Electron—atomic-hydrogen elastic collisions in the presence of a laser field

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We study the "elastic" scattering of fast electrons by atomic hydrogen in the presence of a laser field. Our method takes into account the "dressing" of the target states by including the laser-atom interaction to first order, while the laser-projectile interaction is treated to all orders. This allows us to treat laser fields which, although weak compared to the atomic unit of field strength, can nevertheless be strong by laboratory standards. In the limit of weak fields our results reduce to those obtained recently by a perturbative approach (in the laser-projectile interaction) for all laser frequencies.

I. INTRODUCTION

In considering collisions of electrons with real atoms in the presence of a laser field, one has to take into account three distinct interactions. Firstly, the interaction between the unbound electron and the atomic system takes place, as in the field-free case. Secondly, the radiation field strongly interacts with the unbound electron, inducing stimulated or inverse bremsstrahlung. Thirdly, the electromagnetic wave can modify the atomic states involved in the scattering process. Accurate scattering calculations being quite difficult even in the absence of the laser field, it is clear that there is no hope for an exact treatment of the problem. In order to simplify it, a common approximation is then to neglect the third interaction so that the atomic target is considered to be unaffected by the laser. It is obvious, however, that this simplification is not valid when the intensity of the field is strong, nor when the laser frequency lies close to atomic excitation energies. For that reason, a formalism has been developed recently by Byron and Joachain[1] in order to take into account the "dressing" of atomic states in the scattering process. It makes use of first-order time-dependent perturbation theory for treating the laser-atom interaction, the laser-projectile interaction being taken into account to all orders. This method has the advantage of being valid for all field intensities, provided that the electric field strength is much less than one atomic unit, namely $\mathscr{E}_0 \ll 5\times10^9$ V cm^{-1}. It should also be noted that a low-frequency approximation was used for convenience in Ref. 1. Since higher-frequency lasers are now becoming available, this approximation will be removed in the present work.

Another recent treatment, which has been proposed by Dubois *et al.*,[2] is based on first-order perturbation theory in the external field for both the target *and* the projectile. It amounts to compute the four diagrams displayed in Fig. 1. The first two diagrams, called "electronic," correspond to the scattering with emission or absorption of one photon by the incident electron, while "atomic" diagrams III and IV depict the same process but involve the atomic electron. The calculation of the first two diagrams is a simple matter, since the amplitude factorizes into the field-free amplitude (in first Born approximation) and the standard amplitude for bremsstrahlung. The calculation of diagrams III and IV is, however, much more complicated requiring the use of the Coulomb propagator.[2] This method is, in its present form, restricted to one-photon exchange and is well suited for weak fields. It involves no limitation on the frequency.

In this paper we are interested in providing a unified treatment of electron—atomic-hydrogen scattering in a laser field, namely, a treatment that would be valid (a) to all orders in the external field for the projectile, (b) without limitation on the laser frequency, and (c) for an

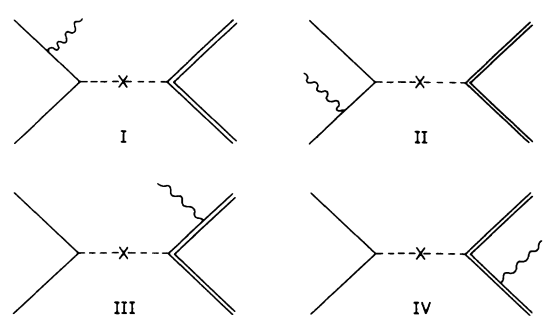

FIG. 1. The four diagrams contributing to one-photon emission or absorption processes occurring during an electron-atom collision in the presence of a laser field in a first-order perturbative approach. The electronic diagrams I and II correspond to processes in which the projectile exchanges one photon with the field, while the atomic diagrams III and IV account for the dressing of the target.

arbitrary number of exchanged photons. It will allow us to test some approximations made in Ref. 1 (low-frequency approximation, closure approximation) and also to check the limits of validity of the perturbative approach of Ref. 2. In order to make contact with this last work, all calculations will be performed here in first Born approximation. However, as shown by Byron and Joachain,[1] the generalization to eikonal-Born series (EBS) amplitudes[3] can be done in the low-frequency limit. The main effects are nevertheless expected to be contained in the first Born term, provided the number of exchanged photons is not zero. We will furthermore neglect exchange effects, which were not considered in Ref. 2, and are small for fast incident electrons.

