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Multiple ionization in relativistic heavy-ion–atom collisions

W. E. Meyerhof, R. Anholt, and Xiang-Yuan Xu*
Department of Physics, Stanford University, Stanford, California 94305

Harvey Gould
Materials and Molecular Research Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

B. Feinberg and R. J. McDonald
Accelerator and Fusion Research Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

H. E. Wegner and P. Thieberger
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

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We show that in relativistic heavy-ion collisions the independent-electron model can be used to predict cross sections for multiple inner-shell ionization in a single collision. Charge distributions of 430- and 955-MeV/amu U^{90+}, U^{89+}, U^{83+}, and U^{68+} beams emerging from thin solid targets were used to obtain single- and multiple-electron stripping cross sections. The probabilities of stripping electrons from the *K*, *L*, or *M* shells were calculated using the semiclassical approximation and Dirac hydrogenic wave functions. The data generally agree with theory. An influence of the Auger effect is seen in U^{68+} collisions.

The transition from a two-body to a many-body system is one of the important areas of study in physics. Since the two-body problem is largely solved in atomic physics, it is advantageous to investigate the atomic few-body problem, statically and dynamically. Theoretically, the simplest model to use is the independent-particle model (IPM), which ignores interaction between the electrons and uses only single-particle wave functions.

We show that in relativistic heavy-ion collisions the IPM can be used to predict cross sections for multiple ionization in a single collision with good accuracy, although some systematic deviations are found which may point to electron-correlation effects.[1,2] According to the IPM, multiple ionization and multiple excitation should follow a binomial distribution. This distribution has been observed in satellite *K* x-ray spectra (simultaneous *K*- and *L*-vacancy production),[3] hypersatellite spectra (simultaneous double *K*-vacancy production),[4] in multiple ionization and multiple capture,[5-7] and in recoil ion measurements.[8,9] To date, detailed comparisons between calculated and measured multiple-ionization or excitation cross sections have been hampered by various side effects. In *K*-*L* satellite experiments, the interpretation of the measurements is sensitive to uncertainties in the fluorescence yields for each multiple-hole configuration. In many charge-changing experiments, where outer-shell ionization is dominant, one cannot use hydrogenic wave functions to describe the initial and final electron states. Also, at ion velocities generally used, wave-function distortion effects such as binding and polarization are present.[10] These effects, themselves the subject of much investigation,[11] tend to obscure possible electron-correlation effects in multiple ionization. Recoil ion measurements have been analyzed by a statistical approach.[12]

At relativistic energies, charge-changing collisions can be well described by relatively simple theories, such as the plane-wave Born approximation (PWBA) for single-electron ionization[13] and the eikonal approximation for single-electron capture.[14,15] Wave-function distortion, target-electron screening, and relativistic effects on ionization are present, but can be calculated accurately.[11] For high-*Z* ions, Dirac hydrogenic wave functions can be used. Hence, one should be able to compute relativistic multiple-ionization cross sections with a high degree of accuracy.

A recent upgrade of the Lawrence Berkeley Laboratory BEVALAC provides uranium ions with any desired charge state up to 1000 MeV/amu.[16] The method we used to determine single- and multiple-stripping cross sections is described in Ref. 17. Uranium ions with incident charge states 90+, 89+, 83+, and 68+ (2, 3, 9, and 24 electrons) accelerated to 955 MeV/amu and with charge states 90+ and 83+ accelerated to 430 MeV/amu were passed through thin Be, C, Mylar, Al, Cu, Ag, and Au foils. For each combination of energy, incident charge state, and target material, charge distributions were determined as a function of target thickness. The stripping cross sections were determined by least-squares fits of the integrated rate equations to the data.[6] Only the near-linear part of the charge-state-population dependence on target thickness[18] was used in order to avoid excited-state effects.[19] Thin targets of Cu, Ag, and Au were deposited on 50-μg/cm^2 C backings; the effect of the backing was taken into account in the cross-section analysis. Target thicknesses were determined (to ±10%) by α-particle energy loss or by x-ray attenuation.

Figures 1 to 3 show our measured results for single- and multiple-electron stripping cross sections divided by Z_t^2.

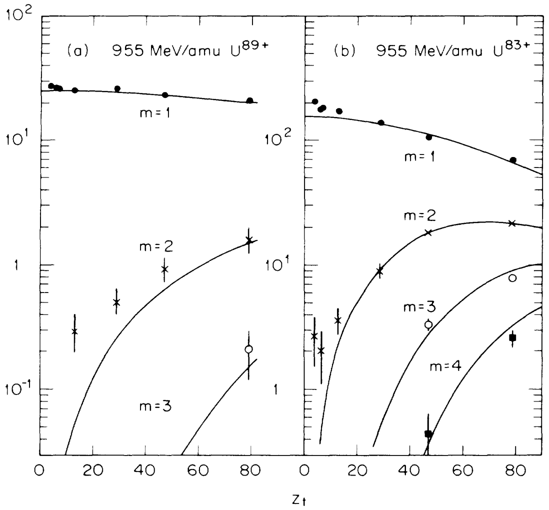

FIG. 1. Single and multiple stripping cross sections for 955-MeV/amu U^{89+} (one L-shell electron) and U^{83+} (seven L-shell electrons) projectiles passing through various target foils as a function of the target atomic number (Z_t). The cross sections in barns have been divided by Z_t^2. On each curve, m indicates the multiplicity of the stripping process. The solid curves show the independent-electron approximation results.

