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# Observation of the "Brownian motion" of the electric field in a laser

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Critical dynamics of the laser field near the threshold of oscillation is investigated and observation of the Brownian motion of the electric field amplitude driven by quantum noise is reported. Experimental results agree well with the nonlinear oscillator model of the laser.

The scaled, slowly varying, complex field amplitude $E(t)$ of the single-mode laser obeys the nonlinear Langevin equation[1]

$$\dot{E}(t)=-2\frac{\partial U}{\partial E^*}(|E|,a)+q(t) , \tag{1}$$

where the potential $U(|E|,a)$ which determines the systematic force is given by

$$U(|E|,a)=-\tfrac{1}{2}a|E|^2+\tfrac{1}{4}|E|^4 . \tag{2}$$

The dimensionless parameter $a$, the so-called pump parameter, is negative below, zero at, and positive above the threshold of laser oscillation. $q(t)$ represents the quantum noise due to spontaneous emission and is taken to be a $\delta$-correlated Gaussian stochastic process with zero mean and $\langle q(t)q^*(t')\rangle=4\delta(t-t')$. Predictions of Eq. (1) have been tested in the transient[2-4] as well as the steady-state[5,6] operation of the laser in photoelectric counting and correlation measurements. These experiments provide clear evidence for the smooth phase transition that the laser fluctuations undergo in passing the threshold of laser oscillation.[7] Like all phase transitions, the width of the region in which this transition occurs is extremely small and is given by[8] $|a|\lesssim 4$. From the shape of the potential $U(|E|,a)$ in the threshold region (Fig. 1) it is clear that

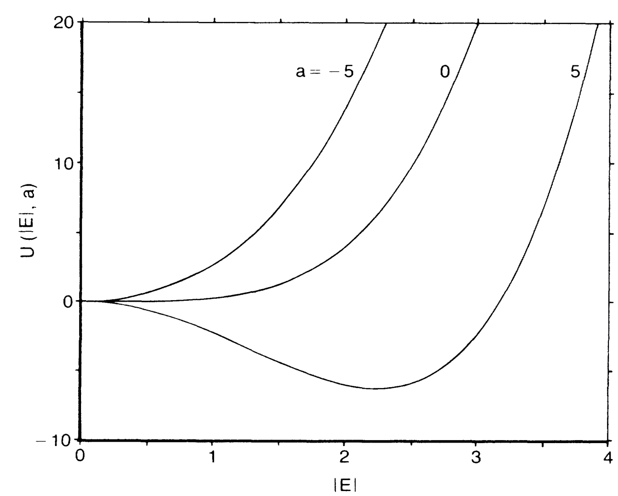


FIG. 1. Shape of the potential $U(|E|,a)$ near threshold.

the systematic forces remain small in the entire critical region and the dynamics of the laser field is dominated by the quantum noise. In this regime of operation, if the laser is suddenly turned on the time evolution of the electric field may be described as a continuous two-dimensional random walk[1] (in the photon-number representation this random walk is a discrete one[9]) of a Brownian particle. Indeed Eq. (1) represents the overdamped motion of a Brownian particle in a potential well. This Brownian motion may be detected by observing the time it takes the laser field to reach a certain reference value starting at the origin initially. This time will be found to fluctuate reflecting the underlying Brownian motion. This, of course, is the classic first-passage-time (FPT) problem.[10] Thus the FPT measurements provide a convenient means of observing the Brownian motion of the electric field in the threshold region. It should be mentioned that the concept of the decay of an unstable state[11-14] does not provide a meaningful framework for describing laser dynamics near threshold. This is because the so-called states of the laser (characterized by the laser intensity $I=|E|^2=0$ and $I=a$) are not well defined and may not even exist. The concept of the FPT, however, can still be fruitfully employed. We wish to describe a simple experiment which provides direct evidence for the random walk performed by the electric field of the laser in a regime dominated by quantum noise.

The probability density of the FPT for the laser field to reach a certain reference value starting with zero field initially obeys the backward Fokker-Planck equation which is easily derived from Eq. (1).[10] Since in our measurements the phase of the electric field is not observed we confine our attention to the laser amplitude $|E|$. In terms of the laser intensity $I=|E|^2$, the equation satisfied by the FPT probability density $P(T;I_0,I_f)\equiv P(T)$ is

$$\frac{\partial P}{\partial T}(T)=2I_0\left(a-I_0+\frac{2}{I_0}\right)\frac{\partial}{\partial I_0}P(T)+4I_0\frac{\partial^2}{\partial I_0^2}P(T) , \tag{3}$$

where $I_0$ is the initial intensity and $I_f$ is the final intensity of the laser, and $P(T)$ satisfies the boundary condition $P(T;I_0,I_f=I_0)=\delta(T)$. This equation has no known analytic solution, but some approximate solutions have been constructed.[11] The expressions for the moments are relatively easy to derive.[10] Thus we have for the mean and variance of the FPT probability density the following ex-

 