II. THEORY

The Hamiltonian of the electron-atom system in the presence of the laser field can be written

$$H=H_f+H_t+V_d\ , \tag{1}$$

where H_f and H_t are, respectively, the Hamiltonians of the unbound electron and of the atomic target in the presence of the laser field, and V_d is the electron-atom interaction in the direct channel.

First of all, the wave function of the unbound electron in the laser field is readily obtained by solving the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\chi(\mathbf{r}_0,t)=H_f\chi(\mathbf{r}_0,t)=\frac{1}{2m}\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]^2\chi(\mathbf{r}_0,t)\ . \tag{2}$$

Assuming a monochromatic, linearly polarized, and single-mode laser field treated in the dipole approximation, we have, working in the Coulomb gauge, $\mathcal{E}(t)=\mathcal{E}_0\sin(\omega t)$ and $\mathbf{A}(t)=\mathbf{A}_0\cos(\omega t)$ with $\mathbf{A}_0=c\mathcal{E}_0/\omega$. The so-called Volkov solution of Eq. (2) then reads

$$\chi_{\mathbf{k}}(\mathbf{r}_0,t)=(2\pi)^{-3/2}\exp\{i[\mathbf{k}\cdot\mathbf{r}_0-\mathbf{k}\cdot\boldsymbol{\alpha}_0\sin(\omega t)-E_kt/\hbar]\}\ , \tag{3}$$

where $\mathbf{k}$ is the electron wave vector, $E_k=\hbar^2k^2/2m$, and $\boldsymbol{\alpha}_0=e\mathcal{E}_0/m\omega^2$.

The next step consists in computing the "dressed" states of the hydrogen atom in the presence of the laser field. This is done by solving the equation

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r}_1,t)=H_t\Phi(\mathbf{r}_1,t)\ . \tag{4}$$

Using first-order time-dependent perturbation theory and removing the low-frequency approximation of Ref. 1, we then obtain for the dressed ground-state wave function of hydrogen

$$\Phi_0(\mathbf{r}_1,t)=\exp(-i\mathbf{a}\cdot\mathbf{r}_1)\exp(-i\omega_0t)\times\left[\psi_0(\mathbf{r}_1)-\sin(\omega t)\sum_n\frac{\omega_{n,0}M_{np,0}}{\hbar(\omega_{n,0}^2-\omega^2)}\psi_{np}(\mathbf{r}_1)-i\cos(\omega t)\sum_n\frac{\omega M_{np,0}}{\hbar(\omega_{n,0}^2-\omega^2)}\psi_{np}(\mathbf{r}_1)\right]\ , \tag{5}$$

where ψ_0 and ψ_{np} are, respectively, the "undressed" ground-state wave function and the nth p state of hydrogen, $\hbar\omega_0$ and $\hbar\omega_n$ the corresponding energies, $\omega_{n,0}=\omega_n-\omega_0$ and the summation includes continuum states. Moreover, one has $\mathbf{a}=e\mathbf{A}/\hbar c$ and

$$M_{np,0}=\mathcal{E}_0\cdot\langle\psi_{np}\,|\,e\mathbf{r}_1\,|\,\psi_0\rangle\ . \tag{6}$$

The S matrix element for direct "elastic" scattering in the presence of the laser field is now given, in first Born approximation, by the expression

$$S_{\text{el}}^{B1}=-\frac{i}{\hbar}\int_{-\infty}^{+\infty}dt\,\langle\chi_{\mathbf{k}_f}(\mathbf{r}_0,t)\Phi_0(\mathbf{r}_1,t)\,|\,V_d\,|\,\chi_{\mathbf{k}_i}(\mathbf{r}_0,t)\Phi_0(\mathbf{r}_1,t)\rangle\ , \tag{7}$$

where $V_d=|\mathbf{r}_0-\mathbf{r}_1|^{-1}-r_0^{-1}$. Performing the time integration and working from now on in atomic units, we obtain

$$S_{\text{el}}^{B1}=(2\pi)^{-1}i\sum_{l=-\infty}^{+\infty}\delta(E_{k_f}-E_{k_i}-l\omega)f_{\text{el}}^{B1,l}\ , \tag{8}$$

where $f_{\text{el}}^{B1,l}$ is the first Born approximation to the elastic scattering amplitude with the transfer of l photons, namely

