Efimov effect and spin-polarized hypernuclear-atom systems

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Evidence is cited which supports the belief that Efimov states should exist in trimers of spin-polarized Λ and Σ-hypernuclear tritium. Macroscopic samples of the two hypernuclear-atom systems should also manifest most of the properties of helium.

More than 15 years ago, Efimov discovered a phenomenon in three-boson systems wherein a multitude of bound states (later labeled "Efimov states") appears should the assumed extant pairwise interaction be just below "resonance" or threshold.[1] Indeed for an interaction at threshold itself (so that the two-body binding energy is zero) the number of these states is infinite. When the interaction strength grows beyond its critical value, just as quickly as in their appearance, these three-body bound states disappear into the continuum. Efimov also derived a simple expression involving the scattering length a and effective range r_0 from the pairwise interaction which, when evaluated, can signal the likelihood of these states and their number. In particular his expression has the form

$$N = \ln(|a|/r_0)/\pi , \quad (1)$$

where N is the number of bound Efimov states expected from the given interaction. Subsequent to Efimov's work and the paper of Amado and Noble which clarified the reasons for this "Efimov effect,"[2] efforts were launched to determine if there exists a physical system which would manifest these loosely bound states.[3–5] The nucleus ^{12}C, pictured as three α particles, was first suggested as a possibility. This was followed by studies on the $(^4\mathrm{He})_3$ trimer and the trimer of spin-polarized tritium $^3\mathrm{H}(\downarrow)$, the latter belonging to the class of "spin-polarized quantum systems" some of whose members have been predicted to possess novel macroscopic properties.[6–8] Of the three, it appeared that the most likely to harbor the Efimov states was the ^{4}He trimer. For a coterie of realistic ^{4}He interatomic potentials, Lim *et al.*, found that the extracted values of N straddled and were close to unity. Detailed Faddeev calculations confirmed that there should be one or even two Efimov states in $(^4\mathrm{He})_3$.[3,4] Later work by Maeda and Lim, and more recently, by Cornelius and Glockle with improved numerical techniques and a more accurate ^{4}He-^{4}He potential disagree on whether the formula in Eq. (1) holds true.[9–11] This is understandable because Efimov-state calculations are extremely demanding of numerical accuracy and sensitive to any approximations made. Maeda and Lim believed that the earlier Faddeev estimates gave one too many for the number of these states, i.e., that a regular excited state had crept into the count, and that the ^{4}He trimer with the Aziz potential does not hold an Efimov state. Cornelius and Glockle's work on the same system, which is thought to be more reliable, shows otherwise; the purported permanent excited state does indeed disappear but with the diatomic potential far from threshold, just as expected for an Efimov state.

The Cornelius-Glockle work not withstanding, it may still transpire that other currently accepted ^{4}He-^{4}He potentials, though close to threshold, are nevertheless too much on the overbinding side and will not support Efimov states in $(^4\mathrm{He})_3$.[12] We may yet lose the one physical system thought to hold the clearest chance of showing the Efimov effect. In this paper, I wish to report that there are nevertheless two others able to replace $(^4\mathrm{He})_3$ should that happen. Not unexpectedly, both are spin-polarized systems. They are the trimers of the two spin-polarized hypernuclear tritium atoms $^3_\Lambda\mathrm{H}(\downarrow)$ and $^3_\Sigma\mathrm{H}(\downarrow)$ which, given the advance of modern technology, should soon be realized experimentally. The hypernuclei exist (e.g., $^3_\Lambda$H is stable against strong-interaction decay, has a lifetime of the order of 10^{-10} sec, and has a total binding energy of 2.355 ± 0.05 MeV, while $^3_\Sigma$H is much less stable and should have a binding energy near that of $^3_\Lambda$H) (Ref. 13) and are obtained by replacing a neutron (mass 939.6 MeV) in the tritium nucleus by the Λ-hyperon Λ (mass 1115.6 MeV) and Σ-hyperon Σ (mass 1193 MeV), respectively. Each of the hyperons is neutral and has spin $\frac{1}{2}$ just like the neutron and thus preserves the bosonic characteristics of the spin-polarized $^3\mathrm{H}(\downarrow)$ system as well as the spin-triplet interaction between the atoms in a strong magnetic field. The only change experienced "macroscopically" from each of the two replacements is a change in the mass of the atoms in the system. The hypernuclear tritium atoms are like tritium atoms only more heavy; we have found a way to "tune" the mass of spin-polarized tritium in the same way that $^3\mathrm{H}(\downarrow)$ is itself a mass-tuned $\mathrm{H}(\downarrow)$. How does this bear on the existence of Efimov states in the trimers of these spin-polarized hypernuclear systems? Earlier,[5,14] we found that the $^3\mathrm{H}(\downarrow)$-$^3\mathrm{H}(\downarrow)$ interaction (chosen to be the Lennard-Jones 6-12 potential of Ref. 15) is close to threshold, within 3% in η (the de Boer parameter) whose computed value for $^3\mathrm{H}(\downarrow)$ is 0.1846 (η_c, the threshold value, is 0.1797). At the same time, for this potential and this system, $a = -130$ Å, $r_0 = 12$ Å, and $N = 0.758$ (see Table I). If we can increase the mass involved we will bring down the value of η; a suitable increase of m will push η closer towards η_c, cause a consequent jump in the value of N beyond 1 and the appearance of Efimov states in the trimer. This does not happen

