Nonrelativistic Schrödinger Green's function for crossed time-dependent electric and magnetic fields

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The retarded nonrelativistic Schrödinger Green's function for an electron in crossed electric and magnetic fields is derived here in closed form. The electric and magnetic fields are taken to be spatially uniform but the electric field may have arbitrary time dependence and arbitrary orientation with respect to the constant magnetic field.

The determination of the Schrödinger Green's function for an electron in crossed spatially uniform electric and magnetic fields was addressed by Schwinger[1] in a relativistic analysis for time-constant fields using a proper-time technique. Furthermore, Feynman and Hibbs[2] have also discussed the equivalent nonrelativistic path integral for uniform and constant electric fields. More recently such high electric field Green's functions have been brought to take on nonlinear transport problems by Thornber[3] and Jauho and Wilkins[4] with provision for time variation of the electric field. In seeking to generalize the nonrelativistic retarded Green's function to include uniform magnetic fields of arbitrary strength and orientation relative to the electric field, we have found it simpler to rederive the nonrelativistic Green's function in crossed fields directly rather than take a nonrelativistic limit of the crossed field Green's function determined by Schwinger relativistically. This nonrelativistic derivation is reported here, following Schwinger's techniques closely, for electric field of arbitrary time dependence crossed with a constant magnetic field.

The nonrelativistic retarded Schrödinger Green's function for an electron in an externally impressed uniform electric field $\mathbf{E}(t)$ having arbitrary time dependence crossed with a constant uniform magnetic field $\mathbf{B}$ of arbitrary orientation, is defined to satisfy the equation ($\hbar=c=1$ throughout this report)

$$[i\partial/\partial t-\mathscr{H}(\mathbf{x},t)]G(\mathbf{x},t;\mathbf{x}',t')=\delta^3(\mathbf{x}-\mathbf{x}')\delta(t-t') \tag{1}$$

with $\mathscr{H}(\mathbf{x},t)=\Pi^2/2m-\mu_0\boldsymbol{\sigma}\cdot\mathbf{B}+eV(\mathbf{x},t)$, where $V(\mathbf{x},t)=-\mathbf{E}(t)\cdot\mathbf{x}$ and $\boldsymbol{\Pi}=-i\nabla-e\mathbf{A}(\mathbf{x})$ with $\mathbf{A}(\mathbf{x})=\frac{1}{2}\mathbf{B}\times\mathbf{x}$. [$\boldsymbol{\sigma}=(\sigma_1,\sigma_2,\sigma_3)$ denotes the Pauli spin matrices, and μ_0 is the Bohr magneton.] For $t>t'$,

$$[i\partial/\partial t-\mathscr{H}(\mathbf{x},t)]G(\mathbf{x},t;\mathbf{x}',t')=0$$

and the retardation condition is characterized by $G(\mathbf{x},t<t';\mathbf{x}',t')\equiv 0$ so that $iG(\mathbf{x},t=t'^{+};\mathbf{x}';t')=\delta^3(\mathbf{x}-\mathbf{x}')$. Employing matrix notation with respect to position space indices (but not time indices), we have $G(\mathbf{x},t;\mathbf{x}',t')=\langle\mathbf{x}\,|\,\hat{G}(t,t')\,|\,\mathbf{x}'\rangle$ and $\mathscr{H}(\mathbf{x},t)\delta^3(\mathbf{x}-\mathbf{x}')=\langle\mathbf{x}\,|\,\hat{H}(t)\,|\,\mathbf{x}'\rangle$ and for $t>t'$

$$[i\partial/\partial t-\hat{H}(t)]\hat{G}(t,t')=0 \tag{2}$$

subject to the retardation condition $i\hat{G}(t=t'^{+},t')=I$. The formal solution of Eq. (2) is given in terms of the time development operator

$$i\hat{G}(t,t')=U(t,t')=\left[\exp\left[-i\int_{t'}^{t}d\bar{t}\,\hat{H}(\bar{t})\right]\right]_{+},$$

