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Holographic observation of period-doubled and chaotic bubble oscillations in acoustic cavitation

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Bubble fields generated in a liquid (water) irradiated with sound of high intensity are investigated by high-speed holographic cinematography at framing rates up to 69 300 holograms per second. The period-doubling route to chaos found previously in the sound output from the liquid also shows up in the bubble oscillations. This observation confirms the view that the nonlinear acoustic response of the liquid is mediated by the bubbles and their nonlinear oscillations.

The progress in dynamical systems theory[1,2] has shed new light on the old problem of acoustic cavitation noise which arises when a liquid is irradiated with a sound of high intensity.[3-7] Esche[8] was the first to observe subharmonics in the spectrum of the response of the liquid. Since that time the physical mechanism behind the generation of these subharmonics is a source of constant debate (see Ref. 9 for a discussion). In the case of normal liquids (e.g., water) it is now widely believed that the bubbles which appear in the liquid beyond a threshold acoustic intensity are the cause for the transformation of the input frequency into its subharmonics and broadband noise. A different mechanism is conjectured to be at work in superfluid helium because conventional vapor bubbles cannot exist in that case.[10]

In this Rapid Communication we show that whole cavitation bubble fields produced in water may undergo period doubling in a sound field, i.e, may oscillate at one half, one fourth, . . . the driving sound field frequency and even chaotically beyond the accumulation point of period doubling. This is achieved with the help of high-speed holographic cinematography at framing rates up to 69 300 holograms per second. The sound output from the liquid is measured simultaneously and compared with the holographic data.

The experimental arrangement is shown in part and schematically in Fig. 1. The liquid is irradiated with sound by applying a periodic signal to a piezoelectric cylinder (PZT 4-material) totally submerged in the liquid. The cylinder has a diameter and length of 76 mm and a wall thickness of 5 mm. It is driven at 23.1 kHz. Cavitation bubbles produced acoustically in this way are small

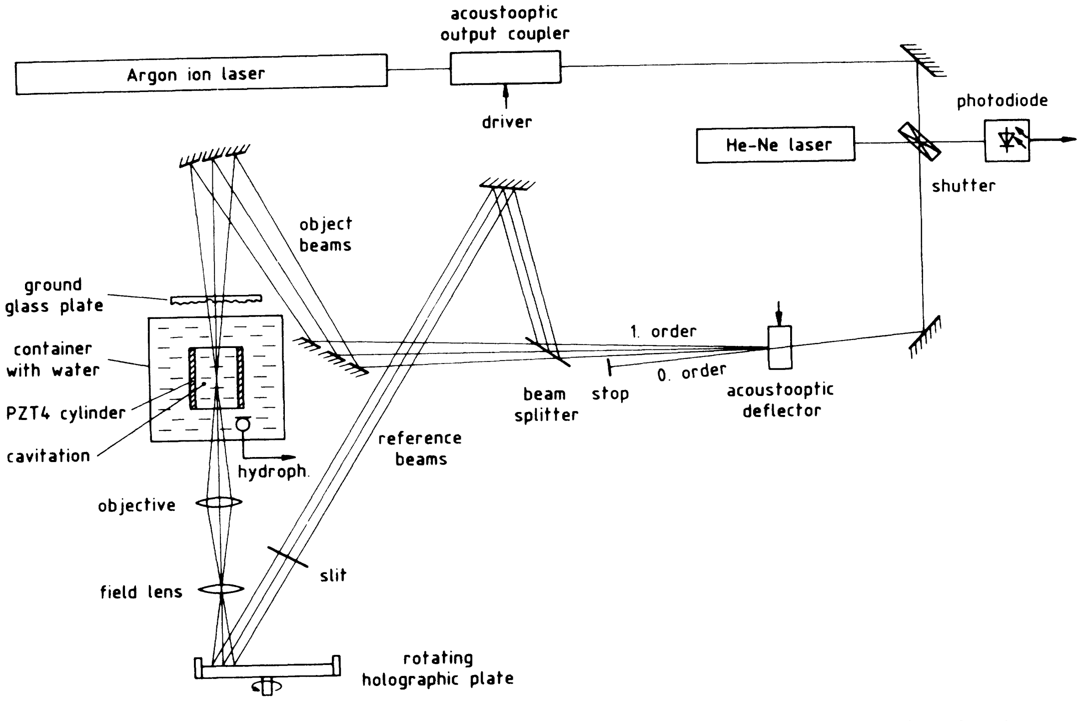

FIG. 1. Arrangement for high-speed holographic cinematography of acoustic cavitation bubble fields at framing rates up to 69 300 holograms per second. See Ref. 6 for further handling of the hydrophone output.