TABLE I. Values of the two-body binding energy E_2, scattering length a, effective range r_0, the Efimov number N, and the ratio L for $^3H(\downarrow)$, $^3_\Lambda H(\downarrow)$, and ^4He for various potentials.

Potential	System	E_2 (K)	a (Å)	r_0 (Å)	N	L
Nosanow	$^3H(\downarrow)$		−130	12	0.76	1.027
(Ref. 15)	$^3_\Lambda H(\downarrow)$	1.06(−3)	134	11	0.80	0.965
($\eta_c=0.1797$)						
Kolos-Wolniewicz	$^3H(\downarrow)$		− 43.2	12.3	0.40	1.080
(Refs. 16 and 17)	$^3_\Lambda H(\downarrow)$		−264	11	1.01	1.014
($\eta_c=0.17094$)						
Aziz	^4He	0.83(−3)	122	7.39	0.89	0.979
(Ref. 11)						
Feltgen	^4He	0.46(−3)	164	7.41	0.986	0.979
(Ref. 12)						
de Boer	^4He		−280	8	1.13	1.010
(Ref. 14)						

for either $^3_\Lambda H(\downarrow)$ or $^3_\Sigma H(\downarrow)$ with this potential. The change in mass wrought by the hyperons swings the de Boer parameter well beyond threshold, further away from it than for $^3H(\downarrow)$. However, this potential is not the realistic spin-triplet $H(\downarrow)$-$H(\downarrow)$ or $^3H(\downarrow)$-$^3H(\downarrow)$ interaction. That designation is accorded the Kolos-Wolniewicz potential for which Uang and Stwalley have manufactured a piecewise cubic-spline fit that they used subsequently to extract the relevant atom-atom scattering parameters for spin-polarized hydrogen and its isotopes.[16,17] Borrowing liberally from Uang and Stwalley's detailed computations and using their numerical techniques, I have generated the entries on Table I. I found that L ($=\eta/\eta_c$) for $^3_\Lambda H(\downarrow)$ is 1.014, i.e., $^3_\Lambda H(\downarrow)$-$^3_\Lambda H(\downarrow)$ is only 1.4% from threshold with N hovering near 1. Likewise, $^3_\Sigma H(\downarrow)$-$^3_\Sigma H(\downarrow)$ is 1.2% from the threshold on the bound side. These are the smallest fractions away from threshold yet encountered in the potentials of physical systems. (The de Boer potential for ^4He-^4He listed on Table I is only 1% from threshold, and does lead to an Efimov state but is not now regarded as a realistic interaction.) Thus trimers of the two spin-polarized hypernuclear atoms $^3_\Lambda H(\downarrow)$ and $^3_\Sigma H(\downarrow)$ are now the clearest candidates for the manifestation of the Efimov effect although the fact that $^3_\Sigma H(\downarrow)$ is much less stable makes the latter system further from the realm of possibility. So, we can note one more remarkable phenomenon to be expected in spin-polarized quantum systems. In concluding it also bears remarking that a majority of the characteristics of collections of helium atoms should apply to the two hypernuclear systems.[9,18] Such topics will be covered in a future report.

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