Propagation of a Gaussian wave packet in an absorbing medium

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Propagation of a Gaussian wave packet in an absorbing medium is examined in order to shed light on the physical basis of group velocity which exhibits a singular behavior. It is found that the velocity of the wave packet, defined as the traveling distance of the peak amplitude divided by its flight time, decreases in the absorption range of frequency, although the group velocity becomes infinite in the same range. Fast pulse propagation, which was observed by Chu and Wong and is characterized by a packet velocity faster than the light velocity, turns out to be a characteristic in the early stage of the flight and is understood in terms of packet distortion due to damping of Fourier-component waves in an anomalous dispersion region. It also turns out that slow pulse propagation characterized by a packet velocity less than the light velocity appears for long traveling distance. The results provided a unified picture of wave-packet propagation in an absorbing medium.

I. INTRODUCTION

A common understanding of wave-packet propagation in a dielectric medium is that the wave energy propagates with the group velocity as long as absorption is negligibly small.[1] In an absorbing medium, however, as was studied by Sommerfeld and Brillouin,[2] group velocity can exceed the light velocity and become infinite in the absorption range of frequency. Similar results have been reported for a plasma with an electron cyclotron absorption.[3,4] The singular behavior of group velocity demands a new physical insight into wave phenomena in an absorbing medium. In fact many studies have been reported on this subject.[5–9]

Recent experiments on semiconductors with absorption have provided two seemingly contradictory results. Ulbrich and Fehrenbach[10] studied propagation of a light pulse in a spatially dispersive medium and observed that the pulse velocity decreases in the absorption range of frequency (referred to as slow pulse propagation). On the other hand, Chu and Wong[11] observed that the maximum amplitude of the light pulse propagates with a velocity faster than the light velocity in a frequency-dispersive medium (fast pulse propagation), which seems to give experimental verification of Garrett and McCumber's[6] conclusion that under certain conditions the peak amplitude does propagate with the group velocity even when it exceeds the light velocity. Crisp[7] proposed the pulse distortion due to asymmetric energy absorption as a mechanism of the fast pulse propagation.

Problems of interest are to reveal whether spatial dispersion is responsible for the slow pulse propagation of a wave packet and whether group velocity is appropriate to describe propagation of a wave packet in an absorbing medium.

We consider a Gaussian wave packet in a dissipative Lorentz medium to examine the propagation of wave packet in an absorbing medium. To describe the wave packet, one has to obtain the asymptotic form of the Fourier integral, in which the integrand is a highly oscillating function. An expansion approximation commonly used assumes a narrow frequency spread of the spectrum. However, the integrand is a highly oscillating function and the contribution is not localized in the neighborhood of the central region of the spectrum because of resultant cancellations of the integrand. The basic assumption that the contribution is localized does not always hold and therefore this approximation is inappropriate for the present problem.

To make a breakthrough in such circumstances, we use the saddle-point method,[12] in which the oscillation of the integrand is suppressed by deforming the contour and one can evaluate the integral with desired accuracy. Moreover, this method has a great advantage that further assumption on the spectrum such as a narrow frequency spread is not needed for evaluations. Using this method, we can examine the propagation of a wave packet in an absorbing medium from a short range to a long range of traveling distance.

In this paper, we present the fundamental characteristics of wave-packet propagation in an absorbing medium. One of the major results is coexistence of the fast pulse propagation and the slow pulse propagation. The fast one is found to be a characteristic in a short traveling distance and the slow one in a long traveling distance. The transition from the fast stage to the slow stage is presented for the first time.

It also turns out that the group velocity does not describe the wave-packet propagation in a long traveling distance. The propagation velocity for an absorbing medium is derived within the framework of the saddle-point method and it is confirmed that there is good agreement with numerical results. The quantitative discussion for the validity of group velocity is presented and it is found that the group velocity is valid only in a short traveling distance.

We propose another mechanism for the fast pulse propagation based on spreading and damping of Fourier-component waves. The transition from the fast stage to the slow stage is explained by the damping of those components in the absorption range of frequency.

The present work provides a unified picture of wave-packet propagation in an absorbing medium from a short range to a long range of traveling distance.

In Sec. II an asymptotic form of a wave packet propagating in an absorbing medium is described with the aid of the saddle-point method. Numerical results are given in Sec. III, where the mechanism behind the fast pulse propagation is also presented. In Sec. IV, we discuss the results in comparison with other works.

II. FORMULATION

One-dimensional propagation of a Gaussian wave packet is studied in a uniform electron Lorentz gas. The refractive index, which characterizes the wave-packet propagation, is related to the dielectric coefficient of the medium as

$$n(\omega)=n'(\omega)+in''(\omega)=\sqrt{\epsilon(\omega)}\ , \tag{1}$$

$$\epsilon(\omega)=1+\frac{\omega_p^2}{\omega_0^2-\omega^2-2i\rho\omega}\ , \tag{2}$$

where ω_p and ρ are an electron plasma frequency and a collision frequency, respectively. The quantity ω_0 is a frequency of resonance at which an absorption takes place. Figure 1 shows the refractive index as a function of frequency. The medium exhibits anomalous dispersion, $\partial n/\partial\omega<0$, and is highly dispersive in the absorption range of frequency.