where $[\;]_{+}$ denotes time ordering. Denoting the time-developed positional states as $|\,\mathbf{x}(t)\rangle=U(t,0)\,|\,\mathbf{x}\rangle$, we have $G(\mathbf{x},t;\mathbf{x}',t')=\langle\mathbf{x}(t)\,|\,\mathbf{x}'(t')\rangle$ and $(i\partial/\partial t)G(\mathbf{x},t;\mathbf{x}',t')=\langle\mathbf{x}(t)\,|\,\hat{H}(t)\,|\,\mathbf{x}'(t')\rangle$ which yields

$$(\partial/\partial t)[G(\mathbf{x},t;\mathbf{x}',t')]/G(\mathbf{x},t;\mathbf{x}',t')=-i\langle\mathbf{x}(t)\,|\,\hat{H}(t)\,|\,\mathbf{x}'(t')\rangle/\langle\mathbf{x}(t)\,|\,\mathbf{x}'(t')\rangle,$$

whence

$$G(\mathbf{x},t;\mathbf{x}',t')=K(\mathbf{x},\mathbf{x}',t')\times\exp\left[-i\int dt\frac{\langle\mathbf{x}(t)\,|\,\hat{H}(t)\,|\,\mathbf{x}'(t')\rangle}{\langle\mathbf{x}(t)\,|\,\mathbf{x}'(t')\rangle}\right], \tag{3}$$

where $K(\mathbf{x},\mathbf{x}',t')$ is independent of t, and is to be determined from the initial retardation condition. Following the techniques of Ref. 1, our program to evaluate Eq. (3) will employ the Heisenberg operator equations of motion for $\hat{\mathbf{x}}(t)$ and $\hat{\boldsymbol{\Pi}}(t)$, and will use the solutions to eliminate the explicit appearance of $\hat{\boldsymbol{\Pi}}(t)$ in favor of $\hat{\mathbf{x}}(t)$ in $\hat{H}(t)$. Setting $t'\to 0$, one must then commute $\hat{\mathbf{x}}(t)$ to the left of $\hat{\mathbf{x}}(0)$ factors in $\hat{H}(t)$ using the canonical commutation relations, and finally the time integrand of Eq. (3) may be determined explicitly with the use of $\langle\mathbf{x}(t)\,|\,\hat{\mathbf{x}}(t)=\langle\mathbf{x}(t)\,|\,\mathbf{x}$ and $\hat{\mathbf{x}}(0)\,|\,\mathbf{x}'(0)\rangle=\mathbf{x}'\,|\,\mathbf{x}'(0)\rangle$.

Taking $\mathbf{B}$ along the x_3 axis and $\mathbf{E}(t)$ arbitrarily oriented, we denote vectors as $\mathbf{V}=(\mathbf{V}_\perp,V_\parallel)$ where $\mathbf{V}_\perp=(V_1,V_2)$ and $V_\parallel=V_3$. Thus $\hat{H}(t)$ takes the form

$$\hat{H}(t)=\frac{\hat{\boldsymbol{\Pi}}_\perp^2}{2m}+\frac{\hat{P}_\parallel^2}{2m}-\mu_0B\sigma_3-e\mathbf{E}_\perp(t)\cdot\hat{\mathbf{x}}_\perp-eE_\parallel(t)\hat{x}_\parallel \tag{4}$$

and the canonical commutation relations are given by $[\hat{x}_\parallel,\hat{P}_\parallel]=i$; $[\hat{x}_{\perp m},\hat{\Pi}_{\perp n}]=i\delta_{mn}$; $[\hat{\Pi}_{\perp m},\hat{\Pi}_{\perp n}]=ieF_{mn}$, where $F=iB\sigma_2$ and $F^{-1}=-i\sigma_2/B$. Forming the Heisenberg equations of motion, we have