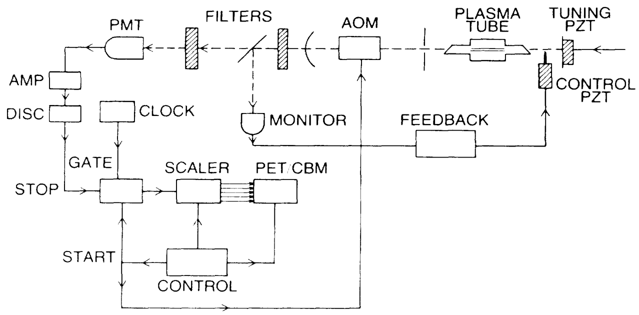


FIG. 2. Outline of the experimental apparatus.

pressions $(I_0=0, I_f=I_r)$,

$$\langle T\rangle = \tfrac{1}{4}\int_0^{I_r} dI_2 \frac{e^{U(I_2)}}{I_2}\int_0^{I_2} dI_1 e^{-U(I_1)} \,, \tag{4}$$

$$\langle T^2\rangle = \tfrac{1}{8}\int_0^{I_r} dI_4 \frac{e^{U(I_4)}}{I_4}\int_0^{I_4} dI_3 e^{-U(I_3)} \times \int_{I_3}^{I_r} dI_2 \frac{e^{U(I_2)}}{I_2}\int_0^{I_2} dI_1 e^{-U(I_1)} \,. \tag{5}$$

Solutions to Eq. (3) were obtained by integrating Eq. (3) numerically. These predictions were tested in the following experiment.

An outline of the experimental setup is shown in Fig. 2. A 20-cm-long standing-wave He:Ne laser operating at 6328 Å in a single longitudinal and transverse mode was used. An intracavity acousto-optic modulator (AOM) served as a $Q$ switch. The entire setup was enclosed in a temperature-controlled oven. The laser cavity was an Invar structure. Once thermal equilibrium was reached the laser was found to be extremely stable, such that frequency drifted no more than 1–2 MHz over a period of about 5 min. To perform the experiment the laser is stabilized at a certain operating point close to the threshold with the help of an electronic feedback loop. A voltage pulse is now applied to the $Q$ switch which extinguishes the laser by increasing the losses inside the cavity. When the extinction is complete, a second voltage pulse applied to the $Q$ switch decreases losses suddenly and turns the laser on. At the same time a gate is opened which allows pulses from a clock to reach a scaler. The growth of laser light is monitored by a photodetector whose output is fed to a discriminator. The discriminator threshold determines the reference intensity or the amplitude $(I_r=|E_r|^2)$. When the photodetector output signal, which is proportional to $|E_r|^2$, crosses the discriminator threshold, an output pulse is generated which stops the gate. The number of clock pulses stored in the scaler, which is a measure of the time it takes the laser field amplitude to grow from a value zero to $|E_r|=\sqrt{I_r}$, is transferred to the multichannel memory of a computer and the scaler is cleared for the next measurement cycle. Once the laser reaches the steady state its intensity is stabilized by the feedback loop. This is to ensure the same operating conditions for each measurement cycle. By repeating this process several thousand times, a histogram is built up, which becomes a measure of the first-passage-time probability density. The data collection sequence is controlled by a PET microcomputer. The laser was turned off for approximately 0.6 ms and on for about 6 ms. The rise time of the AOM was 60 ns which was negligible compared to the typical rise time of the laser ($\sim$50 $\mu$s) near threshold. Data were taken for several different operating points and reference intensities.

The behavior of the experimentally measured mean and the variance of the FPT is shown in Figs. 3 and 4. The full curves are the predictions of Eqs. (4) and (5). The agreement between the two is excellent in the entire threshold region. The slow dependence of the mean and the variance on the operating point above threshold and the rapid dependence below threshold $(\langle I\rangle_{\text{th}}=1.13)$ is attributable

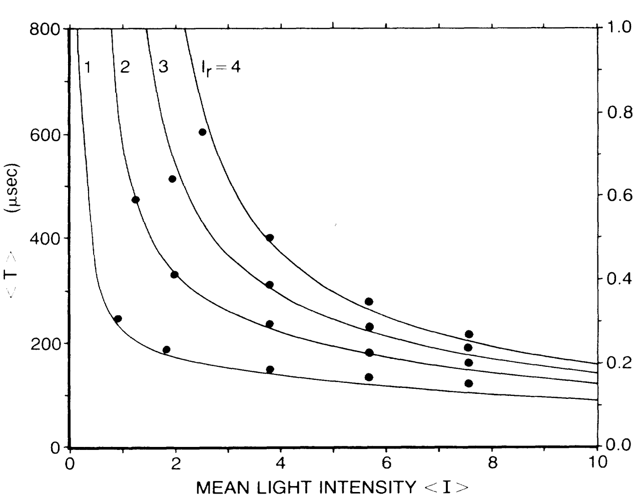


FIG. 3. Comparison of theoretical and experimental mean first passage times for several different reference intensities. The dimensionless mean intensity refers to the steady-state operation. Laser threshold is at $\langle I\rangle=1.13$.