$$f_{\text{el}}^{B1,l}(\boldsymbol{\Delta})=J_l(\boldsymbol{\Delta}\cdot\boldsymbol{\alpha}_0)f_{\text{el}}^{B1}(\boldsymbol{\Delta})-iJ_l'(\boldsymbol{\Delta}\cdot\boldsymbol{\alpha}_0)\sum_n\omega_{n,0}\frac{f_{0,np}^{B1}(\boldsymbol{\Delta})M_{np,0}\ M_{0,np}f_{np,0}^{B1}(\boldsymbol{\Delta})}{\omega_{n,0}^2-\omega^2}\ . \tag{9}$$

Here $\boldsymbol{\Delta}=\mathbf{k}_i-\mathbf{k}_f$ is the momentum transfer, J_l is an ordinary Bessel function of order l,J_l' its first derivative, and f_{el}^{B1}, $f_{np,0}^{B1}$, and $f_{0,np}^{B1}$ are the first Born amplitudes corresponding to the scattering processes $0\to0$, $0\to np$, and $np\to0$ in the absence of the laser field.

The first term on the right-hand side of (9) corresponds to the interaction of the laser field with the incident electron only (in the same way as in potential scattering), while the second one includes dressing effects and thus describes the distortion of the atom by the electromagnetic wave.

It should also be remarked at this point, that the low-frequency approximation of Ref. 1 can simply be recovered by letting ω tend to zero in the denominator of this second term.

Although the expression (9) can be computed exactly in the case of atomic hydrogen, the task would become impossible for more complex atoms; so it is interesting to test the limits of validity of the closure approximation proposed in Ref. 1. This approximation consists in replacing $\omega_{n,0}$ by an average excitation energy $\bar{\omega}$; Eq. (9) then becomes

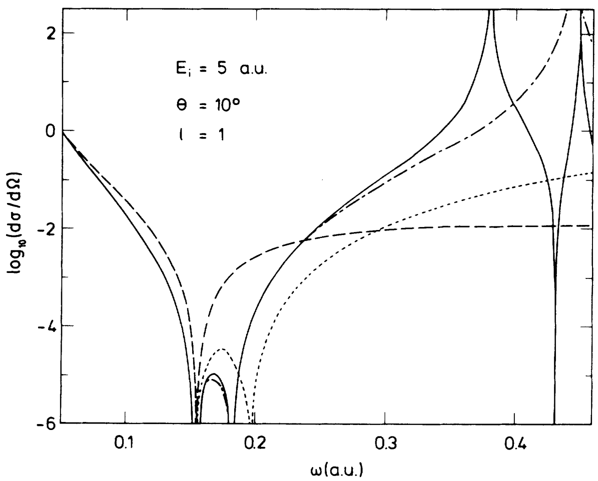

FIG. 2. Variations of $\log_{10}(d\sigma/d\Omega)$ as a function of the laser angular frequency ω for an incident electron energy $E_i=5$ a.u., a scattering angle $\theta=10°$, and the absorption of one photon ($l=1$). The electric field $\mathscr{E}$ is assumed to be weak and is chosen to be parallel to the incident electron wave vector $\mathbf{k}_i$. The cross section has been normalized as in Ref. 2 in order to be independent of the field intensity. Solid line, full calculation using Eqs. (9) and (11) which exactly coincides with the first-order perturbative results of Dubois *et al.* (Ref. 2); dashed-dotted line, same calculation but using the closure approximation of Eq. (10); dotted line, calculation using the low-frequency approximation; dashed line, calculation neglecting the dressing of the target.

$$f_{\rm el}^{B1,l}(\Delta)=J_l(\mathbf{\Delta}\cdot\boldsymbol{\alpha}_0)f_{\rm el}^{B1}(\Delta)+\frac{4\bar{\omega}}{\Delta^2(\bar{\omega}^2-\omega^2)}J_l'(\mathbf{\Delta}\cdot\boldsymbol{\alpha}_0)\mathscr{E}_0\cdot\nabla_\Delta \times\langle 0|\exp(i\mathbf{\Delta}\cdot\mathbf{r}_1)|0\rangle\ . \quad (10)$$

Finally, the first Born differential cross section is obtained as

$$\frac{d\sigma_{\rm el}^{B1,l}}{d\Omega}=\frac{k_f(l)}{k_i}|f_{\rm el}^{B1,l}|^2\ . \quad (11)$$