The profile of the wave packet at an arbitrary time and position is expressed by the inverse Fourier integral[13]

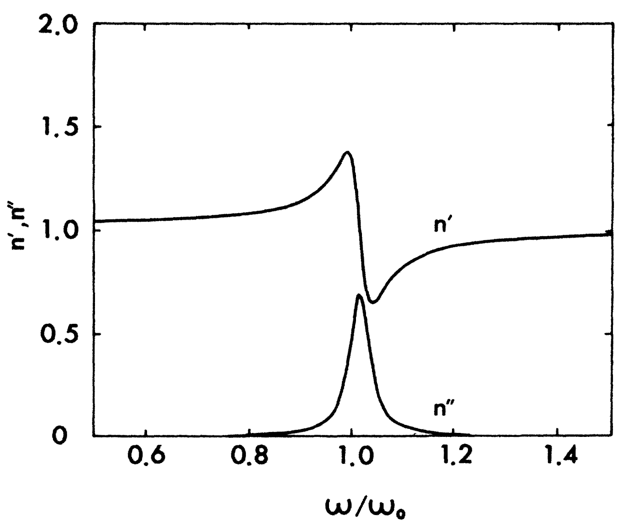

FIG. 1. Refractive index as a function of frequency. $\omega_p/\omega_0=0.25$, $\rho/\omega_0=0.02$.

$$U(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega\, e^{-i\omega t}[A(\omega)e^{i[n(\omega)\omega/c]x}+B(\omega)e^{-i[n(\omega)\omega/c]x}]\ . \tag{3}$$

Quantities $A(\omega)$ and $B(\omega)$ are related to the shape of the wave packet imposed at the origin and its derivative with respect to spatial coordinates;

$$\left.\begin{matrix}A(\omega)\\ B(\omega)\end{matrix}\right\}=\frac{1}{2}\frac{1}{\sqrt{2\pi}}\times\int_{-\infty}^{\infty}dt\, e^{i\omega t}\left[U(0,t)\mp\frac{ic}{n(\omega)\omega}\frac{\partial U(0,t)}{\partial x}\right]\ . \tag{4}$$

We will consider a Gaussian modulated wave packet given by

$$U(0,t)=e^{-t^2/2\Delta^2}\cos(\omega_c t)\ , \tag{5}$$

where Δ is the pulse width and ω_c is the frequency of the carrier wave.

Taking a right-going wave (positive x direction) in Eq. (3) and assuming $\partial U(0,t)/\partial x=0$ for simplicity, we have

$$U(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega\, A(\omega)e^{i[n(\omega)\omega/c]x-i\omega t}\ , \tag{6}$$

where $A(\omega)$ is given by

$$A(\omega)=\frac{\Delta}{4}(e^{-(\omega-\omega_c)^2\Delta^2/2}+e^{-(\omega+\omega_c)^2\Delta^2/2})\ .$$

Noting that $n(-\omega)=[n(\omega)]^*$, we find the second term of Eq. (6) to be the complex conjugate of the first. Then, $U(x,t)$ is finally given by

$$U(x,t)=\frac{\Delta/4}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega\, e^{-(\omega-\omega_c)^2\Delta^2/2}\times e^{i[n(\omega)\omega/c]x-i\omega t}+\text{c.c}\ . \tag{7}$$

We restrict our attention to the asymptotic behavior of the wave packet, which is obtained with the aid of the saddle-point method. This method is relevant to describe a wave packet propagating in a highly dispersive medium. To make use of this method $n(\omega)$ is analytically continued to a complex ω plane. Equation (7) is then rewritten as

$$U(x,t)=\frac{\overline{\Delta}/4}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\overline{\omega}\, e^{(x/x_0)P(\overline{\omega})}+\text{c.c.}\ , \tag{8}$$

where $\overline{\omega}=\omega/\omega_0$, $x_0=c/\omega_0$, and the $P(\overline{\omega})$ is given by

$$P(\overline{\omega})=i\left[n(\overline{\omega})\overline{\omega}-\overline{\omega}\frac{ct}{x}\right]-\frac{1}{2}\frac{x_0}{x}(\overline{\omega}-\overline{\omega}_c)^2\overline{\Delta}^2\ , \tag{9}$$

where $\overline{\omega}_c=\omega_c/\omega_0$, $\overline{\Delta}=\Delta\omega_0$. Since x/x_0 is the propagation distance normalized by the wavelength in the vacuum (except for the factor 2π) and is a large quantity, the asymptotic behavior of Eq. (8) is determined by the saddle points of $P(\overline{\omega})$, denoted $\overline{\omega}_s$, which are defined by

$$\left.\frac{\partial P}{\partial\bar{\omega}}\right|_{\bar{\omega}_s}=i\left[\frac{\partial n(\bar{\omega})\bar{\omega}}{\partial\bar{\omega}}-\frac{ct}{x}\right]-\left.\frac{x_0}{x}(\bar{\omega}-\bar{\omega}_c)\Delta^2\right|_{\bar{\omega}_s}=0\ . \tag{10}$$

The saddle points $\bar{\omega}_s$ are usually complex quantities and the path of integration is deformed so as to pass through the saddle points. After an integration, we obtain the asymptotic form of the wave packet:

$$U(x,t)=\sum_{\bar{\omega}_s}\frac{\bar{\Delta}/4}{\left[\frac{x}{x_0}\left|\frac{\partial^2 P}{\partial\bar{\omega}^2}\right|_{\bar{\omega}_s}\right]^{1/2}}e^{-(\bar{\omega}_s-\bar{\omega}_c)^2\bar{\Delta}^2/2}\times\exp\left[\frac{x}{x_0}i\left[n(\bar{\omega}_s)\bar{\omega}_s-\bar{\omega}_s\frac{ct}{x}\right]+i\alpha_s\right]+\text{c.c.}\ , \tag{11}$$

where α_s is the angle between the path of integration and the real axis at the saddle points $\bar{\omega}_s$. When more than one saddle point is present, Eq. (11) is considered as a summation with respect to the saddle points, which depends on the path of integration.

