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Remeasurement of the Rydberg constant

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We report a remeasurement of the Rydberg constant via a single-photon determination of the Balmer-α wavelength. The new value reflects the redefinition of the meter, a reexamination of corrections, and improvements of the experimental apparatus. The result is $R = 109\,737.315\,69(7)$ cm^{-1}, where $c = 299\,792\,458$ m/s by definition. This result does not significantly disagree with the preceding measurement by Amin *et al.*, significantly disagrees with the measurement by Hildum *et al.*, and agrees very well with a recent measurement by Biraben and Julien.

I. INTRODUCTION

The Rydberg constant is a fundamental quantity which is an "auxiliary constant." These constants have such relatively high precision that they are taken as exact in least-squares adjustments of the physical constants.[1] Recent, very precise measurements of two-photon transitions in hydrogen[2,3] and positronium[4] motivate a more precise value for R. Recently, the extension of direct frequency measurements into the optical domain has lead to a redefinition of the meter, such that the speed of light is exactly 299 792 458 m/s.[5] Primary wavelength and frequency standards at 633 nm are precise to 1.6 parts in 10^{10} (one standard deviation), although the BIPM value[6] has an error of 3.4 parts in 10^{10} (one standard deviation).

Amin, Caldwell, and Lichten[7,8] measured the wavelength of the Balmer-α line to a precision of one part in 10^9. This paper reports a remeasurement of this quantity with improvement of the apparatus to avoid systematic effects. The present measurement is much freer of possible systematic effects and advances the errors to a few parts in 10^{10} level, closer to the goal of an accuracy comparable to the present realization of the definition of length.

II. EXPERIMENTAL METHODS AND APPARATUS

The basic plan of the experiment is the same as that by Amin *et al.*,[7,8] a measurement of the wavelengths of the four possible Balmer-α lines. For each isotope of H and D, both lines are from the single hyperfine level $F = I + \frac{1}{2}$ of the metastable $2S$ state to $3P_{1/2}$ and $3P_{3/2}$. The tables of Erickson,[9] which give calculated values of the term values, are used with corrections for the most recent values of the fine structure constant α,[10] the electron-to-proton mass ratio,[11] and the Rydberg constant.[7,8] The difference between the measured and calculated values of the wavelengths then gives four independent values of the Rydberg constant for infinite mass.

TABLE I. Corrections to R_∞ (parts in 10^{10}).

Correction[a]	Transition H, $3P_{1/2}$	H, $3P_{3/2}$	D, $3P_{1/2}$	D, $3P_{3/2}$
Phase shift[b]	147	147	146	146
Refractive index[b]	34	34	34	34
$2S$ hfs	972	972	299	299
$3P$ hfs	32	−32	10	−10
Optical pumping	0	0	−2	−1
Second-order Doppler	6	6	3	3
Photon recoil	−10	−10	−5	−5
Total	1000[c]	936[c]	305[c]	286[c]
	1157[b]	1093[b]	461[b]	442[b]

[a]For both sets of data, unless noted.
[b]Phase-shift data only.
[c]Virtual-mirror data only.

TABLE II. Calculation of the Rydberg constant R_∞. Corrections given in Table I.

Upper State	H, $3P_{1/2}$	H, $3P_{3/2}$	D, $3P_{1/2}$	D, $3P_{3/2}$
		Wave number[a] $1/\lambda = 15230\ \mathrm{cm}^{-1}+$		
Phase shift	3.256812	3.365219	7.401728	7.510166
Virtual mirrors	3.256807	3.365224	7.401736	7.510173
Theory[b]	3.256787	3.365198	7.401688	7.510129
		Rydberg constant[c] $R_\infty = 109737\ \mathrm{cm}^{-1}+$		
Phase shift	0.315618	0.315592	0.315715	0.315715
Virtual mirrors	0.315583	0.315633	0.315774	0.315767
Combined by atoms	H		D	
Phase shift	0.315605		0.315715	
Virtual mirrors	0.315608		0.315770	
		Grand average		
Phase shift		0.315660 ± 0.000032		
Virtual mirrors		0.315689 ± 0.000048		

