A_\nu)A_\alpha \mathrm{Tr}(Q^2\partial_\beta U U^{-1}+Q^2U^{-1}\partial_\beta U+Q\partial_\beta U Q U^{-1}/2-QUQ\partial_\beta U^{-1}/2)\\ &+(\text{other meson terms})\ , \qquad (54)\end{aligned}$$

with $D_\mu U=\partial_\mu U-ieA_\mu[Q,U]$.

In the two-flavor approximation the field $U$ consists of space-time dependent SU(2) matrices, and

$$Q=\begin{bmatrix}\frac{2}{3} & 0\\ 0 & -\frac{1}{3}\end{bmatrix}. \tag{55}$$

This Lagrangian is invariant under the finite gauge transformation

$$\begin{aligned} U(x)&\to e^{+ie\alpha(x)Q}U(x)e^{-ie\alpha(x)Q}\ ,\\ A_\mu(x)&\to A_\mu(x)+\partial_\mu\alpha(x)\ . \end{aligned} \tag{56}$$

The baryons are represented by soliton solutions to the field equations deduced from (53). Terms in the Lagrangian, not explicitly written, involving mesons are needed to stabilize the soliton solutions and realistically describe baryon physics. Under the simple assumption that the soliton field has the maximum possible symmetry, it is possible to write the eigenfunctions to the above Lagrangian in the form

$$U(x)=A(t)\exp[iF(r)\boldsymbol{\tau}\cdot\mathbf{r}]A^{-1}(t)\ , \qquad (57)$$

where $F(0)=\pi$, and $F(\infty)=0$ describes a one-baryon state. Quantization of this field yields wave functions identical in mathematical form to the collective wave functions for an axially symmetric rigid rotor, with the isospin degree of freedom corresponding to the projection of angular momentum on the symmetry axis. The nucleon states correspond to the states of total angular momentum $\frac{1}{2}$, and the $\Delta$ states correspond to the states of total angular momentum $\frac{3}{2}$. The additional meson interaction terms do not affect this basic structure, only the detailed form of the soliton field function $F(r)$ is changed. Because the Skyrmion behaves as a rigid body, we might anticipate that it produces acausal behavior.

We will write the Hamiltonian corresponding to the above Lagrangian as an expansion in powers of $e$

$$H=H_0+H_1+H_2\ , \qquad (58)$$

where

$$H_1=ej_\mu A^\mu \qquad (59)$$

is the usual first-order electromagnetic coupling. The vector current $j_\mu$ contains both an isovector piece, which comes from the Skyrme part of the Lagrangian (53), and an isoscalar piece which comes from the anomaly part of the Lagrangian. The first-order term enters only in Figs. 1(a) and 1(b), while the second-order term enters in Fig. 1(c).

Matrix elements of the isovector current constructed from the above Lagrangian explicitly fail to satisfy the commutation relations demanded by microscopic causality. In the notation of Adkins, Nappi, and Witten, the angular integral of the time component of the isovector current is[28]

$$\int d\Omega\, V^{k,0}(\mathbf{r})=(4\pi i/3)\Lambda(r)\mathrm{Tr}[A^{-1}(t)\tau^k\partial_0 A(t)]\ , \qquad (60)$$

where $\Lambda(r)=F_\pi^2\sin^2[F(r)]+$(small corrections). By taking matrix elements of this operator in any nucleon state, the trace term can be rewritten in terms of the isospin operator, so that

$$\int d\Omega\, V^{k,0}(\mathbf{r})=(4\pi i/3)\Lambda(r)I_k\ , \qquad (61)$$

where $I_k$ is the $k$th component of the isospin operator. Replacing the isovector current operator by its matrix elements constitutes a semiclassical approximation to the Witten-Skyrme theory. From this we deduce the commutation relations for the matrix elements of the isovector charge densities

$$\left[\int d\Omega\, V^{i,0}(\mathbf{r}_1),\ \int d\Omega\, V^{j,0}(\mathbf{r}_2)\right] = -(16\pi^2 i/9)\Lambda(r_1)\Lambda(r_2)\epsilon_{ijk}I_k \qquad (62)$$

which do not vanish if $r_1\neq r_2$. Although this might be regarded as an unsatisfactory result of the semiclassical soliton, we instead view it as an outstanding triumph of the Skyrme-Witten theory, as we will find that it reproduces the data on the polarizability difference for the proton.