The saddle points are numerically determined by using Eqs. (1), (2), and (10). For the present case there are five saddle points in the complex ω plane. Three of them are in the right half of the plane and the other two are in the left half of the plane. The latter make no contribution to $U(x,t)$ and therefore can be omitted from consideration. Figure 2 shows the contour map of $\mathrm{Re}[P(\bar{\omega})]$ and the loci of the saddle points as a function of time. Figure 2(a) is for an absorption-free case, in which the carrier frequency ω_c is far from the characteristic frequency of absorption, and Fig. 2(b) is for an absorption dominant case. There is a branch cut just below the real axis, which comes from the dielectric property of the medium. The path of integration is so chosen that it passes through the appropriate saddle points from valley to valley on the contour map, which is indicated by a solid line. Fortunately, only one saddle point is enough to calculate $U(x,t)$ in the present cases. Contribution from the other saddle points is less than several percent, at most, in all the cases.

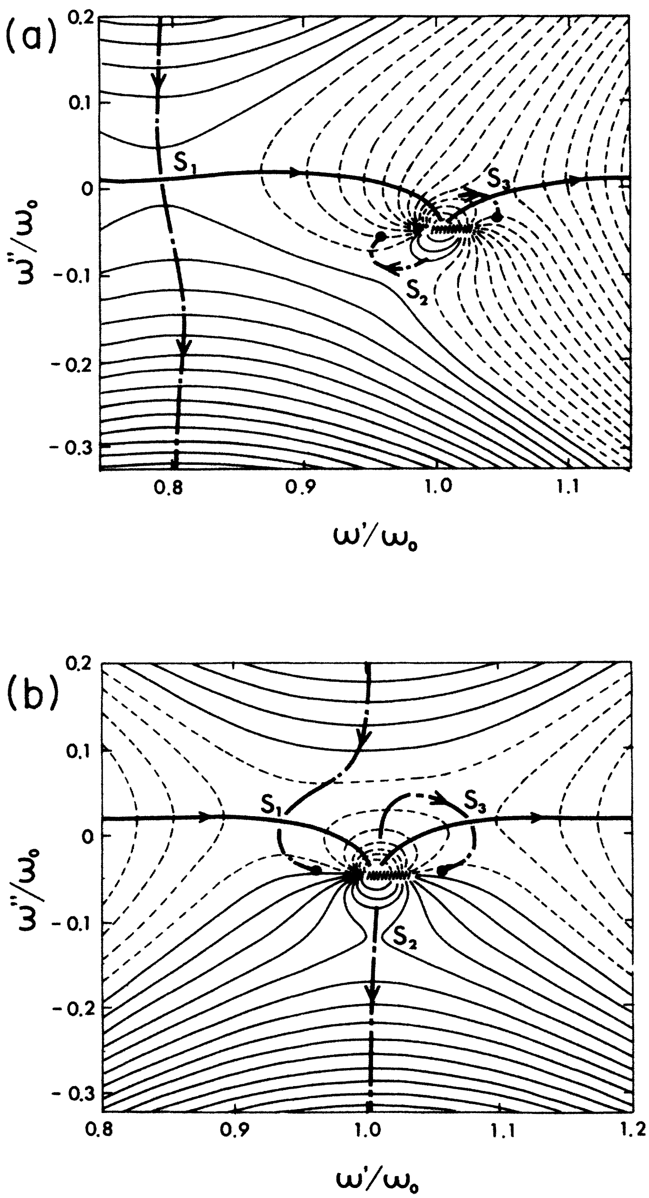

FIG. 2. (a) Contour map of $\mathrm{Re}[P(\bar{\omega})]$ for an absorption-free case. Dashed lines indicate contours of negative region (valley) and solid lines indicate contours of positive region (hill). S_1, S_2, and S_3 are saddle points. The hatched line indicates the branch cut. —•—, path of integration; —·—·—·, locus of saddle point as a function of time. (b) Contour map of $\mathrm{Re}[P(\bar{\omega})]$ for an absorption dominant case. —•—, path of integration; —·—·—·, locus of saddle point as a function of time.

III. RESULTS

Figures 3 and 4 show the time development of the wave packet (envelope) observed at a fixed position for different carrier frequencies ω_c. The abscissa is normalized time t/t_0 where t_0 is $1/\omega_0$, and the observation position is $x/x_0=120$. The maximum amplitudes are normalized to unity. The pulse width is $\Delta\omega_0=50$ for both figures. Figure 3 is for a moderate anomalous-dispersion case $[(\partial n/\partial\bar{\omega})/n\sim-10^1]$, and Fig. 4 is for a strong anomalous-dispersion case $[(\partial n/\partial\bar{\omega})/n\sim-10^2]$. When the carrier frequency is far from the frequency of absorption, the wave packet propagates without any deformation. On the other hand, when the carrier frequency is near the frequency of absorption, the wave packet takes a longer time to reach the observation point and exhibits the deformation from its initial waveform. The degree of profile deformation is stronger for the strong anomalous-dispersion case.