[a]Based on $f(^3\mathrm{He}\text{-}^{20}\mathrm{Ne}{:}^{127}\mathrm{I}_2,g) = 473612340.492(74)$ MHz (Ref. 6), $c = 299792458$ m/s (Ref. 6), $\lambda(^3\mathrm{He}\text{-}^{20}\mathrm{Ne}{:}^{127}\mathrm{I}_2,g) = 632991.23010(10)$ pm (Ref. 6).
[b]$R_{\mathrm{theor}} = 109737.31544\ \mathrm{cm}^{-1}$ (Ref. 7), $1/\alpha = 137.035963$ (Ref. 10), Lamb shift for H is 1057.8514(19) MHz (Ref. 18), for D is 1059.235(27) MHz (Ref. 19), and for m_p/m_e is 1836.152701(37).
[c]$R_\infty = R_{\mathrm{theor}}\lambda_{\mathrm{theor}}/\lambda$.

As an improvement, the present experiment uses a National Bureau of Standards (NBS) iodine-stabilized He-Ne standard laser, identical to the one used in the realization of the definition of the meter.[5] This particular laser has been compared with other standards on a worldwide basis, and its operating properties have been thoroughly investigated. The NBS standard laser can be stabilized to any one of the *d*, *e*, *f*, *g*, *h*, *i*, and *j* peaks of ^{127}I. To avoid problems caused by frequency modulation of the standard laser, a second single frequency He-Ne laser is used as the operating standard. This laser is offset from the mean frequency of the standard by ±50 MHz. The offset is accomplished by heterodyning the standard and offset laser in a photodiode, amplifying the beat note, counting its frequency, and comparing it to a rf that is controlled by a crystal oscillator.

The dye laser, one made by NBS in Boulder, is in a ring configuration, uses DCM dye, and is stabilized to a cavity by the method of Hänsch and Couillaud.[12] The dye-laser frequency is then locked to the atomic line by sinusoidal modulation of the frequency.

The étalon for comparing the dye laser with the standard is improved with a new mount which presses the mirrors against three 1.6-mm ($\frac{1}{16}$-in.) diameter Pyrex balls, which bear on a solid Cervit spacer. This eliminates distortion of the mirror centers. The etalon has an aperture of 2 cm. These improvements eliminate diffraction corrections caused by mirror curvature or finite aperture. A new 1-m spacer and an old 12-cm spacer give a virtual-mirror spacing of 88 cm, and therefore greater precision to the wavelength measurement.

Corrections due to nonlinearities in pressure scanning are reduced by minimizing the volume of the tubes between the gas leak and the wavelength measuring etalon. Also, the chart recorder is replaced by a digital oscilloscope interfaced via a RS-232 connection to a small home computer, which is programmed in machine language to allow data transfer at 1200 Baud. The digital recorder introduces a new problem caused by its fast (200-μs) sampling time. This introduces a stroboscopic effect on the data, caused by the beating of the dye-laser frequency modulation waveform (15 Hz) and the sampling frequency of 10 Hz. This slow beat is eliminated by synchronizing the two waveforms.

A new electron-gun collimating magnet with less stray field and magnetic shielding reduce the field in the transition region to 2×10^{-6} T (20 mG), thus eliminating Zeeman corrections and shifts due to unequal magnetic sublevel populations. A low backstreaming pump and low vapor pressure pump oil, in conjunction with continuous trapping, reduce impurity coating on the detector and retroflecting prism. The mounting for the tungsten oven

TABLE III. Error estimates (parts in 10^{10}).

Statistical	5
Recording and pressure scanning	3
Optical pumping, and light shifts	2
Wavelength standard	1.6
Nonuniform mirror coatings	1
Second-order Doppler, photon recoil, 2*S* hfs, index of refraction, and minor effects	1
Root-mean-square error	6.5

TABLE IV. Comparison with other measurements.

Reference	$R = 109\,737\ \mathrm{cm}^{-1}+$
Petley *et al.*[a]	$0.315\,29 \pm 0.000\,85$
Goldsmith *et al.*[b,c]	$0.315\,04 \pm 0.000\,32$
Amin *et al.*[b,d]	$0.315\,44 \pm 0.000\,11$
Hildum *et al.*[e]	$0.314\,92 \pm 0.000\,22$
Biraben and Julien[f]	$0.315\,69 \pm 0.000\,06$
Present result	$0.315\,69 \pm 0.000\,07$

[a]Reference 15.
[b]Corrected for the new definition of the meter.
[c]Reference 16.
[d]References 7 and 8. Also revised upward by 6 parts in 10^{10} to correct the sign for diffraction correction.
[e]Reference 2.
[f]Reference 17.