$$\frac{d\hat{\boldsymbol{\Pi}}_\perp}{dt}=i[\hat{H}(t),\hat{\boldsymbol{\Pi}}_\perp]=\frac{e}{m}F\hat{\boldsymbol{\Pi}}_\perp+e\mathbf{E}_\perp\;;\quad\frac{dP_\parallel}{dt}=eE_\parallel\,, \tag{5}$$

$$\frac{d\hat{\mathbf{x}}_\perp}{dt}=i[\hat{H}(t),\hat{\mathbf{x}}_\perp]=\frac{\hat{\boldsymbol{\Pi}}_\perp}{m}\;;\quad\frac{d\hat{x}_\parallel}{dt}=\frac{\hat{P}_\parallel}{m}\,. \tag{6}$$

The equation for $d\hat{\Pi}_\perp/dt$ is readily solved by setting $\hat{\Pi}_\perp(t)=\exp[(e/m)Ft]\hat{\eta}(t)$, with the result

$$\hat{\Pi}_\perp(t)=e^{(e/m)Ft}\left[\hat{\Pi}_\perp(0)+\int_0^t d\bar{t}\,e^{-(e/m)F\bar{t}}e\mathbf{E}_\perp(\bar{t})\right]. \tag{7}$$

Substituting Eq. (7) into Eq. (6) for $d\hat{\mathbf{x}}_\perp/dt$, one may integrate directly to obtain the solution for $\hat{\mathbf{x}}_\perp(t)$ in terms of $\hat{\Pi}_\perp(0)$. Alternatively, this result may be used to express $\hat{\Pi}_\perp(0)$ in terms of $\hat{\mathbf{x}}_\perp(t)$ as

$$\hat{\Pi}_\perp(0)=eF(e^{(e/m)Ft}-1)^{-1}\left[\hat{\mathbf{x}}_\perp(t)-\hat{\mathbf{x}}_\perp(0)-\frac{1}{m}\int_0^t d\bar{t}\,e^{(e/m)F\bar{t}}\int_0^{\bar{t}}d\bar{\bar{t}}\,e^{-(e/m)F\bar{\bar{t}}}e\mathbf{E}_\perp(\bar{\bar{t}})\right]. \tag{8}$$

Using the fact that $F=iB\sigma_2$ and noting that σ_2 is idempotent ($\sigma_2^2=1$), we have $\exp[\pm(e/m)Ft]=\exp(\pm i\sigma_2\omega_c t)=\cos(\omega_c t)\pm i\sigma_2\sin(\omega_c t)$, where $\omega_c=eB/m$ is the cyclotron frequency. Consequently Eqs. (7) and (8) yield

$$\hat{\Pi}_\perp(t)=e^{i\sigma_2\omega_c t}\int_0^t d\bar{t}\,e^{-i\sigma_2\omega_c\bar{t}}e\mathbf{E}_\perp(\bar{t})+im\omega_c\sigma_2 e^{i\sigma_2\omega_c t}(e^{i\sigma_2\omega_c t}-1)^{-1}\left[\hat{\mathbf{x}}_\perp(t)-\hat{\mathbf{x}}_\perp(0)-\frac{1}{m}\int_0^t d\bar{t}\,e^{i\sigma_2\omega_c\bar{t}}\int_0^{\bar{t}}d\bar{\bar{t}}\,e^{-i\sigma_2\omega_c\bar{\bar{t}}}e\mathbf{E}_\perp(\bar{\bar{t}})\right]. \tag{9}$$

A corresponding treatment of $\hat{P}_{\parallel}(t)$, which does not involve the magnetic field, yields

$$\hat{P}_{\parallel}(t)=\frac{1}{t}\left[m[\hat{x}_{\parallel}(t)-\hat{x}_{\parallel}(0)]+et\int_0^t dt'E_{\parallel}(t')-e\int_0^t d\bar{t}\int_0^{\bar{t}}d\bar{\bar{t}}\,E_{\parallel}(\bar{\bar{t}})\right]. \tag{10}$$