and fast-moving objects which are not readily photographed. As they appear and move in a three-dimensional volume they are difficult to focus on and to keep in focus during their motion. Holography presents a way out of this difficulty. We therefore decided to investigate the possibilities of a holographic equivalent of the rotating-drum or mirror camera systems of conventional photography. Our latest achievement is a high-speed holographic movie camera described in detail in Ref. 11 and briefly in Fig. 1. It is based on the series of coherent light pulses that can be obtained from a cavity-dumped argon-ion laser in conjunction with a suitable hologram framing scheme. In the present investigation a series of about 200 light pulses of about 30 ns duration each is used to form the same number of holograms on a rotating holographic plate in the following way. The series is started by opening the shutter (see Fig. 1) and thus passing the light of the He-Ne laser to the photodiode which with some intermediate electronics triggers the driver of the output coupler. The acousto-optic deflector operates at three distinct frequencies. From one light pulse to the next the frequencies are switched successively. This mode of operation deflects the light pulses into three different directions. Each light pulse is then split into reference and object beam which meet again at the rotating holographic plate. The object beams, on their way to the holographic plate, pass a ground glass plate where they are scattered and serve as illumination of the cavitation bubbles inside the piezoelectric cylinder. A magnified image of the bubble field in the cylinder is produced by an objective in the region of the field lens. The field lens serves as light collector and is used because of light-intensity problems due to the low energy per pulse. But even then the size of the individual holograms is limited to a few mm^2. The lateral resolution in this geometry was experimentally determined to about 35 μm. When the holographic plate is rotated at a speed of about 250–300 revolutions per second, holographic framing rates of about 70 000 holograms per second can be realized.

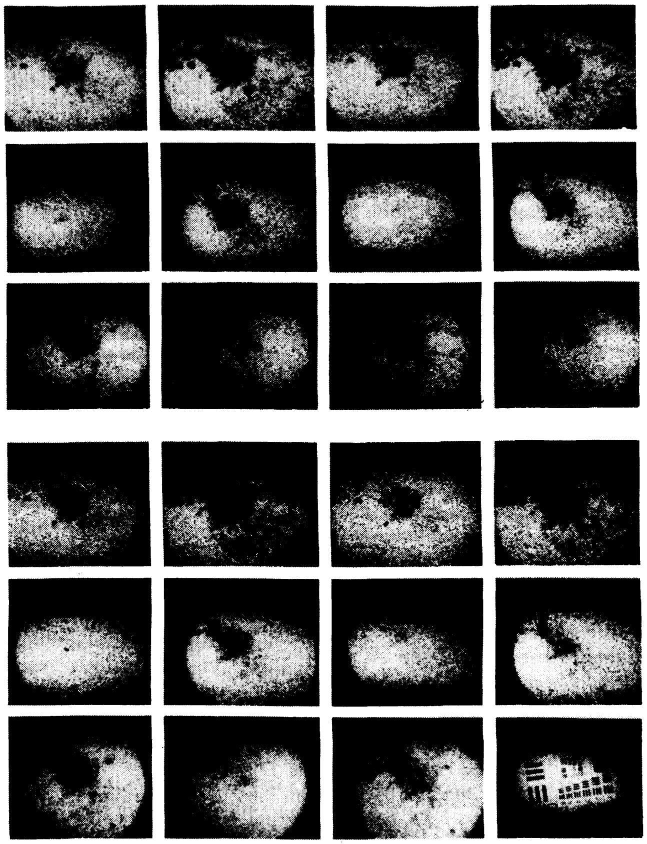

FIG. 2. Reconstructed images from a holographic series taken at 69 300 holograms per second of bubbles inside a piezoelectric cylinder driven at 23.1 kHz. Second period doubling has taken place with indications of the third period doubling.

Simultaneously to the hologram recording, the sound output (noise) from the liquid is monitored by a hydrophone and stored digitally as described in Ref. 6. The optic (holographic) and the acoustic measurements cover the same time interval. Within this instrumentation the period-doubling route to chaos can be followed not only acoustically but also optically by watching the bubble oscillations directly. Figure 2 shows an example of an oscillating bubble field which already has undergone the second period doubling to period 4 (with respect to the period of the driving sound field). Just 23 of the 200 holograms have been taken, and just one plane each of the reconstructed three-dimensional images is reproduced in this figure. Frame 24 is taken from the hologram of a resolution chart obtained in the same way as the pictures of the bubble field. The thickness of the two groups of three thick black bars is 500 μm. The holographic series has been taken at 69 300 Hz, exactly three times the driving frequency. Thus three holograms are taken per period of the driving sound field. This has been proven most convenient as the best phase of the driving period for observing period-doubled oscillations is not known beforehand. With three holograms per period the chance to get a good phase is considerably improved, and the time of experimentation is lowered, respectively. According to this mode of operation, three pictures are grouped together vertically (belonging to one period of the driving). Thus the horizontal rows correspond to the bubble field at a given phase of the driving, and eight periods are covered, four in the upper block and four in the lower block of 3×4 frames. Careful inspection shows that period 4 is clearly present and, maybe, period 8 has already set in. This finding is confirmed in Fig. 3, where the power spectrum of the noise is given which has been simultaneously taken. It shows strong lines at $\frac{1}{4} f_a$ (f_a is the driving frequency) and its harmonics and first indications of $\frac{1}{8} f_a$ and its harmonics.