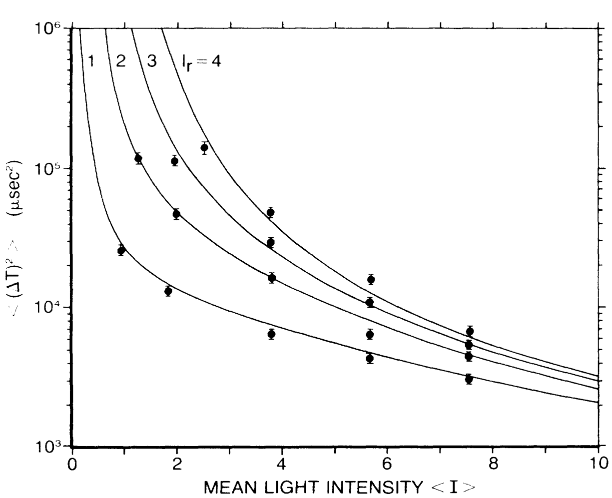


FIG. 4. Comparison of theoretical and experimental variance of the first passage time for several different reference intensities. The dimensionless mean intensity refers to the steady-state operation.

to the small but important change that the potential undergoes for small $|E|$ near threshold. Below threshold the small systematic forces always act toward the origin, whereas above threshold they act away from the origin. As a result the particle takes longer to reach a given reference point below threshold than it does to reach the same reference point above threshold. The convergence of various curves for higher operating points is an indication of the onset of the so-called scaling regime.[13,14] In that regime the growth of laser field amplitude may be describable in terms of the decay of an unstable state.

We have also compared the measured and the predicted FPT probability densities. Figure 5 shows an example of such a comparison. The full curve was obtained by solving Eq. (3) numerically and the dashed curve represents the predictions for the Ornstein-Uhlenbeck process.[11,12] Similar agreements are obtained throughout the threshold region. The details of these and other investigations will be presented elsewhere. Comparison of the data involves two scale constants, one for the light intensity $\langle I \rangle$ and one that converts measured values of $T$ to the dimensionless values used in the theory. Both scale factors were determined by plotting $\langle T \rangle$ vs $\langle I \rangle$ on a double logarithmic graph. The scale factor for $\langle I \rangle$ was found to be consistent with an independent determination based on the steady-state fluctuation measurements. Thus our comparison procedure involves only the time scale factor which was determined to be $800 \pm 20$ $\mu$s.

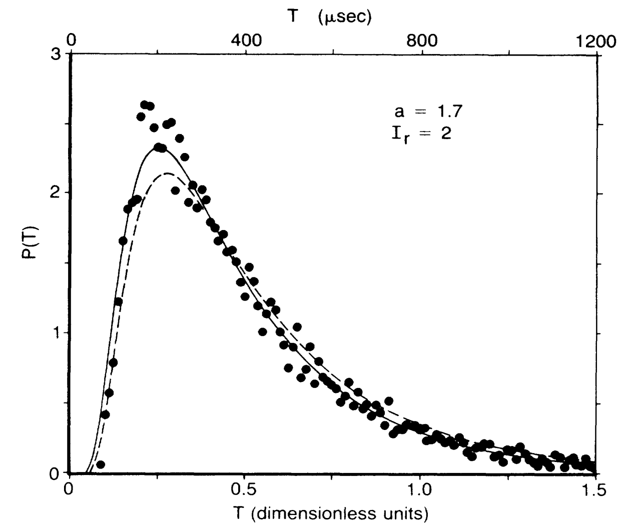


FIG. 5. Comparison of the experimentally measured FPT probability density with theoretical predictions. The full curve is obtained by solving Eq. (3) numerically. The broken curve is the prediction for the Ornstein-Uhlenbeck process (Ref. 12).

The laser threshold transition is often described as an analog of the second-order phase transition in a ferromagnet.[7] Therefore the laser dynamics, near threshold, constitute an example of critical dynamics. The growth of the field amplitude during the turn on is analogous to the growth of magnetization in a ferromagnet when it is suddenly quenched below the Curie point. Our measurements were carried out close to the threshold where the gain and loss differ by no more than a few parts in $10^4$. This type of measurement carried out in the vicinity of threshold will be of great interest in the study of critical dynamics of the order parameter in the theory of phase transitions.

These measurements also provide direct evidence for the random walk of the electric field of the laser in a regime where its dynamics is dominated by quantum noise. The agreement between the experimental results and theoretical predictions based on the nonlinear oscillator model of the laser also provides a very sensitive test of the laser theory. In fact the eigenvalue method[15] for computing the solution of Eq. (3), which works so well for the steady-state measurements, fails because in the transient operation, such as that encountered in the experiments described here, the entire spectrum of eigenvalues and eigenfunctions is important. On the other hand, for the steady-state measurements only the first few eigenvalues and eigenfunctions need to be calculated with any accuracy.

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