In order to proceed with the construction of $\hat{H}(t)$, we need $[\hat{\Pi}_\perp^2(t)+\hat{P}_{\parallel}^2(t)]/2m$. Since we deal with forms such as $\hat{\Pi}_\perp(t)=g(\sigma_2)\hat{\mathbf{V}}_\perp$ it is useful to note that $\hat{\Pi}_\perp^2=\hat{\mathbf{V}}_\perp g(-\sigma_2)g(\sigma_2)\hat{\mathbf{V}}_\perp$ (since σ_2 is antisymmetric under transposition) whence we obtain

$$\begin{aligned}\hat{\Pi}_\perp^2(t)=&\int_0^t d\bar{t}\int_0^t dt'e\mathbf{E}_\perp(\bar{t})\exp[i\sigma_2\omega_c(\bar{t}-t')]e\mathbf{E}_\perp(t')\\&+\frac{m^2\omega_c^2}{4\sin^2(\omega_c t/2)}\{\hat{\mathbf{x}}_\perp^2(t)+\hat{\mathbf{x}}_\perp^2(0)-2\hat{\mathbf{x}}_\perp(t)\cdot\hat{\mathbf{x}}_\perp(0)+[\hat{\mathbf{x}}_\perp(t)\cdot\hat{\mathbf{x}}_\perp(0)-\hat{\mathbf{x}}_\perp(0)\cdot\hat{\mathbf{x}}_\perp(t)]\}\\&+\frac{\omega_c^2}{4\sin^2(\omega_c t/2)}\int_0^t d\bar{t}\int_0^{\bar{t}}d\bar{\bar{t}}\int_0^t dt'\int_0^{t'}dt''e\mathbf{E}_\perp(\bar{\bar{t}})\exp[i\sigma_2\omega_c(\bar{\bar{t}}-\bar{t}+t'-t'')]e\mathbf{E}_\perp(t'')\\&-\frac{m\omega_c}{\sin(\omega_c t/2)}\int_0^t d\bar{t}\,e\mathbf{E}_\perp(\bar{t})\exp[-i\sigma_2\omega_c(\bar{t}+t/2)][\hat{\mathbf{x}}_\perp(t)-\hat{\mathbf{x}}_\perp(0)]\\&-\frac{m\omega_c^2}{2\sin^2(\omega_c t/2)}\int_0^t d\bar{t}\int_0^{\bar{t}}d\bar{\bar{t}}\,e\mathbf{E}_\perp(\bar{\bar{t}})\exp[-i\sigma_2\omega_c(\bar{t}-\bar{\bar{t}})][\hat{\mathbf{x}}_\perp(t)-\hat{\mathbf{x}}_\perp(0)]\\&-\frac{\omega_c}{\sin(\omega_c t/2)}\int_0^t dt'\int_0^t d\bar{t}\int_0^{\bar{t}}d\bar{\bar{t}}\,e\mathbf{E}_\perp(t')\exp[i\sigma_2\omega_c(\bar{t}-\bar{\bar{t}}+t'-t/2)]e\mathbf{E}_\perp(\bar{\bar{t}}).\end{aligned} \tag{11}$$

Here

$$[\hat{\mathbf{x}}_\perp(t)-\hat{\mathbf{x}}_\perp(0)]^2=\hat{\mathbf{x}}_\perp^2(t)+\hat{\mathbf{x}}_\perp^2(0)-2\hat{\mathbf{x}}_\perp(t)\cdot\hat{\mathbf{x}}_\perp(0)+[\hat{\mathbf{x}}_\perp(t)\cdot\hat{\mathbf{x}}_\perp(0)-\hat{\mathbf{x}}_\perp(0)\cdot\hat{\mathbf{x}}_\perp(t)]$$