As is known from chaos theory,[1,2] the intervals for higher periods quickly become smaller. Thus only period doubling to period 8 could be observed for sure in these experiments. Totally chaotic oscillations near the accumulation point of period doubling were difficult to obtain. In our experiments such oscillations seemed to gradually switch between nearly periodic oscillations of a different, usually high period.

The result of the investigations strongly suggests an inti-

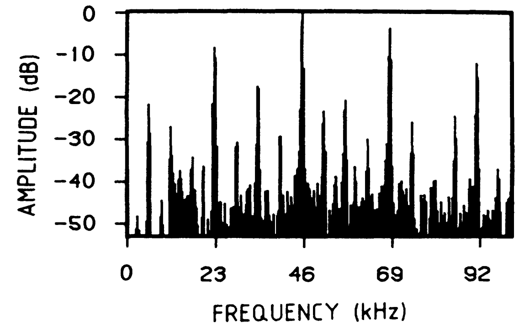

FIG. 3. The power spectrum of the noise corresponding to Fig. 2. Lines at $\frac{1}{8}f_a$ ($f_a = 23.1$ kHz) and their harmonics show up, indicating that the third period doubling has just taken place.

mate connection between bubble oscillations and cavitation noise putting this conjecture on a firm basis. Moreover, the pictures show that obviously the *whole bubble field* undergoes period doubling and not just a subset of the bubbles in the field. This is quite a remarkable result as bubbles of different sizes (as present in the fields) have different resonance frequencies and different properties with respect to their nonlinear oscillation amplitudes and frequencies. Thus single spherical bubble theory predicts different thresholds for the set in of subharmonics, period doubling, and the occurrence of strange attractors when the bubbles are of different sizes.[4,6,7,12] This difficulty can only be overcome by assuming that a high degree of cooperation among the bubbles is present forcing them into unison play. This finding and explanation is in agreement with recent measurements and calculations of the fractal dimension of the noise attractor.[5] A surprisingly low value of about 2.5 has been obtained for the dimension of the noise attractor pointing to a correspondingly low number of relevant degrees of freedom of the system despite its thousands of oscillating bubbles. Only strong cooperation among the bubbles will reduce the dynamics of the whole bubble field to the motion on a 2.5-dimensional attractor in phase space.

It will be left to further investigations to elucidate the nature of the forces acting to yield this effect.

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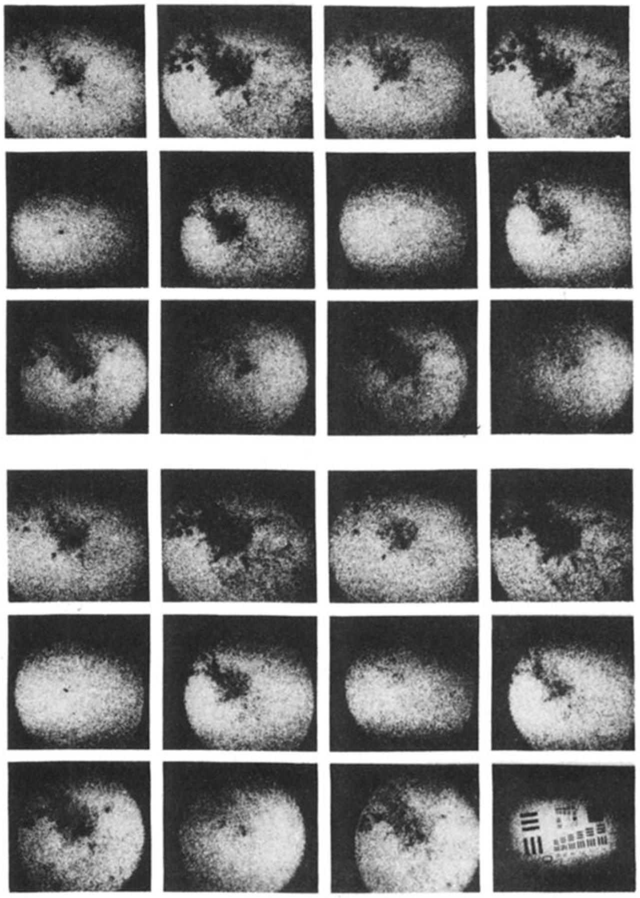

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