For 955-MeV/amu U^{68+} ions, up to sixfold ionization in a single collision ionization could be observed. The present U^{90+} single-ionization cross sections agree with measurements of Gould *et al.*[20] made at 437 and 962 MeV/amu. The solid curves in Figs. 1, 2, and 3(a) were calculated using the IPM. If $p_s(b)$ is the one-electron ionization probability in shell s at an impact parameter b, the probability of ionizing n electrons out of a total of N electrons in the shell is given by the binomial distribution[7]

$$P_s(n,N)=\frac{N!}{n!(N-n)!}p_s^n(1-p_s)^{N-n} . \tag{1}$$

If electrons can be ejected from more than one shell, e.g., from three shells, the cross section for stripping m electrons is given by[1,2]

$$\sigma_m=\sum_{\substack{n_1,n_2,n_3\\(n_1+n_2+n_3=m)}}\int_0^\infty P_1(n_1,N_1)P_2(n_2,N_2)\times P_3(n_3,N_3)2\pi b\,db , \tag{2}$$

where the subscripts 1, 2, and 3 refer to the three shells considered.

To compute σ_m, for p_s we use the semiclassical approximation (SCA) formulation of Hansteen, Johnson, and Kocbach.[21] In this theory, at a given reduced projectile velocity v/v_s (v_s is the Bohr velocity in shell s) and reduced impact parameter b/a_s (a_s is the Bohr radius of shell s), p_s scales approximately as $(Z_t/Z_s)^2$, where in the present case of projectile ionization, Z_t and Z_s are the target atomic number and the screened-projectile atomic number, respectively. In applying the tables of Hansteen *et al.*[21] to the ionization of already highly stripped U ions, we use the electron binding energies computed by Carlson, Nestor, Wasserman, and McDowell[22] and Slater screened charges Z_s.[23] Instead of using the cross-section scaling correction factor μ defined by Hansteen *et al.*,[21] we simply normalize the calculated SCA cross sections to the PWBA.[13]

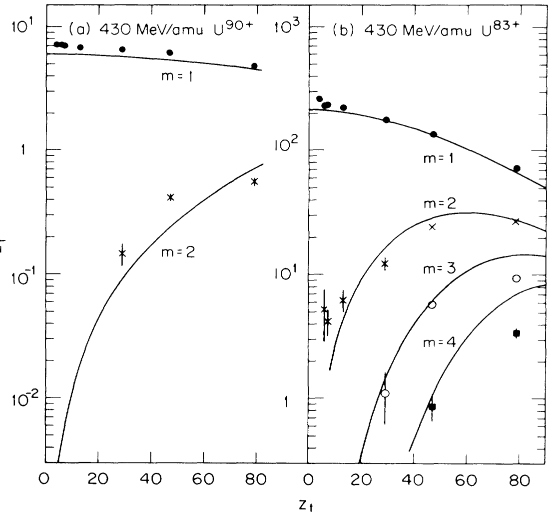

FIG. 2. Same as Fig. 1, for 430-MeV/amu U^{90+} and U^{83+} projectiles.

As shown in Ref. 11, at relativistic projectile velocities ($\beta=v/c\gtrsim0.3$), relativistic wave-function effects on the ionization cross section become quite small. On the other hand, Amundsen and Aashamar[24] have shown that relativistic-velocity effects on the impact-parameter dependence of the ionization probability are also small as long as $\beta\lesssim0.9$. Hence, in the present regime, the use of normal-

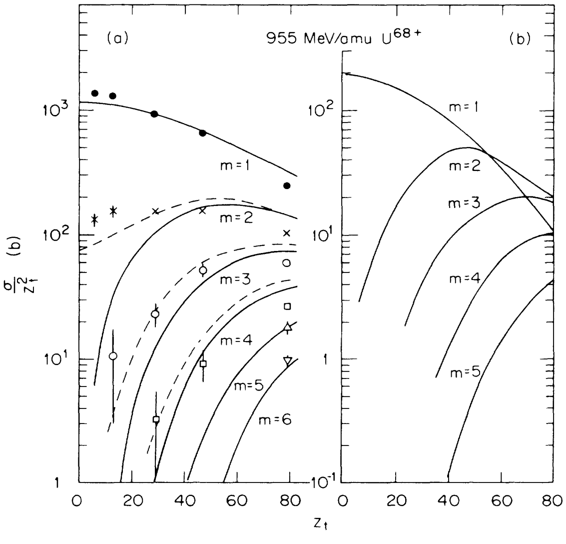

FIG. 3. (a) Same as Fig. 1, for 955-MeV/amu U^{68+} ions. The dashed curves include the computed influence of the *LMM* Auger effect. (b) Theoretical cross sections for multiple ionization if one vacancy is in the K or L shell.

ized nonrelativistic SCA probabilities should be valid. Binding effects and screening effects are also negligible here.[25]

As is well known, for large values of Z_t the SCA breaks down, giving values of p_s that can exceed unity. Although the probabilities at small impact parameters are very large (which results in large multiple-ionization cross sections), for relativistic U they never exceed unity, partly because Z_t/Z_s never exceeds unity. In the actual calculations, ionization from the 1*s*, 2*s*, 2*p*, 3*s*, 3*p*, and 3*d* shells are taken into account.

