Thick-target bremsstrahlung spectra generated by the β particles of ^{90}Sr-^{90}Y and ^{99}Tc

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External bremsstrahlung generated by the β particles of the radioactive sources ^{90}Sr-^{90}Y (2274 keV) and ^{99}Tc (292 keV) in targets of Cu, Mo, Ag, Cd, and Pb sufficiently thick to stop all β particles has been studied using a 4.35×5.08-cm^2 NaI(Tl) scintillation spectrometer. The spectra unfolded using the Liden and Starfelt procedure are compared with the Bethe-Heitler, Elwert-corrected Bethe-Heitler, Morgan-corrected Bethe-Heitler, and Tseng and Pratt theories. The spectra generated by the high end-point energy source agree mostly with the Tseng and Pratt theory whereas those of the low end-point energy source agree mostly with the Elwert-corrected Bethe-Heitler theory. Deviations at high-energy regions of the spectra still exist.

External bremsstrahlung (EB) is a continuous energy electromagnetic radiation emitted when charged particles are deflected in the Coulomb field of the nuclei in the target material. One can use monoenergetic electrons or β particles to produce EB. When the target thickness is an appreciable fraction of the mean range of the electrons, the radiation is referred to as thick-target EB. Study of thick-target EB is important from the point of view of radiation transport, material processing, medical therapy, etc.

Sommerfeld[1] worked out the theory of EB for nonrelativistic electrons neglecting the effect of screening. The theory of EB for relativistic electrons was developed by Bethe and Heitler[2] (BH) using Dirac's exact relativistic wave equation and the plane-wave approximation. The major drawback of this theory is the use of the Born approximation in which the effect of the Coulomb field on the wave functions of the incident and scattered electrons is disregarded. This renders it less accurate for high-momentum transfer collisions and for high-Z materials. Elwert[3] gave a multiplicative Coulomb correction factor f_E to the BH cross section as

$$f_E=\frac{\beta_0\{1-\exp[-(2\pi Z/137\beta_0)]\}}{\beta\{1-\exp[-(2\pi Z/137\beta)]\}}\ , \qquad (1)$$

where $\beta_0=p_0/E_0=v_0/c$ is the ratio of the incident electron velocity to the velocity of light in vacuum. $\beta=p/E=v/c$ is the ratio of the scattered electron velocity to the velocity of light in vacuum. This is valid in the low-energy region.

Morgan, Jr.[4] has given a multiplicative Coulomb correction factor f_M to the BH cross section as

$$f_M=\cosh\left[\frac{k}{T_0}\left[\frac{Z}{0.394Z+9.47\beta_0}\right]\right]\ , \qquad (2)$$

where T_0 is the kinetic energy of incident electron. This correction factor is valid from 0.1 to 2 MeV.

Tseng and Pratt[5] have made EB cross-section calculations based on the description of the atom as a static spherically symmetric charge distribution of infinite mass, using four different central potentials, viz., point Coulomb, Thomas-Fermi, modified Thomas-Fermi, and modified Hartree-Fock-Slater, and by using a partial-wave expansion procedure. They claim that their results are closer to experimental results of thin targets and monoenergetic electrons.

The theories discussed above give cross sections which are applicable to thin-target EB spectra. However, Bethe and Heitler[2] have given an expression for the EB produced in targets thick enough to stop all incoming electrons, as

$$n(E',k)=N\int_{1+k}^{E'}\frac{d\sigma(E,k)dE}{-dE/dx}\ , \qquad (3)$$

where $n(E',k)$ is the number of bremsstrahlung photons produced at the energy k per unit energy interval by an electron of total energy E', N is the number of atoms per unit volume of the target material, $-dE/dx$ is the total electron energy loss per unit path length in the target, and $d\sigma(E,k)$ is the EB cross section for the production of EB of energy k per unit energy interval by an electron of energy E. When β particles are used to produce EB, then Eq. (3) has to be integrated over the β spectrum, keeping in mind that a β particle of certain energy will not produce an EB photon above that energy. In such a case the number of EB photons $S(k)$ of energy k per unit energy interval per β particle is given by

$$S(k)=\frac{\int_{1+k}^{E_{\max}}n(E,k)P(E)dE}{\int_{1}^{E_{\max}}P(E)dE}\ , \qquad (4)$$

where $E_{\max}$ is the maximum total energy of the beta particles in m_0c^2 units and $P(E)$ is the β spectrum.

In the present paper, the theoretical evaluation of $S(k)$ [Eq. (4)] using BH cross sections has been called the "BH theory." The evaluation of the same using BH cross sections with the Elwert multiplicative factor has been called the "Elwert-Bethe-Heitler (EBH) theory;" with the Morgan correction factor it has been called the "Morgan-Bethe-Heitler (MBH) theory." For the "Pratt theory" the tabulated cross section values of Pratt *et al.*[6] are used.