III. RESULTS AND DISCUSSION

We will now show that the results of Ref. 2 can be recovered from our calculations, provided that the Bessel functions in (9) can be expanded to first order. First of all, some comparisons can be made on analytical grounds only. Let us consider Eq. (9), but with the dressing neglected. From (11), the differential cross section is then obtained as

$$\left[\frac{d\sigma_{\rm el}^{B1,l}}{d\Omega}\right]_{\text{no dressing}}=\frac{k_f(l)}{k_i}J_l^2(\mathbf{\Delta}\cdot\boldsymbol{\alpha}_0)4\frac{(\Delta^2+8)^2}{(\Delta^2+4)^4}\ . \quad (12)$$

Expanding the Bessel function to first order for $l=1$ and adopting for our cross section the same intensity-independent normalization as in Ref. 2, we obtain

$$\left[\frac{d\sigma_{\rm el}^{B1,l}}{d\Omega}\right]_{\text{no dressing}}=8\pi\alpha\frac{(\mathbf{\Delta}\cdot\mathscr{E}_0)^2}{\omega^3\mathscr{E}_0^2}k_f\frac{(\Delta^2+8)^2}{(\Delta^2+4)^4}\ , \quad (13)$$

in agreement with Eq. (24) of Ref. 2, α being the fine-structure constant. This shows explicitly the correspondence between the first term on the right-hand side of (9), in which dressing effects are neglected, and the amplitude corresponding to diagrams I and II of Fig. 1 in perturbation theory.

Let us now turn to the part of the scattering amplitude containing dressing effects. Using expression (10) in the low-frequency limit, for analytical simplicity, expanding the Bessel function as above and adopting again the same normalization as in Ref. 2, we obtain for the cross section in which the electronic term would be neglected

$$\left[\frac{d\sigma}{d\Omega}\right]_{\text{dressing}}=8\pi\alpha\omega k_f\left[\frac{128}{\bar{\omega}}\right]^2\frac{(\mathbf{\Delta}\cdot\mathscr{E}_0)^2}{\Delta^4\mathscr{E}_0^2(\Delta^2+4)^6}\ . \quad (14)$$

In the soft-photon limit, Dubois *et al.* give for the cross section corresponding to diagrams III and IV taken alone, the analytical result [formulas (8) and (20) of Ref. 2]

$$\left[\frac{d\sigma}{d\Omega}\right]_{\rm III+IV}=8\pi\alpha\omega k_f(96)^2\frac{(\mathbf{\Delta}\cdot\mathscr{E}_0)^2}{\Delta^4\mathscr{E}_0^2}\frac{(\Delta^2+12)^2}{(\Delta^2+4)^8}\ , \quad (15)$$

and both expressions are again identical provided we can approximate $(\Delta^2+12)^2/(\Delta^2+4)^2\cong 9$, since the right value for $\bar{\omega}$ is $\frac{4}{9}$. Furthermore, as shown in Figs. 2 and 3, the agreement between our results and those of Ref. 2 remains excellent, even when increasing the frequency. (The geometry is chosen as in Ref. 2, with the polarization vector of the field parallel to the incoming electron momentum.)

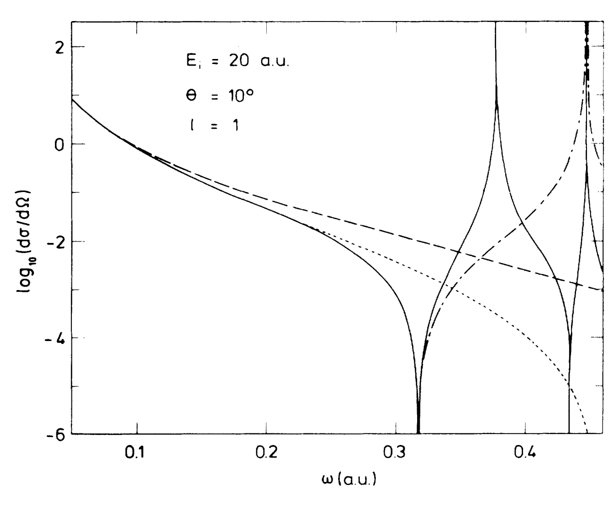

FIG. 3. Same as Fig. 2, but for an incident electron energy $E_i=20$ a.u.