Our primary interest on wave-packet propagation in an absorbing medium is to know whether the group velocity properly describes the propagation of the wave packet or not. The packet velocity determined by the traveling distance divided by the flight time of the peak amplitude is plotted in Fig. 5. The group velocity calculated from the dispersion equation is also shown by a dashed line for comparison. As seen in the figure, the packet velocity in the absorption-free region coincides with the group veloci-

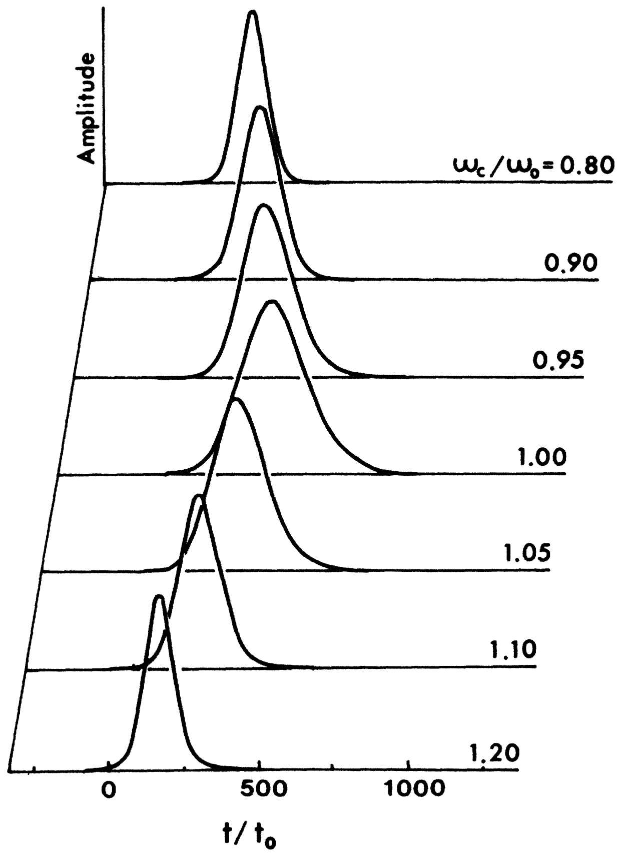

FIG. 3. Time development of wave packets. The maximum amplitudes are normalized to unity and the observation position is $x/x_0=120$. $\rho/\omega_0=0.02$, $\omega_p/\omega_0=0.25$.

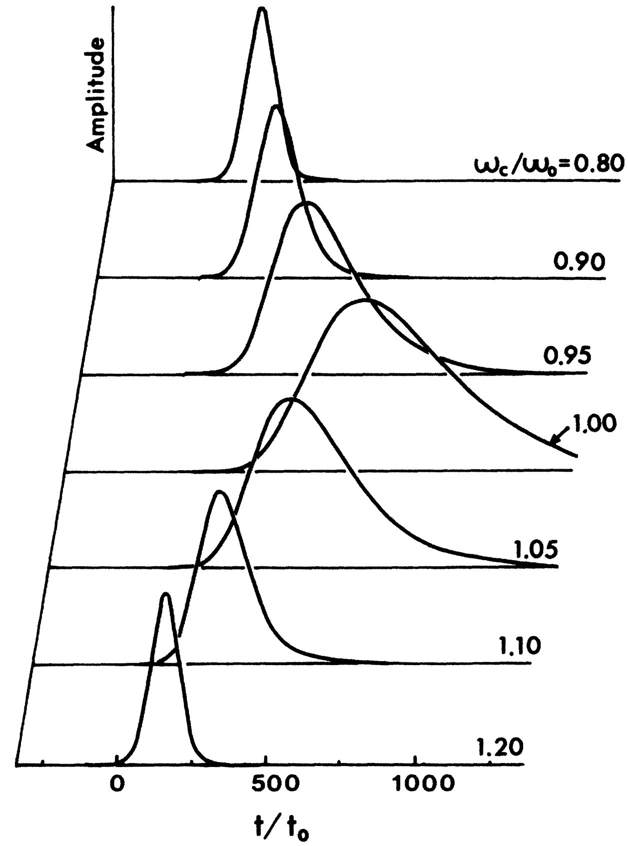

FIG. 4. Time development of wave packets. The maximum amplitudes are normalized to unity and the observation position is $x/x_0=120$. $\rho/\omega_0=0.005$, $\omega_p/\omega_0=0.25$.

ty, but it decreases down to the one-third of the light velocity in the absorption range of frequency, showing a remarkable deviation from the group velocity. This means that the group velocity is no longer the propagation velocity of the wave packet.

The variation of the peak amplitude as a function of propagation distance is shown in Fig. 6 for different carrier frequencies ω_c. When the frequency ω_c is far from the frequency of absorption, the amplitude exhibits an exponential damping (straight line). In the absorption range of frequency, however, the amplitude decays in a nonexponential manner particularly in a shorter distance. The results may be related to the packet deformation. Since the medium is highly dispersive in this frequency range, a decrease in amplitude caused by spreading of Fourier components, which is always accompanied by the packet deformation, takes place in addition to the pure damping due to absorption. Consequently, the amplitude indicates the nonexponential damping in the frequency range $\omega_c/\omega_0 \approx 1$.