and the commutator required to bring $\hat{\mathbf{x}}_\perp(t)$ to the left of $\hat{\mathbf{x}}_\perp(0)$ may be evaluated using Eq. (8) in the form

$$\hat{\mathbf{x}}_\perp(t)=\hat{\mathbf{x}}_\perp(0)-\frac{i\sigma_2}{m\omega_c}(e^{i\sigma_2\omega_c t}-1)^{-1}\hat{\Pi}_\perp(0)+\frac{1}{m}\int_0^t d\bar{t}\,e^{i\sigma_2\omega_c\bar{t}}\int_0^{\bar{t}}d\bar{\bar{t}}\,e^{-i\sigma_2\omega_c\bar{\bar{t}}}e\mathbf{E}_\perp(\bar{\bar{t}}) \tag{12}$$

whence the equal time canonical commutation relations yield

$$[\hat{\mathbf{x}}_\perp(t)\cdot\hat{\mathbf{x}}_\perp(0)-\hat{\mathbf{x}}_\perp(0)\cdot\hat{\mathbf{x}}_\perp(t)]=-\frac{2i}{m\omega_c}\sin(\omega_c t). \tag{13}$$

The result for $\hat{\Pi}_\perp^2(t)$ is

$$\begin{aligned}\hat{\Pi}_\perp^2(t)=&\int_0^t d\bar{t}\int_0^t dt'e\mathbf{E}_\perp(\bar{t})\exp[i\sigma_2\omega_c(\bar{t}-t')]e\mathbf{E}_\perp(t')+\frac{m^2\omega_c^2}{4\sin^2(\omega_c t/2)}\left[\hat{\mathbf{x}}_\perp^2(t)-\hat{\mathbf{x}}_\perp^2(0)-2\hat{\mathbf{x}}_\perp(t)\cdot\hat{\mathbf{x}}_\perp(0)-\frac{2i}{m\omega_c}\sin(\omega_c t)\right]\\&+\frac{\omega_c^2}{4\sin^2(\omega_c t/2)}\int_0^t d\bar{t}\int_0^{\bar{t}}d\bar{\bar{t}}\int_0^t dt'\int_0^{t'}dt''e\mathbf{E}_\perp(\bar{\bar{t}})\exp[i\sigma_2\omega_c(\bar{\bar{t}}-\bar{t}+t'-t'')]e\mathbf{E}_\perp(t'')\\&-\frac{m\omega_c}{\sin(\omega_c t/2)}\int_0^t d\bar{t}\,e\mathbf{E}_\perp(\bar{t})\exp[-i\sigma_2\omega_c(\bar{t}+t/2)][\hat{\mathbf{x}}_\perp(t)-\hat{\mathbf{x}}_\perp(0)]\end{aligned}$$

$$-\frac{m\omega_c^2}{2\sin^2(\omega_c t/2)}\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, e\mathbf{E}_\perp(\bar{\bar{t}})\exp[-i\sigma_2\omega_c(\bar{t}-\bar{\bar{t}})][\hat{\mathbf{x}}_\perp(t)-\hat{\mathbf{x}}_\perp(0)]$$

$$-\frac{\omega_c}{\sin(\omega_c t/2)}\int_0^t dt'\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, e\mathbf{E}_\perp(t')\exp[i\sigma_2\omega_c(\bar{t}-\bar{\bar{t}}+t'-t/2)]e\mathbf{E}_\perp(\bar{\bar{t}}) \;. \tag{14}$$

