Second-harmonic generation by coalescence of two counterpropagating surface waves in a laser-produced plasma

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A mechanism is proposed in which second-harmonic emission is generated due to the coalescence of two surface waves counterpropagating along the critical surface of a laser-produced plasma. The waves are resonantly excited by a laser beam focused upon the inhomogeneous plasma whose density profile is steepened in the vicinity of the critical density. The necessary condition for this mechanism to operate is, however, the presence of a dc magnetic field in the critical region.

I. INTRODUCTION

Second-harmonic emission from a laser-produced plasma is a very powerful tool for diagnosing the critical region where the electron plasma frequency equals the frequency of the heating radiation. This emission results (see, e.g., Ref. 1) from coalescence of either a transverse (incident electromagnetic) and a longitudinal (plasma) wave or two longitudinal waves. The plasma waves are generated by the incident heating wave which (if some necessary requirements are met) can decay into a plasma wave and an ion-acoustic wave (the parametric decay instability) and/or by linear conversion (resonance absorption).

Under certain circumstances, which are formulated in this paper, a plasma density profile steepened by the ponderomotive force can also support so-called surface waves. These are localized in a vicinity of the density step and propagate along the critical surface. The presence of a weakly inhomogeneous plasma allows an obliquely incident p-polarized electromagnetic wave to excite these surface waves. In conditions when laser radiation is focused upon a target with the beam axis normal to the target surface, the two specular incident "rays" excite two counterpropagating surface waves. In the absence of a dc magnetic field these waves drive second harmonic currents which are exactly in opposite phase and, therefore, the net second harmonic current vanishes. However, if a dc magnetic field normal to the plane of incidence is present the two counterpropagating surface waves have different dispersion properties and also different damping rates due to linear conversion into the upper hybrid modes. As a consequence, a net second harmonic current arises and this is accompanied by second harmonic emission. In what follows the electromagnetic quantities characterizing the two counterpropagating surface waves are evaluated in terms of the specularly incident pump waves and the relevant dispersion functions. The latter contain all the characteristics of the plasma density step. Then, the second-harmonic response function is calculated which represents a source of the second-harmonic emission. Finally, the intensity of this emission and the corresponding coefficient of conversion are calculated.

II. FORMULATION OF THE PROBLEM

Let us consider an inhomogeneous plasma with a steep density gradient in a vicinity $0 \leq x \leq a$ of the critical density (see Fig. 1). Further, let us assume the conditions when:

(a) there are two p-polarized electromagnetic waves (e.g., from a normally incident focused beam) that are specularly incident (with respect to the plasma density gradient) upon the plasma with the angle of incidence θ such that $\sin^2\theta > \epsilon_1$ where ϵ_1 is the plasma permittivity ϵ at $x=0$;

(b) a weak dc magnetic field $\mathbf{H}_0 = (0,0,H_0(x))$ is present in the transition regime $0 \leq x \leq a$ $(\Omega/\omega \ll 1$, $\Omega = eH/mc$, e,m are the electron charge and mass, and c is the speed of light);

(c) the plasma density step is such that it allows for the presence of leaky surface modes, i.e.,

$$0 < \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} < 1$$

where $\epsilon_2 = \epsilon\,(x=a)$;

(d) the size of the transition region is such that $ka \ll 1$ where k is the wave number of the surface waves

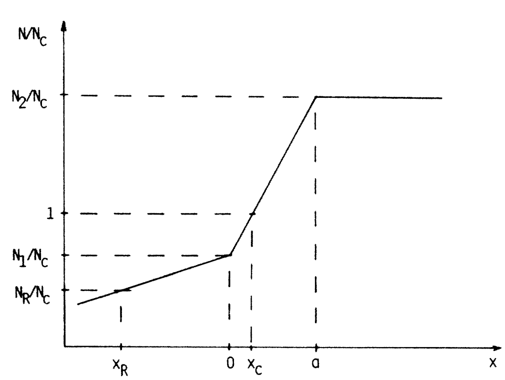

FIG. 1. A schematic diagram of a plasma density profile supporting surface waves.

resonantly excited by the incident waves;

(e) the underdense plasma ($N < N_1$) is weakly inhomogeneous, i.e., $k_0 L_R = k_0 N_R (dN/dx)_{N_R}^{-1} \gg 1$ where N_R is the plasma density at the turning point $\epsilon_R = \sin^2\theta$, $k_0 = \omega/c$, and ω is the frequency of the incident radiation.