To calculate the polarizability of the Skyrmion, we consider the change in energy in static electric and magnetic fields in second-order perturbation theory:

$$\Delta U=\langle 0|H_2|0\rangle+\sum_n \frac{|\langle n|H_1|0\rangle|^2}{E_n-E_0}\ . \qquad (63)$$

Once again, we may identify the first-order terms with Figs. 1(a) and 1(b), while the second-order terms correspond to Fig. 1(c). Motivated by this expression, we will define first- and second-order electric polarizability contributions, $\alpha_1$ and $\alpha_2$, respectively, where $\alpha_1$ is the coefficient of $\mathbf{E}\cdot\mathbf{E}/2$ arising from the iterated first-order perturbation, and $\alpha_2$ arises from the diagonal second-order perturbation. Because the off-diagonal matrix elements which appear in (63) for an electrostatic field are exactly the same as those which appear in the electric dipole approximation to the transition matrix element for photon absorption, we may use a multipole analysis of photopion production to evaluate $\alpha_1$ directly from experimental data. In this approach we find

$$\alpha_1=(1/2\pi^2)\int_0^\infty \sigma_{E1}(\omega')d\omega'/\omega'^2 \qquad (64)$$

and in the same way

$$\beta_1=(1/2\pi^2)\int_0^\infty \sigma_{M1}(\omega')d\omega'/\omega'^2\ , \qquad (65)$$

where $\sigma_{E1}$ and $\sigma_{M1}$ are the electric and magnetic dipole parts of the total photoabsorption cross section. Because of the $1/\omega'^2$ weighting, most of the contribution to the integrals comes from energy regions where the long-wavelength approximation is good enough to trust this estimate of the static matrix elements. The results using the analysis of Pfeil and Schwela[29] below $\omega'=0.45$ GeV, and of Walker[30] up to 1.2 GeV, are

$$\alpha_1=3.7\times10^{-4}\ \mathrm{fm}^3\ , \qquad (66)$$

$$\beta_1=7.1\times10^{-4}\ \mathrm{fm}^3\ . \qquad (67)$$

The net result is that the sum

$$\alpha_1+\beta_1+e^2\langle r^2\rangle/3m=14.1\times10^{-4}\ \mathrm{fm}^3 \qquad (68)$$

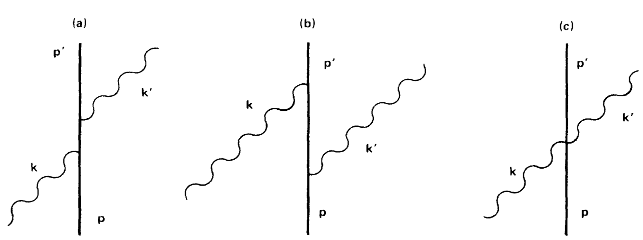


FIG. 1. Feynman diagrams for Compton scattering or electromagnetic energy shifts, (a) and (b) represent the iterated first-order interaction terms, (c) represents a second-order interaction term.