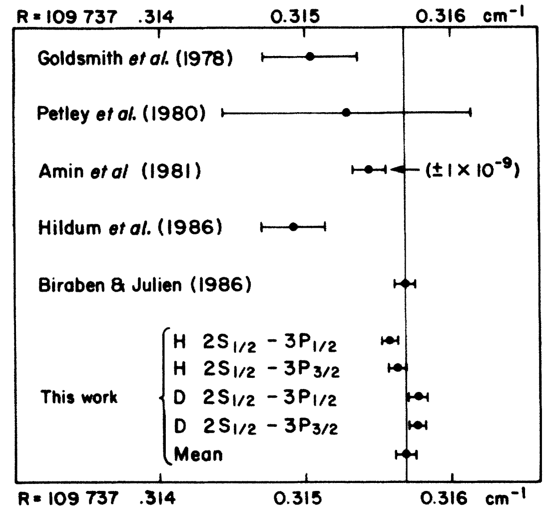

FIG. 1. Comparison of recent values for the Rydberg constant.

source now operates under zero tension, lengthening the average oven lifetime to 40 h.

In effect, the entire experiment has been rebuilt, within the limitation that the basic plan was unchanged. The quoted error is now below one part in 10^9. Several systematic sources of error have been eliminated or greatly reduced; this gives us considerably more confidence in this new measurement.

III. RESULTS AND CORRECTIONS

Table I shows the corrections (in parts in 10^{10}). Corrections due to nonlinear scans, magnetic field, wavelength standard, and diffraction, all of which were present in the former experiment, have been eliminated.

Table II contains the measured and corrected wave numbers for each of the four lines and the calculated values for the Rydberg constant. The error given for R is simply the statistical error (1σ) for the mean of the four values, each taken with equal weight.

The measured $3P$ *fs* of 3250.3(4) MHz for H and 3250.9(4) MHz for D agree well with theoretical values[9] of 3250.1 and 3251.0 MHz. The isotope shift of 124 262.3(3) MHz disagrees with the theoretical value[9] of 124 261.6 MHz.

Table III shows the error estimates for R. The final value for the Rydberg constant, taken by the method of virtual mirrors, is $R = 109\,737.315\,69(7)$, with an error (1σ) of approximately 6.5 parts in 10^{10}.

The method of virtual mirrors eliminates problems caused by phase shifts in the reflecting coatings by subtracting order numbers for two different spacings. A detailed description and evaluation of the alternative phase-shift method has been given in Refs. 13 and 14. The results of this method, shown in Table II, are 2.6 parts in 10^{10} lower than the virtual-mirror data and actually have a higher precision. If we were to repeat our previous procedure of taking a weighted average of the two methods, our final value would be 1 part in 10^{10} lower. We have chosen not to do so, as we feel the virtual-mirror data are more valid.

IV. COMPARISON WITH OTHER RESULTS

Table IV and Fig. 1 compare our present and previous results with those of other groups. In the case of work published before 1986, the values have been revised to allow for the redefinition of the meter.[6] Our present and previous results differ by 2 parts in 10^9, which is 1.7 standard deviations of the difference and is not significant. Likewise, the considerably less precise value by Petley, Morris, and Shawyer[15] does not differ significantly (0.5σ). The work by Goldsmith, Weber, and Hänsch,[16] which is less precise than the present work, differs significantly (2σ), and the recent, also less precise, two-photon measurement by Hildum *et al.*[2] differs significantly (3.4σ) from this value. On the other hand, the recent atomic beam two-photon determination by Biraben and Julien[15] is of comparable precision to the present measurement and is in perfect agreement. At present, we are measuring the wavelength of the Balmer-β transition at 486 nm as an additional check on the Rydberg constant and the Bohr formulas, and to achieve a higher accuracy. These new results will be presented shortly.

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