A survey of the literature[7–18] on β-induced thick-target EB shows that there is a systematic deviation of the thick-target experimental EB spectra from the Born approximation theory of Bethe and Heitler[2] and the Elwert-corrected BH theory. The deviation of the experimental results with theory has been shown to increase with increasing EB photon energy and atomic number of the target material.[11,16,19] In this work, the thick-target EB spectra produced by β particles of ^{90}Sr-^{90}Y and ^{99}Tc in various targets are compared with the theories of BH,

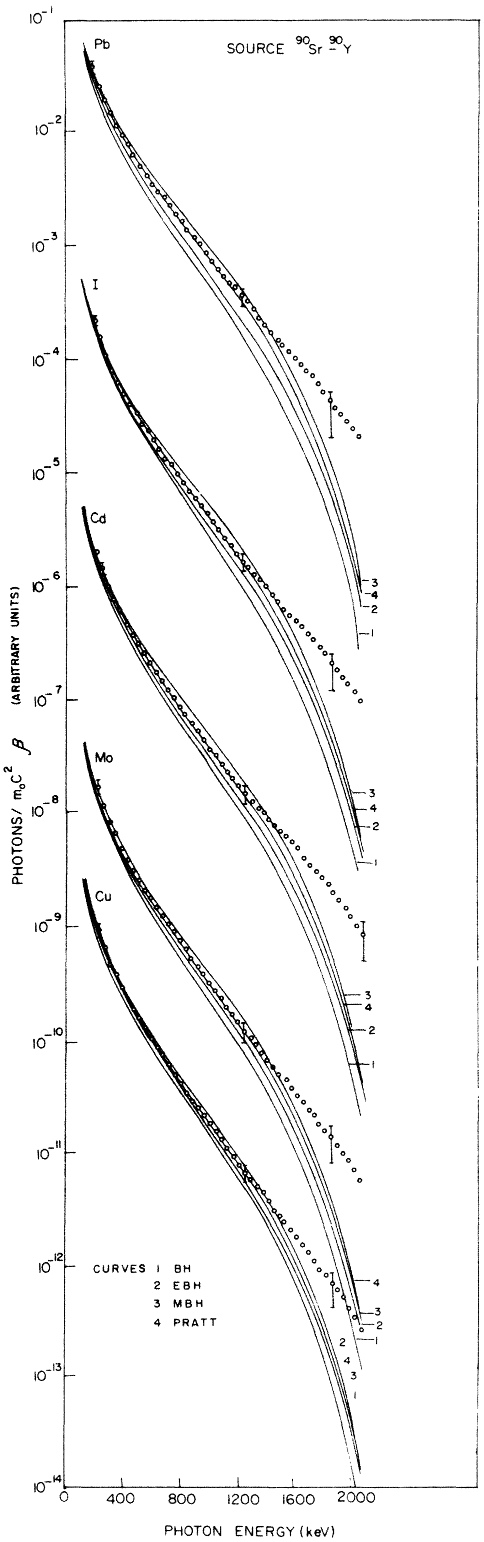

FIG. 1. The experimental distributions of EB produced in different materials by the β particles of ^{90}Sr-^{90}Y along with the four theoretical calculations.