is, within errors, completely consistent with the dispersion integral. If microscopic causality were valid, then the contribution of the second-order terms could be shown to contribute nothing, and the dispersion relation would be verified. Microscopic causality, when used with the values of $\alpha_1$ and $\beta_1$ from (66) and (67), and the general relation $\beta_2=-\alpha_2/2$ which holds for any Dirac or Klein-Gordan particle (as shown in Refs. 19 and 31), or for the Skyrmion [as we will show later in Eq. (80)] thus implies that the value for the polarizability difference is

$$\alpha+e^2\langle r^2\rangle/3m-\beta=-0.1\times10^{-4}\ \text{fm}^3\ , \tag{69}$$

as opposed to the value determined by Akhmedov and Fil'kov:

$$26\pm1.9\times10^{-4}\ \text{fm}^3\ . \tag{70}$$

This result is substantially model independent, since the values of the first-order polarizabilities were directly determined by the photopion production data. The substantial polarizability difference observed constitutes the foundation of our claim that there is direct experimental evidence for the violation of microscopic causality.

## THEORETICAL CALCULATION OF ACAUSAL POLARIZABILITY

Let us now see what the Witten-Skyrme theory has to say. In order to make use of the solutions to the Lagrangian in the absence of electromagnetism, we choose the time-independent gauge

$$A_\mu=(-\mathbf{r}\cdot\mathbf{E},-\mathbf{r}\times\mathbf{B}/2)\ , \tag{71}$$

$$\mathbf{E}\cdot\mathbf{B}=0 \tag{72}$$

to describe uniform electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. We also specify that the fields be perpendicular. We then find the contribution to the polarizability from the second-order term in the Lagrangian for any solution having the symmetry of the Skyrme ansatz will not depend on the anomaly part of the Lagrangian, and will have the leading term

$$(F_\pi^2e^2/32)[(\mathbf{r}\cdot\mathbf{E})^2-(\mathbf{r}\times\mathbf{B})^2/4]\text{Tr}(1-\tau_3U\tau_3U^{-1})\ . \tag{73}$$

For the Lagrangian of Adkins and Nappi,[32] which incorporates explicitly the $\omega$-vector meson, this is the only relevant second-order electromagnetic term in the Lagrangian. To evaluate this term, we write the time-dependent $U$ matrices in the form[33]

$$U(x)=\exp\{i\tau_ie_{ij}[\alpha\beta\gamma]r_jF(r)/r\}\ . \tag{74}$$

The $3\times3$ matrix, $e_{ij}[\alpha\beta\gamma]$, represents a rotation in 3-space by the Euler angles $\alpha\beta\gamma$. Performing the trace operation in (73), the interaction becomes

$$(F_\pi^2e^2/8)[(\mathbf{r}\cdot\mathbf{E})^2-(\mathbf{r}\times\mathbf{B})^2/4]\times\sin^2[F(r)][1-(e_{3k}r_k/r)^2]\ . \tag{75}$$

Using the collective variable wave function for the nucleons expressed in terms of the rotation matrices

$$\begin{aligned}|n\uparrow\rangle&=i/2\pi D^{1/2}_{1/2\,1/2}(\alpha\beta\gamma)\ ,\\ |n\downarrow\rangle&=-i/2\pi D^{1/2}_{1/2\,-1/2}(\alpha\beta\gamma)\ ,\\ |p\uparrow\rangle&=i/2\pi D^{1/2}_{-1/2\,1/2}(\alpha\beta\gamma)\ ,\\ |p\downarrow\rangle&=-i/2\pi D^{1/2}_{-1/2\,-1/2}(\alpha\beta\gamma)\ ,\end{aligned} \tag{76}$$

we can take matrix elements of the interaction in any nucleon state, and we find the expectation value

$$(F_\pi^2e^2/12)[(\mathbf{r}\cdot\mathbf{E})^2-(\mathbf{r}\times\mathbf{B})^2/4]\sin^2F(r)\ . \tag{77}$$