A similar treatment of $\hat{P}_{\parallel}^2(t)$, which does not involve the magnetic field, yields (note that $[\hat{x}_{\parallel}(t),\hat{x}_{\parallel}(0)]=-it/m$)

$$\hat{P}_{\parallel}^2(t)=m^2t^{-2}\left[\hat{x}_{\parallel}^2(t)+\hat{x}_{\parallel}^2(0)-2\hat{x}_{\parallel}(t)\hat{x}_{\parallel}(0)-it/m+\frac{e^2}{m^2}\left[\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, E_{\parallel}(\bar{\bar{t}})\right]^2-\frac{2e}{m}[\hat{x}_{\parallel}(t)-\hat{x}_{\parallel}(0)]\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, E_{\parallel}(\bar{\bar{t}})\right]$$

$$+\frac{2me}{t}\left[\hat{x}_{\parallel}(t)-\hat{x}_{\parallel}(0)-\frac{e}{m}\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, E_{\parallel}(\bar{\bar{t}})\right]\int_0^t dt' E_{\parallel}(t')+\left[e\int_0^t dt' E_{\parallel}(t')\right]^2 . \tag{15}$$

$\hat{H}(t)$ may now be formed from Eq. (4) using Eqs. (14) and (15). The time integrand of Eq. (3) involving

$$\langle \mathbf{x}(t)\,|\,\hat{H}(t)\,|\,\mathbf{x}'(t')\rangle/\langle \mathbf{x}(t)\,|\,\mathbf{x}'(t')\rangle$$

may then be determined [using the fact that $\hat{\mathbf{x}}(t)$ has already been systematically commuted to the left of $\hat{\mathbf{x}}(0)$ factors] employing the relations $\langle \mathbf{x}(t)\,|\,\hat{\mathbf{x}}(t)=\mathbf{x}(t)\,|\,\mathbf{x}$ and $\hat{\mathbf{x}}(0)\,|\,\mathbf{x}'(0)\rangle=\mathbf{x}'\,|\,\mathbf{x}'(0)\rangle$. With this we find the Green's function for uniform electric field having arbitrary time dependence crossed with a constant magnetic field of arbitrary orientation to be given as ($t>0$, set $t'\to 0$)

$$G(\mathbf{x},t;\mathbf{x}',t'=0)$$

$$=K(\mathbf{x},\mathbf{x}')t^{-(1/2)}\exp\Bigg[-i\int_0^t dt\Bigg\{\frac{1}{2m}\int_0^t d\bar{t}\int_0^t dt' e\mathbf{E}_\perp(\bar{t})\exp[i\sigma_2\omega_c(\bar{t}-t')]e\mathbf{E}_\perp(t')$$

$$+\frac{m\omega_c^2}{8\sin^2(\omega_c t/2)}\left[(\mathbf{x}_\perp-\mathbf{x}'_\perp)^2-\frac{2i}{m\omega_c}\sin(\omega_c t)\right]$$

$$+\frac{\omega_c^2}{8m\sin^2(\omega_c t/2)}\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\int_0^t dt'\int_0^{t'} dt''\, e\mathbf{E}_\perp(\bar{\bar{t}})\exp[i\sigma_2\omega_c(\bar{\bar{t}}+t'-\bar{t}-t'')]e\mathbf{E}_\perp(t'')$$

$$-\frac{\omega_c}{2\sin(\omega_c t/2)}\int_0^t d\bar{t}\, e\mathbf{E}_\perp(\bar{t})\exp[-i\sigma_2\omega_c(\bar{t}+t/2)](\mathbf{x}_\perp-\mathbf{x}'_\perp)$$

$$-\frac{\omega_c^2}{4\sin^2(\omega_c t/2)}\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, e\mathbf{E}_\perp(\bar{\bar{t}})\exp[-i\sigma_2\omega_c(\bar{t}-\bar{\bar{t}})](\mathbf{x}_\perp-\mathbf{x}'_\perp)$$

$$-\frac{\omega_c}{2m\sin(\omega_c t/2)}\int_0^t dt'\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, e\mathbf{E}_\perp(t')\exp\left[i\sigma_2\omega_c\left[\bar{t}-\bar{\bar{t}}+t'-\frac{t}{2}\right]\right]e\mathbf{E}_\perp(\bar{\bar{t}})$$