It is interesting to note that the velocity of the peak amplitude is strongly affected by the deformation of the packet profile. One of these examples is shown in Fig. 7, in which the observation position is $x/x_0=30$ and an arrow indicates the traveling time of the light pulse in the vacuum. The peak amplitude propagates with a velocity faster than the light velocity (fast pulse propagation), which was experimentally observed by Chu and Wong.[11]

After propagating in a long distance, the packet velocity decreases to be less than the light velocity. Figure 8 shows the time development of the same wave packet ob-

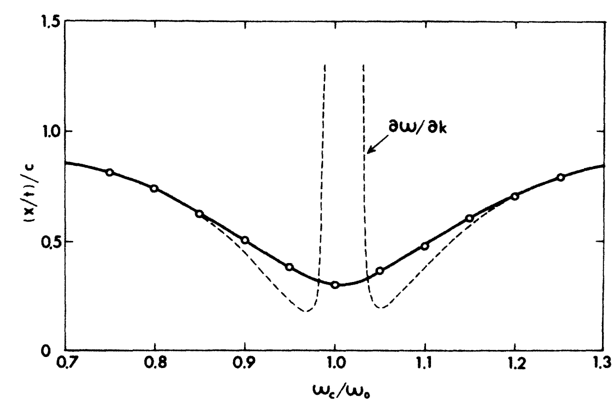

FIG. 5. Velocity of wave packet as a function of carrier frequency. The group velocity calculated from the dispersion equation is shown by the dashed line. The solid line indicates Eq. (20). The parameters are the same as in Fig. 3.

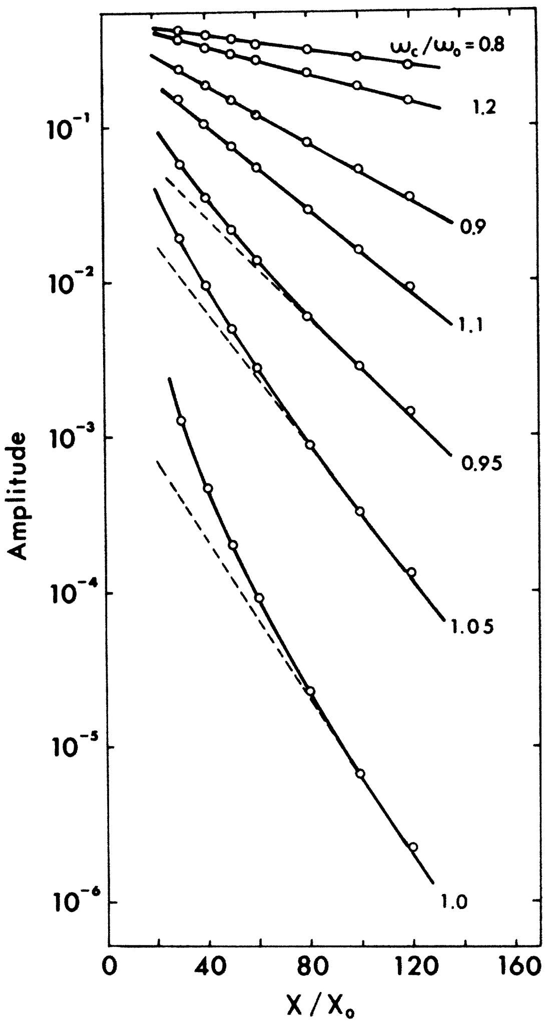

FIG. 6. Amplitude variation (maximum amplitude) as a function of propagation distance. The parameters are the same as in Fig. 3.

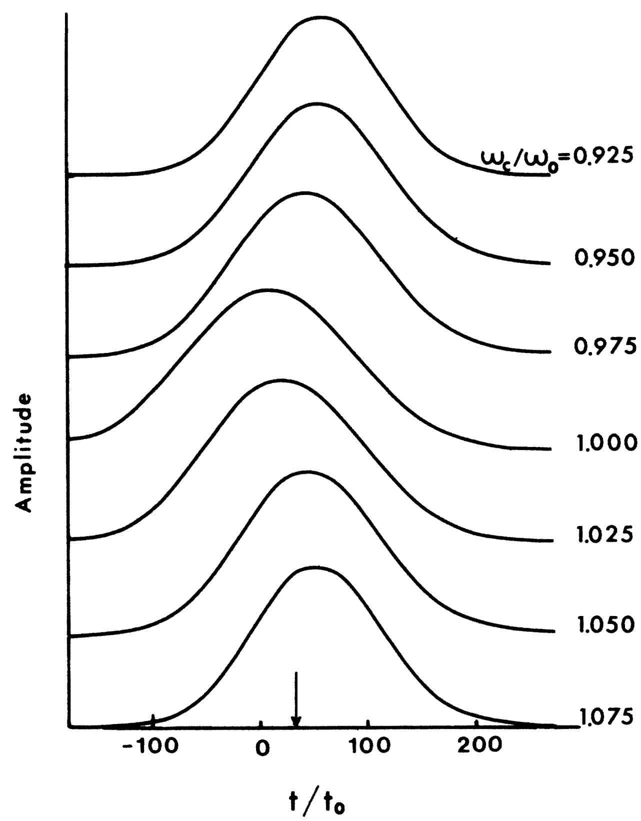

FIG. 7. Time development of wave packets (fast pulse propagation). The observation position is $x/x_0=30$. The arrow indicates the traveling time of a light pulse in the vacuum. $\Delta\omega_0=50$, $\rho/\omega_0=0.05$, $\omega_p/\omega_0=0.25$.