The data shown in Figs. 1 and 2 are overall in good agreement with the IPM for multiple ionization in the *K* and *L* shells. In U^{68+} [Fig. 3(a)], where *M*-shell ionization is dominant, the measured cross sections exceed the IPM cross sections, especially at low Z_t. We attribute the discrepancy to the *LMM* Auger effect, as discussed below.

Major evidence of multiple-ionization effects in these collisions is found not only in the multiple-ionization cross sections themselves but also in the falloff of the reduced single-electron ionization cross section σ_1/Z_t^2 with increasing Z_t. We emphasize that binding and other perturbing field effects are nearly negligible in these collisions (especially for *L*- and *M*-shell ionization); therefore, the single-ionization cross sections should vary, as in the PWBA, as Z_t^2, and σ_1/Z_t^2 should be a constant for a given degree of ionization.[11,13] The falloff in σ_1/Z_t^2 is mainly due to the role of the unionized electrons. Requiring that only one electron be ionized, e.g., in a nine-electron ion (U^{83+}), requires that eight electrons not be ionized, so that one has terms such as $(1-p_K)^2(1-p_L)^6$ for the *K* and *L* electrons. Since p_L and p_K are close to unity at large Z_t, these factors become quite small. If more electrons are present initially, the terms $(1-p_s)$ are raised to even higher powers, so that the cross-section falloff becomes even more significant, in agreement with the observed results (in 955-MeV/amu U^{89+}, U^{83+}, and U^{68+}, σ_1/Z_t^2 drops by factors $\sim\frac{1}{1.3}$, $\sim\frac{1}{3}$, and $\sim\frac{1}{4}$, respectively, over the Z_t range investigated).

Disagreement with the IPM for multiple ionization is most apparent in U^{68+} collisions at low Z_t. Here, *L*-shell ionization followed by *LMM* Auger transitions can occur, giving an apparent increase in the multiple ionization cross sections. The theoretical cross sections for single *K*- or *L*-shell ionization accompanied by any degree of *M*-shell multiple ionization in 955-MeV/amu U^{68+} collisions are shown in Fig. 3(b). The single-ionization cross section for creating a *K*- or *L*-shell vacancy in U^{68+} collisions is smaller than the overall single-electron loss cross section, mainly due to the *M* shell. The relative multiple-ionization cross sections are similar to those found in *K* x-ray satellite experiments:[3,4] the large probability $p_M(b\approx 0)$ comes into play and the factors $p_M^n(1-p_M)^{N-n}$ in Eq. (1) can cause double and triple ionization to be more likely than single ionization.

The calculated double-ionization cross section in U^{68+} collisions must be incremented by the theoretical single *K*- or *L*-shell ionization cross section multiplied by the *L*-shell Auger yield ω_A, and similarly for the other multiple-ionization cross sections. (The *K*-shell vacancy contribution to the inner-shell electron-loss cross section is small, and converts mostly to *L* vacancies.) In the present case, only *LMM* Auger transitions[26] and *LM* x-ray transitions[27] can occur, all other electrons being absent from U^{69+} ions. Assuming that vacancies in the 14-electron *M* shell are statistically distributed, we use Larkin's prescription[28] to correct the Auger yield for fewer *M*-shell electrons, and obtain $\omega_A=0.386$ for single *L*-shell ionization, instead of the average value 0.455 for a fully populated U ion with a single *L* vacancy.[26] The total multiple-electron loss cross sections, including Auger transitions, are shown by the dashed lines in Fig. 3(a), and are in better agreement with experiment. For U^{89+} and U^{90+} [Figs. 1(a) and 2(a)], Auger transitions can make no contribution. For U^{83+} collisions [Figs. 1(b) and 2(b)], the reduced *K*-shell cross section is only about 2 b and the *K*-shell Auger yield is less than 5%, so the Auger contribution is below the scale of the figures. Nevertheless, systematic deviations from the calculations remain for $m=2$ at low Z_t. These deviations may point to possible correlation effects.[1,2]

In conclusion, multiple-electron ionization in charge-changing collisions has been observed for the first time in collisions that are amenable to calculations. Overall, the present data are in good agreement with the independent-electron approximation, suggesting that electron correlation effects must be small, at least for high-*Z* targets, where larger cross sections make the data most accurate. The data at low Z_t for U^{68+} provide an opportunity for measuring Auger yields in systems where one can specify that only one shell is active, instead of summing over all shells, as is done in single-vacancy atoms.

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[*] Permanent address: Department of Physics, Tsinghua University, Beijing, People's Republic of China.

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