$$+\frac{m}{2t^2}\left[(x_{\parallel}-x'_{\parallel})^2+\left[\frac{e}{m}\right]^2\left[\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, E_{\parallel}(\bar{\bar{t}})\right]^2-\frac{2e}{m}(x_{\parallel}-x'_{\parallel})\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, E_{\parallel}(\bar{\bar{t}})\right]$$

$$+\frac{e}{t}\left[x_{\parallel}-x'_{\parallel}-\frac{e}{m}\int_0^t d\bar{t}\int_0^{\bar{t}} d\bar{\bar{t}}\, E_{\parallel}(\bar{\bar{t}})\right]\int_0^t dt' E_{\parallel}(t')+\frac{e^2}{2m}\left[\int_0^t dt' E_{\parallel}(t')\right]^2-\mu_0 B\sigma_3-e\mathbf{E}(t)\cdot\mathbf{x}\Bigg\}\Bigg] . \tag{16}$$

Of course the retarded Green's function has $G(\mathbf{x},t<0;\mathbf{x}',t'=0)\equiv 0$. The time-independent constant $K(\mathbf{x},\mathbf{x}')$ is determined by magnetic gauge considerations similar to those of Refs. 1 and 5, with the result

$$K(\mathbf{x},\mathbf{x}')=c\exp[(ie/2)\mathbf{x}_\perp\cdot\mathbf{B}\times\mathbf{x}'_\perp-i\phi(\mathbf{x})+i\phi(\mathbf{x}')] ,$$

where $\phi(\mathbf{x})$ is an arbitrary gauge function. It is to be noted that $K(\mathbf{x},\mathbf{x}')$ embodies all of the $(\mathbf{x}_\perp+\mathbf{x}'_\perp)$ dependence associated with the lack of spatial translational invariance induced by the magnetic field. Other $(\mathbf{x}_\perp+\mathbf{x}'_\perp)$ dependence arises in conjunction with the electric field. Finally, the constant c is determined by the initial condition $G(\mathbf{x},t=0^+;\mathbf{x}',t'=0)=-i\delta^3(\mathbf{x}-\mathbf{x}')$.

In the special case of time-independent constant and uniform electric as well as magnetic fields, the Green's function for arbitrary orientation takes the form ($t>0;t'\to 0$),

$$G(\mathbf{x},t;\mathbf{x}',t'=0)=\left[\frac{im}{2\pi}\right]^{1/2}\frac{m\omega_c}{4\pi t^{1/2}\sin(\omega_c t/2)}$$

$$\times\exp\{(ie/2)\mathbf{x}_\perp\cdot\mathbf{B}\times\mathbf{x}'_\perp-i\phi(\mathbf{x})+i\phi(\mathbf{x}')-(ie^2E_\perp^2t/2m\omega_c)$$

$$+(im/2t)(x_{\parallel}-x'_{\parallel})^2-(ie^2E_{\parallel}^2t^3/24m)+(ie/2)\mathbf{E}\cdot(\mathbf{x}+\mathbf{x}')t$$

$$+(ie^2E_\perp^2/2m\omega_c)\alpha(t)+i\mu_0B\sigma_3t-(ie^2E_\perp^2/8m)\beta(t)+(im\omega_c/4)\cot(\omega_c t/2)(\mathbf{x}_\perp-\mathbf{x}'_\perp)^2$$

$$+[(ie^2/m\omega_c^2)-(ie^2t/2m\omega_c)\cot(\omega_c t/2)](\mathbf{x}_\perp-\mathbf{x}'_\perp)\cdot\mathbf{E}_\perp\times\mathbf{B}\}\ , \quad (17)$$

where

$$\alpha(t)=\int_0^t dt\, t\cot(\omega_c t/2) \quad\text{and}\quad \beta(t)=\int_0^t dt\, t^2/\sin^2(\omega_c t/2)\ .$$

For special cases in which either one or the other of the electric and magnetic fields vanishes, this reduces to the well-known results of Refs. 2–5.

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