served at the position $x/x_0=120$, where the slow pulse propagation is clearly seen. This type of propagation was observed by Ulbrich and Fehrenbach[10] in a spatially dispersive medium. The important result of Figs. 7 and 8 is that both types of propagation are observed in one wave packet traveling in a long distance. This means that the two types of propagation are characteristics of the wave phenomena depending on the traveling distance. It should be also emphasized that the slow pulse propagation is observed in a medium without spatial dispersion, which shows that the spatial dispersion is not essential for the slow pulse propagation. The transition from the fast pulse propagation to the slow one is shown in Fig. 9 as a function of traveling distance. The packet velocity in the slow propagation region remains unchanged.

To make clear the mechanism of the fast pulse propagation, we will consider a wave packet with carrier frequency $\omega_c/\omega_0=1.0$. Figures 10 and 11 show the profile of the wave packet at different observation points and its spectrum obtained by fast Fourier transform (FFT). For both figures, the maximum amplitudes are normalized to unity. As seen in Fig. 10, the wave packet suffers a considerable distortion in the early stage of propagation, in which the peak amplitude shifts relative to the front end of the wave packet. After the observation point $x/x_0=30$, the packet is restored to a Gaussian-like profile. According to the distortion of the profile in a real space, the spectral distribution also changes with the traveling distance. In the early stage of propagation, the spectrum is asymmetric. After a certain traveling distance in which Fourier components in the absorption range fully die out, the spectral distribution becomes a Gaussian-like profile again (this means that the profile in a real space is also Gaussian-like profile). From these results the mechanism of the fast pulse propagation is considered to be as follows. Fourier components of the wave packet propagate with each phase velocity and tend to spread out during the propagation. In this process, the components in the absorption range of frequency suffer stronger damping than those in the normal dispersion region, resulting in an asymmetric profile of the spectrum. This causes the peak to shift towards the front end of the wave packet and therefore the peak amplitude seems to propagate faster than the light velocity.

The transition point from the fast stage to the slow

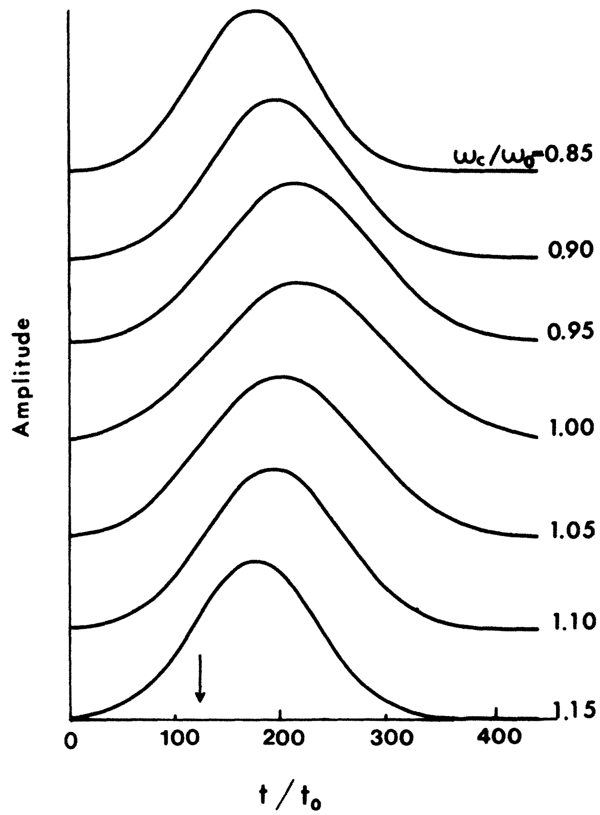

FIG. 8. Time development of wave packets (slow pulse propagation). The observation position is $x/x_0=120$. The arrow indicates the traveling time of a light pulse in the vacuum. The parameters are the same as in Fig. 7.

stage may be estimated as a decay length of those components in the absorption region:

$$|U_A/U_N| \lesssim 10^{-1}, \tag{12}$$

where U_A stands for Fourier-component waves in an anomalous dispersion region (absorption region) and U_N

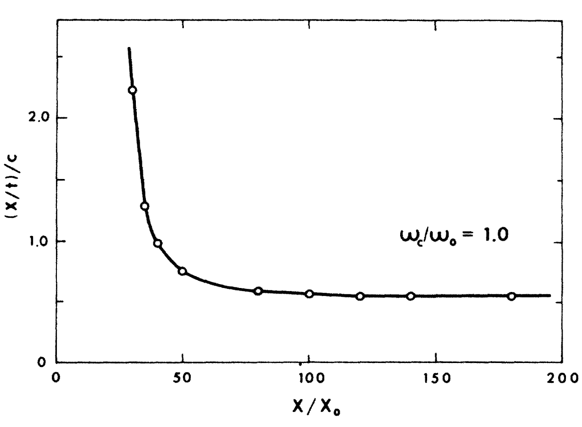

FIG. 9. Variation of packet velocity as a function of propagation distance. The carrier frequency is $\omega_c/\omega_0=1$. The parameters are the same as in Fig. 7.