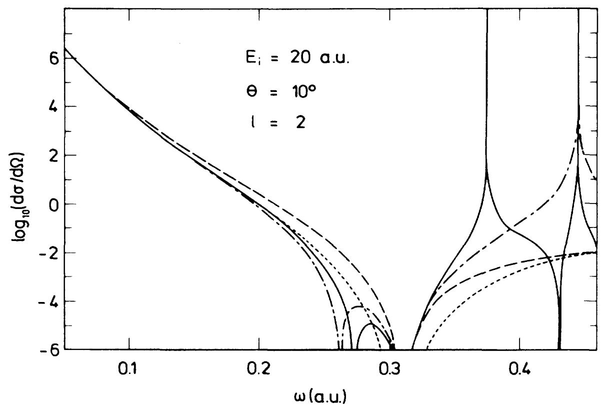

FIG. 4. Variation of $\log_{10}(d\sigma/d\Omega)$ as a function of the laser angular frequency ω for an incident electron energy $E_i=20$ a.u., a scattering angle $\theta=10°$, and the absorption of two photons ($l=2$). The electric field is assumed to be weak and is chosen to be parallel to $\mathbf{k}_i$. The cross section has been normalized by dividing the result (11) by the squared average intensity. Solid line, full calculation using Eqs. (9) and (11); dashed-dotted line, same calculation but using the closure approximation of Eq. (10); dotted line, calculation using the low-frequency approximation; dashed line, calculating neglecting the dressing of the target.

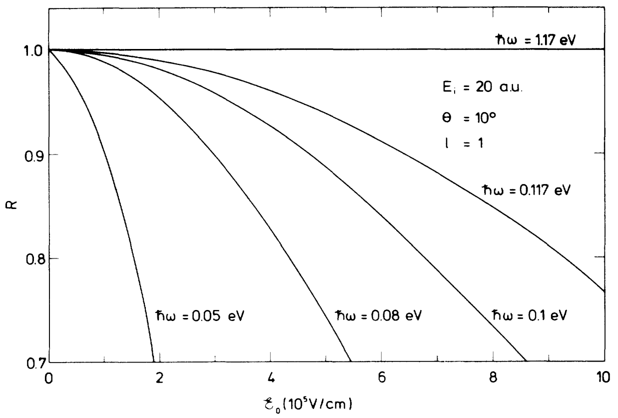

FIG. 6. The ratio R of Eq. (16) as a function of the electric field strength $\mathscr{E}_0$, for various values of the laser photon energy $\hbar\omega$, in the case of one-photon absorption. The incident electron energy is $E_i=20$ a.u. and the scattering angle is $\theta=10°$. The electric field is chosen to be parallel to $\mathbf{k}_i$.

Dressing effects clearly increase with frequency, and as discussed in Refs. 1 and 2, can even dominate the cross section, especially at small scattering angles. Abrupt changes in the cross sections also appear when the laser frequency is matching an atomic frequency, as could be expected on physical grounds and also from the presence of the poles in (9). As discussed in Ref. 2, new zeros occur in the cross sections due to destructive interferences between atomic and electronic amplitudes. The figures also show the limits of validity of the low-frequency and of the closure approximations. The latter is shown to be quite good, except in the neighborhood of atomic frequencies.

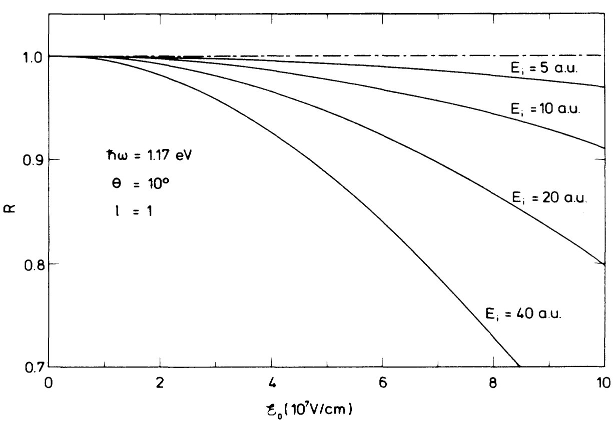

FIG. 5. The ratio R of Eq. (16) as a function of the electric field strength $\mathscr{E}_0$, for various values of the incident electron energy in the case of one-photon absorption ($l=1$). The scattering angle is $\theta=10°$ and the laser photon energy is $\hbar\omega=1.17$ eV. The electric field is chosen to be parallel to $\mathbf{k}_i$.