Since the soliton field $F(r)$ is spherically symmetric, we may do the angular integrals easily, to find

$$H_{\text{int}}=-F_\pi^2e^2(\mathbf{E}^2-\mathbf{B}^2/2)(\pi/9)\int_0^\infty r^4\sin^2F(r)dr \tag{78}$$

so that the polarizability is simply

$$\alpha_2=(2F_\pi^2e^2\pi/9)\int_0^\infty r^4\sin^2F(r)dr\ , \tag{79}$$

$$\beta_2=-\alpha_2/2\ . \tag{80}$$

The relation (80) which we have found for the Skyrmion is a very general feature. It has previously been noted[19,31] that the same relation holds for any particle which obeys either the Dirac equation or the Klein-Gordan equation. The implied relation that $\alpha_2+\beta_2=(\alpha_2-\beta_2)/3$ is an important ingredient in going from Eq. (68) to Eq. (69) in the preceding text.

The value of the integral for the soliton solution of Adkins and Nappi has been integrated numerically with the result 1.03 $\text{fm}^5$. Using Adkins and Nappi's value for $F_\pi$ of 124 MeV, the predicted second-order contributions to the polarizability are

$$\alpha_2=20.8\times10^{-4}\ \text{fm}^3,\quad \beta_2=-10.4\times10^{-4}\ \text{fm}^3\ . \tag{81}$$

The result for $\beta_2$ is completely consistent with the recent analysis by Nyman[34] of the magnetic polarizability of the Skyrmion, using the original Skyrme Lagrangian. Nyman's main result was the relation

$$\beta_2=-\tfrac{1}{4}e^2\langle r^2\rangle_{\text{iso}}/(M_\Delta-M_N) \tag{82}$$

which is independent of the parameters of the original Skyrme model. In more elaborate versions of the Skyrme-Witten Lagrangian, such as the one we have used based on the analysis of Adkins and Nappi which includes the effects of the $\omega$ meson, the precise proportionality of $\beta_2$ to the mean square isovector radius no longer holds, but what does remain valid is the proportionality to the mean square radius of the Skyrme field, $\sin^2[F(r)]$. Our prediction for the polarizability difference, using the experimentally determined values for $\alpha_1$, $\beta_1$, and $\langle r^2\rangle$, and the theoretical values for $\alpha_2$ and $\beta_2$, is

$$\alpha+e^2\langle r^2\rangle/3m-\beta=31.1\times10^{-4}\ \text{fm}^3\ , \tag{83}$$

which agrees, within the 30% errors typical of the Skyrmion theory, with the data:

$$(26\pm1.9)\times10^{-4}\ \text{fm}^3\ . \tag{84}$$

We emphasize that our value of $\beta_1$ differs from the value used by Nyman, since we do not make the narrow resonance approximation in representing the effect of the $\Delta$.

This essentially perfect agreement with the data is a remarkable result, as it was certainly not put into the Skyrme model *ab initio*.

Several analyses of the nucleon polarizability have been made in the framework of the quark bag model,[35–37] but in view of the phenomenological nature of the bag models, it is never clear whether proper treatment of the bag degrees of freedom is important. As a result, none of these treatments can unequivocably challenge the dispersion relation.

We expect that other dispersion relations should also be violated. We find that this is indeed the case. Returning to the photopion production data, it is very clear that the isovector part of the magnetic dipole amplitude for transitions to $j=\frac{1}{2}$ states, $M_{1-}^{(3/2)}$, is in serious disagreement[38] with the relevant dispersion relation. Ironically, this fact was noted by the authors of Ref. 38, and the statement was made: "Dispersive calculations should be repeated in order to see why their predictions are wrong."

Further evidence, of an indirect and tentative nature, that the polarizabilities we have found are the correct ones can be found in the fact that a neutron magnetic polarizability as large as expression (67) would be incompatible with conventional treatments of neutron stars, as pointed out by Bernabeu, Ericson, and Fontan.[39] Only by including the $\beta_2$ contribution is the total magnetic polarizability brought down to a level which is consistent with the conventional treatment of neutron stars. We will discuss in a later paper that the electrostatic polarizability of the neutron implied by the calculation above resolves a long standing mystery concerning the unusual smallness of the neutron charge radius. We will also discuss in the following article the resolution of a long standing problem concerning the shape and angular distribution of Compton scattering in the $\Delta$ resonance region.