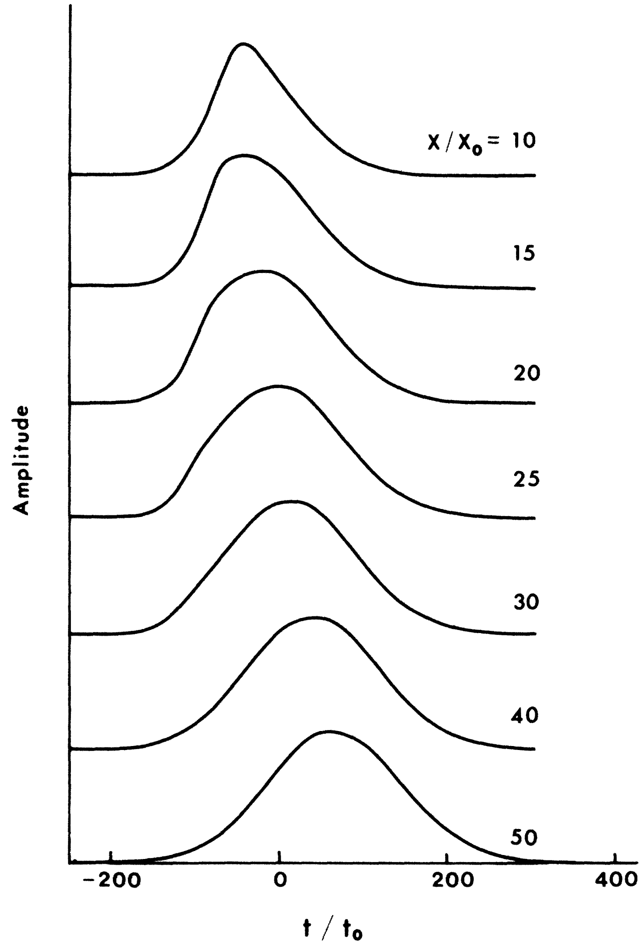

FIG. 10. Distortion of a wave packet. The carrier frequency is $\omega_c/\omega_0=1$. The parameters are the same as in Fig. 7.

for those in the normal dispersion region (see Fig. 12). Taking U_A at the center of the spectrum ($\omega=\omega_0$) and U_N at the edge of the normal dispersion region ($\omega=\omega_0+\rho$), we obtain

$$\left|\frac{U_A}{U_N}\right| \approx \frac{e^{-[n''(\omega_0)\omega_0/c]x}}{e^{-\Delta^2\rho^2/2}e^{-[n''(\omega_0+\rho)\omega_0/c]x}} \lesssim 10^{-1} \tag{13}$$

in which the damping of U_N components is also taken into account. Since the quantity ρ is the half width at half maximum of the absorption band, i.e., $n''(\omega_0+\rho)\approx n''(\omega_0)/2$, this condition is finally written by

$$-\rho^2\Delta^2+\frac{n''(\omega_0)\omega_0}{c}x \gtrsim 4.6 . \tag{14}$$

In the present case, the parameters are $\rho/\omega_0=0.05$, $\Delta\omega_0=50$, and $n''(\omega_0)\approx 0.3$, which gives the transition point:

$$x/x_0 \gtrsim 36 . \tag{15}$$

This agrees well with the result presented in Fig. 9 in which the packet velocity drastically changes to a low value, less than the light velocity, around $x/x_0=30$–40.

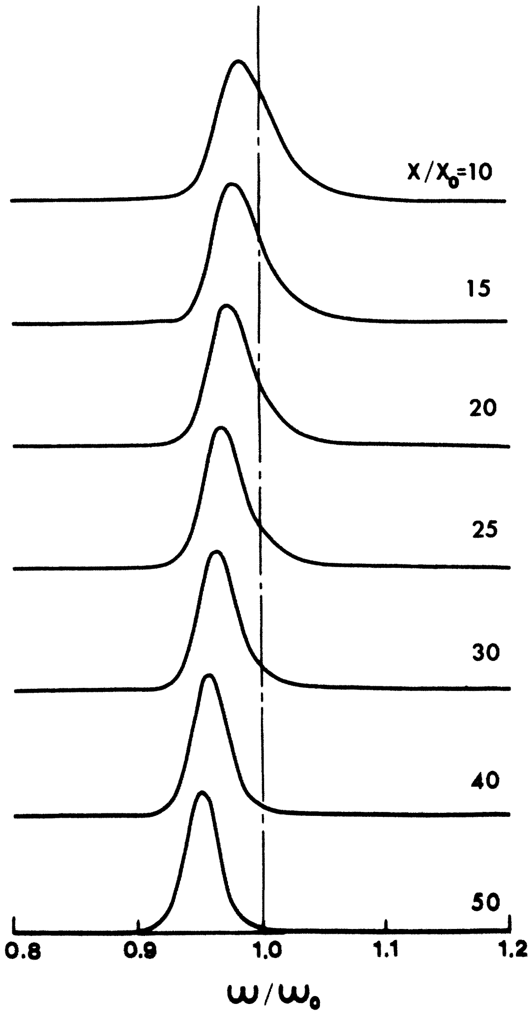

FIG. 11. Fourier spectra of a wave packet at different traveling distance. The maximum amplitudes are normalized to unity. The carrier frequency is $\omega_c/\omega_0=1$. The parameters are the same as in Fig. 7.