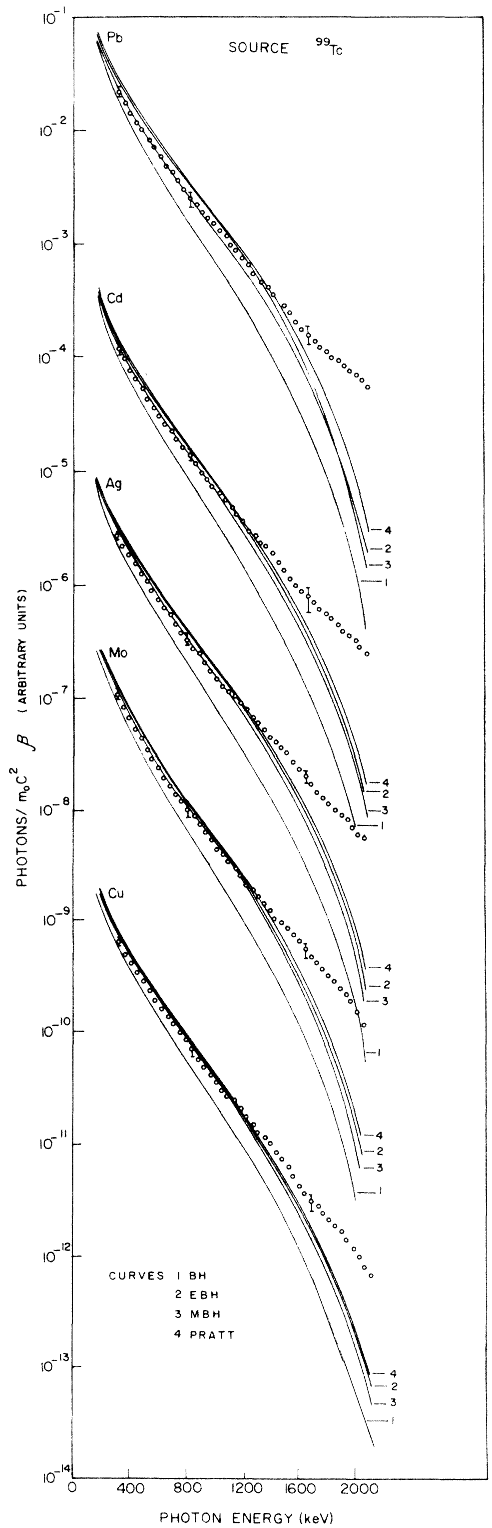

FIG. 2. The experimental distributions of EB produced in different materials by the β particles of ^{99}Tc along with the four theoretical calculations.

EBH, MBH, and Pratt. The experimental procedure is reported elsewhere.[19,20]

The procedure due to Liden and Starfelt[21] has been adopted to unfold the experimental pulse-height distribution. In addition, corrections for self-absorption in the target material, absorption in the Perspex β stopper, the aluminum can, etc., are also applied as detailed elsewhere.[19,20] Finally the true photon spectrum is represented as the number of photons/m_0c^2 β particles.

The experimental distributions of EB produced in different materials by the β particles of ^{90}Sr-^{90}Y, along with the four theoretical calculations, are shown in Fig. 1. In this case the theoretical distributions for ^{90}Sr (546 keV) and ^{90}Y (2274 keV) are evaluated separately and then added together.

Another source which has been used[19] is ^{99}Tc. It is a low-end-point energy (292-keV) β source. Figure 2 shows the EB spectra of Cu, Mo, Ag, Cd, and Pb produced by the β particles of ^{99}Tc along with the four theoretical distributions.

In the case of ^{90}Sr-^{90}Y the error due to counting statistics is less than 2% up to 600 keV, less than 8% from 600 to 1000 keV, less than 12% up to 1500 keV, less than 16% up to 1800 keV, and reaches 21% at 2000 keV. The error involved in the estimation of crystal detection efficiency varies from 1% at 200 keV to about 6% at 2000 keV. The error due to Compton electron distribution correction is from 1% at 200 keV to 3% at 2000 keV. The error due to finite energy resolution correction is less than 2% throughout the energy region under consideration. Errors due to the correction for absorption in the Perspex beta stopper, the target, and the aluminum can of the crystal do not exceed 2% throughout the investigated energy region. The error in the determination of source strength is found to be 8%. Hence the overall error varies from 9% at 200 keV to about 24% at 2000 keV.

In the case of ^{99}Tc the error due to counting statistics is less than 1% at 40 keV and less than 12% at 200 keV. The error due to finite energy resolution correction is less than 2% throughout the energy region. The errors involved in the correction due to Compton electron distribution at 40, 100, and 200 keV are 0.5%, 1.0%, and 1.5%, respectively. The error due to crystal detection efficiency correction is found to vary from 1% at 40 keV to 5% at 200 keV. The error due to absorption correction is less than 2% throughout the energy region considered. The error due to source strength determination is 8%. Hence the overall error varies from 9% at 40 keV to about 16% at 200 keV.

In the case of EB of ^{90}Sr-^{90}Y produced in Cu, Mo, and Cd, the experimental results are in good agreement with the Pratt theory up to the energy of 1200 keV and show positive deviation thereafter. In the case of I and Pb the experimental values agree fairly with the Pratt theory up to 1160 and 1080 keV, respectively, and deviate positively thereafter. This positive deviation increases in general, with increasing photon energy and atomic number of the target material.

In the case of ^{99}Tc it can be observed that the experimental results are in good agreement with the EBH theory up to 130 keV for Cu, Mo, and Ag, up to 120 keV for Cd, up to 115 keV for Pb, and deviate positively thereafter. The positive deviation increases with increasing energy and atomic number.

The above results show that the spectra produced by the β particles of high-end-point energy agree mostly with the Pratt theory, whereas the spectra generated by a low-end-point energy β source agree mostly with the EBH theory. However, the general trend of increased deviation with increasing energy and increasing atomic number persists.

The deviations between experiment and theory, especially at the high-energy regions of the spectra, could be understood qualitatively as follows. The thick-target calculation of EB spectra assume isotropic production of bremsstrahlung, because in a thick target multiple collisions may be expected to smear out the angular dependence. But single radiative collisions of electrons in which all the energy is lost still retain angular dependence, especially at higher energies of the β spectrum. The emission in the forward direction increases as the energy of the electrons increase, i.e., as we go towards the end-point energy of the β spectrum. This in part must explain the observed excess in the higher-energy region of the spectra.

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