Figure 4 shows the corresponding graph for the absorption of two photons. Since this process would be of second order in perturbation theory, there is no sense in adopting the same normalization as in Ref. 2 and as in Figs. 2 and 3. The cross section plotted here is simply that given by (11), divided by the squared average intensity $I^2=\mathscr{E}_0^4/(8\pi)^2$.

Figures 5 and 6 display the ratio

$$R=\left[\frac{d\sigma^{B1,l=1}}{d\Omega}\right]_{\text{all orders}}\Bigg/\left[\frac{d\sigma^{B1,l=1}}{d\Omega}\right]_{\text{first order}} \qquad (16)$$

as a function of the electric field strength for various energies of the incident electron (and fixed ω) and for various frequencies ω (and fixed incident energy), respectively. It shows the limits of validity of the first-order perturbative approach, which turns out to be an expansion in $\mathbf{\Delta}\cdot\mathscr{E}_0/\omega^2$ rather than in $\mathscr{E}_0$, as it appears in the argument of the Bessel functions. For instance, the perturbative approach is seen to fail even for weak fields if the laser frequency is too low or if the momentum transfer is too high.

Finally, the possibility of treating in our model the exchange of an arbitrary number of photons allows us to investigate the sum rule derived by Krüger and Jung[4] in the soft photon limit. Neglecting, in that limit, the dependence of $\mathbf{k}_f$ and $\mathbf{\Delta}$ on the number of exchanged photons and summing the cross section (11) over l, one has from (9)

$$\sum_{l=-\infty}^{+\infty} \frac{d\sigma_{\rm el}^{B1,l}}{d\Omega} = |f_{\rm el}^{B1}(\Delta)|^2 \sum_{l=-\infty}^{+\infty} |J_l(\Delta\cdot\alpha_0) - \gamma(\mathscr{E}_0,\omega,\Delta)J_l'(\Delta\cdot\alpha_0)|^2 , \tag{17}$$

where

$$\gamma(\mathscr{E}_0,\omega,\Delta) = i[f_{\rm el}^{B1}(\Delta)]^{-1} \times \sum_n \frac{\omega_{n,0}[f_{0,np}^{B1}(\Delta)M_{np,0} + M_{0,np}f_{np,0}^{B1}(\Delta)]}{\omega_{n,0}^2 - \omega^2} . \tag{18}$$

This quantity is independent of l and real, since the amplitudes $f_{0,np}^{B1}$ and $f_{np,0}^{B1}$ are purely imaginary. The summation in (17) can then readily be done, giving

$$\sum_{l=-\infty}^{+\infty} \frac{d\sigma_{\rm el}^{B1,l}}{d\Omega} = \frac{d\sigma_{\rm el}^{B1,\rm ff}}{d\Omega}[1 + \tfrac{1}{2}\gamma^2(\mathscr{E}_0,\omega,\Delta)] , \tag{19}$$

where $d\sigma_{\rm el}^{B1,\rm ff}/d\Omega$ denotes the first Born differential cross section in the absence of the laser field. Using again the closure approximation (10), we obtain for γ the following compact expression:

$$\gamma(\mathscr{E}_0,\omega,\Delta) = \frac{128\varpi}{\varpi^2 - \omega^2} \frac{\mathscr{E}_0\cdot\Delta}{\Delta^2(\Delta^2+8)(\Delta^2+4)^2} . \tag{20}$$

This shows explicitly that the dipole distortion of the atomic target has the consequence of changing significantly the sum rule, the effect being quadratic both in the electric field strength and in the inverse of the momentum transfer. Those conclusions are in agreement with the work of Beilin and Zon,[5] in which the Bethe-Born approximation has been used.

IV. CONCLUSION

We have shown that, provided the Bessel functions can be expanded to first order, the agreement between our treatment and the perturbative approach of Ref. 2 is complete. Our method is, however, more general since it allows us to investigate higher field strengths and/or lower laser frequencies. It also provides information about the bremsstrahlung of an arbitrary number of photons. As an application, the sum rule is investigated and our conclusions agree with previous work. The limits of validity of the closure approximation are also checked; this is an important point since that approximation becomes necessary when studying more complex systems than atomic hydrogen.

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