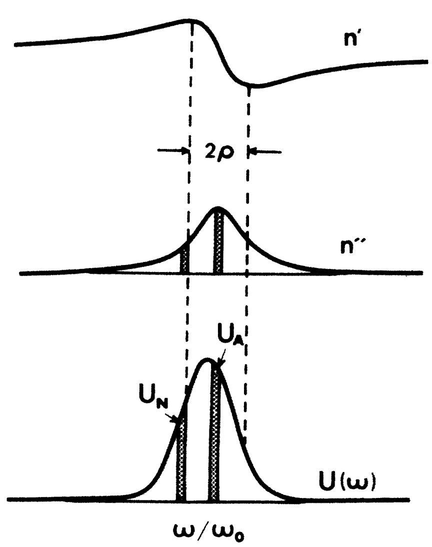

FIG. 12. Schematic of refractive index of an absorbing medium and a Fourier spectrum of wave packet with a carrier frequency $\omega_c/\omega_0=1$.

IV. DISCUSSION

We first discuss the propagation velocity of the wave packet within the framework of the saddle-point method. The saddle point corresponding to the peak amplitude is determined by the minimum of the exponent of Eq. (11):

$$\mathrm{Re}\left[\frac{\partial}{\partial t}P(\varpi_s)\right]=\mathrm{Re}\left[\frac{\partial P}{\partial t}+\frac{\partial P}{\partial \varpi_s}\frac{\partial \varpi_s}{\partial t}\right]=0\,. \tag{16}$$

Noting that $\partial P/\partial\varpi_s=0$, we obtain $\mathrm{Im}(\varpi_s)=0$ from Eq. (16), i.e., the peak amplitude corresponds to the saddle point on the real axis. Then the definition of the saddle point gives

$$\left.\frac{\partial n\varpi}{\partial\varpi}\right|'_{\varpi_s}-\frac{ct}{x}=0\,, \tag{17}$$

$$\left.\frac{\partial n\varpi}{\partial\varpi}\right|''_{\varpi_s}+\frac{x_0}{x}\bar{\Delta}^2(\varpi'_s+\varpi_c)=0 \tag{18}$$

for the peak amplitude, where a prime and a double prime stand for the real and imaginary parts, respectively.

When the absorption is negligibly small, $(\partial n\varpi/\partial\varpi)''_{\varpi_s}\approx 0$, Eq. (18) gives $\varpi_s=\varpi_c$. In this case, Eq. (17) gives the group velocity:

$$\frac{x}{t}=\frac{c}{\left.\dfrac{\partial n\varpi}{\partial\varpi}\right|'_{\varpi_c}}\,. \tag{19}$$

In the absorption dominant case, however, $(\partial n\varpi/\partial\varpi)''_{\varpi_s}\neq 0$ and the saddle point is not equal to the carrier wave frequency ϖ_c. Then, the pulse velocity is generally given by

$$\frac{x}{t}=\frac{c}{\left.\dfrac{\partial n\varpi}{\partial\varpi}\right|'_{\varpi_s}}\,. \tag{20}$$

This quantity usually differs from the group velocity and is a function of propagation distance through the saddle-point frequency ϖ_s which is a function of traveling distance. The solid line in Fig. 5 indicates Eq. (20) and there is a good agreement with the numerical results (open circles).

In a short-range limit such that $(x_0/x)\bar{\Delta}^2\gg 1$, Eq. (18) gives $\varpi_s\approx\varpi_c$ and Eq. (20) may become the group velocity. However, the above condition is necessarily violated as the packet travels in a long distance $(x_0/x)\bar{\Delta}^2\lesssim 1$, in which the pulse velocity is again given by Eq. (20). The group velocity is valid only in a short traveling distance. Chu and Wong[11] observed that the pulse propagates with the group velocity in a frequency-dispersive medium. Since the parameter $(x_0/x)\bar{\Delta}^2$ of the experiment is estimated to be of the order of 10^6 (10^1 in our case), their experiment corresponds to this short-range limit. The result is quite consistent with our theoretical result.

Crisp[7] proposed the pulse distortion as the mechanism of the fast pulse propagation: more energy is absorbed from the rear half of the pulse than from the front half because of finite response time of the medium. The essential feature of this mechanism is that it is independent of the traveling distance. So, this effect always acts on the wave packet during the propagation. As seen in Fig. 9, however, the pulse velocity changes with the propagation distance and the slow pulse propagation appears in a long traveling distance. This implies that there is another distortion mechanism which dies out beyond the certain distance. We propose the spreading of Fourier component waves as a mechanism of the fast pulse propagation. The transition from the fast stage to the slow stage is explained by the damping of component waves.

It might be worth pointing out the validity of group velocity for a nonabsorbing medium. The group velocity is usually derived by expanding the exponent of the Fourier integral around the carrier-wave frequency (expansion approximation). As discussed above, the saddle-point frequency is equal to the carrier-wave frequency in the absorption-free case. In this condition, there is no difference between the saddle-point method and the expansion approximation as far as peak amplitude is concerned. The validity of group velocity for a nonabsorbing medium is again assured by the saddle-point method.

The slow pulse propagation[10,14–16] has been observed only in a spatially dispersive medium. In this paper we present the coexistence of the fast and the slow pulse propagation in a frequency-dispersive medium. The fast one is a characteristic of short traveling distance and the slow one in a long traveling distance. This result strongly suggests that both types of propagation are fundamental characteristics of wave-packet propagation in an absorbing medium and therefore could be found at the same time in a frequency-dispersive medium.

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