## QUESTIONS OF SELF-CONSISTENCY

An important point of self-consistency is that the Witten picture of Skyrmions was motivated by current algebra. The practical applications of current algebra always involve the assumption that the equal time commutators of currents at spacelike separation vanish. Thus the discovery that this assumption is *a priori* invalid puts the entire subject of current algebra on a shaky foundation. In order to save it, we must suppose that many of the assumed commutation relations are at least approximately right. In this sense, we see that the semiclassical Witten-Skyrme theory transcends its origins, in that its range of applicability is apparently broader than the foundations on which it was based.

A more general objection to our discovery that the semiclassical Witten-Skyrme theory violates microscopic causality, as shown by the commutation relations of Eq. (62), is that the Witten-Skyrme theory is grounded on causal behavior, so how could it lead to causality violating results? However, by considering the foundations of Witten's derivation of the Skyrme theory, we see that it rests on a principle of least action, with an action that is chosen on the basis of very general symmetry and topological grounds. *The demand that it be causal is nowhere made.* It remains to be checked whether or not any given action is causal. There are several known examples of theories based on least action principles which do not satisfy causality. The fact that acausal behavior in classical electrodynamics can be described in terms of an action principle is well illustrated by the work of Wheeler and Feynman,[40] Hoyle and Narlikar,[41] and Dirac[42] concerning the Fokker action. In the work of Kramer and Palmer,[43] it appears that they derive equal time commutation relations for the Witten-Skyrme theory which *do* obey microscopic causality. However, on close inspection, these authors assume microscopic causality implicitly at the outset in their equation 4, and so their argument is circular. We have introduced causality violation by not assuming that the local equal time commutation relations obey microscopic causality, although we retain the original global commutation relations.

On another point of consistency, we have found a violation of the Kramers-Kronig dispersion relation for light scattering by protons, yet the corresponding dispersion relation for light scattering by atoms is very successful. Why hasn't a corresponding violation of the dispersion relation been seen in atoms? The reason lies in the relative size of the causality violation compared to the usual Rayleigh scattering terms. For atoms, the normal polarizability is given approximately by

$$\alpha = e^2/m\omega^2 , \tag{85}$$

where $\omega$ is typically a few electron volts. The acausal polarizability can be no more than

$$\alpha = e^2\langle r^2\rangle/m , \tag{86}$$

where $\langle r^2\rangle$ is typically $10^{-7}$ $\mathrm{eV}^{-2}$, and thus insignificant changes in the Kramers-Kronig dispersion relation for atoms are expected.

A similar argument applied to nuclei would imply that there should be microscopic acausality effects in nuclei. In fact there has been a long standing problem with "missing $M1$ strength" and "missing Gamov-Teller strength" in that sum rules for both of these quantities are not fulfilled. Since these sum rules are based, in part, on the assumption of microscopic causality, we would argue that there is no requirement to satisfy the sum rules in the first place.

## SUMMARY

We have found evidence, based on the observed angular distribution of Compton scattering, and the measured photoabsorption cross section of protons, that the difference of the electrostatic and magnetostatic polarizability of the proton is much larger than can be accommodated under the assumption of microscopic causality. We have been able to theoretically reproduce this difference using a semiclassical version of the Witten-Skyrme model of nucleons.

## ACKNOWLEDGMENTS

I wish to thank N. Snyderman for a comment which helped clarify the origin of the causality violating behavior of the semiclassical skyrmion. I wish to thank P. Springer for discussions concerning the "observability" of classical preacceleration. I wish to thank J. Cramer and G. Miller for numerous criticisms on early forms of this manuscript. This work was performed under the auspices of the U.S. Department of Energy under Contract No. W-7405-Eng-48.

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