Emission of the second harmonic arising from coalescence of two counterpropagating surface waves is then governed by the following wave equation:[2]

$$\frac{d^2 E_{2y}}{dx^2} + k_2^2 E_{2y} = R(x) , \tag{1}$$

where

$$k_2 = 2k_0 \left[1 - \frac{v(1-v/4)}{4-u^2-v} \right]^{1/2} ,$$

$u = \Omega/\omega$, $v = N(x)/N_c$, and

$$R(x) = -\frac{\Omega}{c^2} \frac{v}{4-u^2-v} 4\pi j_{2x} - i\frac{2\omega}{c^2} 4\pi j_{2y} \tag{2}$$

is the source function,[2] and the components of the second harmonic current j_{2x} and j_{2y} have the following form:

$$j_{2x} = -i\frac{e}{4\pi m\omega^3} \left[\frac{i\omega E_x^+ - \Omega E_y^+}{1-u^2} \frac{d}{dx} \left[\frac{v}{1-u^2} (i\omega E_x^- - \Omega E_y^-) \right] + \frac{i\omega E_x^- - \Omega E_y^-}{1-u^2} \frac{d}{dx} \left[\frac{v}{1-u^2} (i\omega E_x^+ - \Omega E_y^+) \right] \right] + \frac{eN}{\omega^2} \frac{i2\omega F_x - \Omega F_y}{4-u^2} ,$$

$$j_{2y} = -i\frac{e}{4\pi m\omega^3} \left[\frac{i\omega E_y^+ + \Omega E_x^+}{1-u^2} \frac{d}{dx} \left[\frac{v}{1-u^2} (i\omega E_x^- - \Omega E_y^-) \right] + \frac{i\omega E_y^- + \Omega E_x^-}{1-u^2} \frac{d}{dx} \left[\frac{v}{1-u^2} (i\omega E_x^+ - \Omega E_y^+) \right] \right] + \frac{eN}{\omega^2} \frac{\Omega F_x + 2i\omega F_y}{4-u^2} .$$

Finally, the functions F_x and F_y are defined as follows:

$$F_x = \frac{e^2}{m^2\omega^2(1-u^2)} \left[-\frac{i}{\omega}(\Omega E_x^+ - i\omega E_y^+) \frac{dE_y^-}{dx} - (i\omega E_x^+ - \Omega E_y^+) \frac{d}{dx} \frac{i\omega E_x^- - \Omega E_y^-}{\omega^2(1-u^2)} - \frac{i}{\omega}(\Omega E_x^- - i\omega E_y^-) \frac{d}{dx} E_y^+ - (i\omega E_x^- - \Omega E_y^-) \frac{d}{dx} \frac{i\omega E_x^+ - \Omega E_y^+}{\omega^2(1-u^2)} \right] ,$$

$$F_y = \frac{e^2}{m^2\omega^2(1-u^2)} \left[\frac{i}{\omega}(i\omega E_x^+ - i\Omega E_y^+) \frac{d}{dx} E_y^- - (i\omega E_x^+ - \Omega E_y^+) \frac{d}{dx} \frac{\omega E_x^- + i\omega E_y^-}{\omega^2(1-u^2)} + \frac{i}{\omega}(i\omega E_x^- - \Omega E_y^-) \frac{d}{dx} E_y^+ - (i\omega E_x^- - \Omega E_y^-) \frac{d}{dx} \frac{\Omega E_x^+ + i\omega E_y^+}{\omega^2(1-u^2)} \right] .$$

The components $E_x^\pm, E_y^\pm$ of the electric vectors of the counterpropagating surface waves are solutions to the following set of equations:

$$\frac{d}{dx} \left[\frac{\epsilon}{n^2-\epsilon} \frac{dE_y}{dx} \right] - k_0^2 \left[\epsilon + \frac{g^2}{n^2-\epsilon} - \frac{n^2}{k} \frac{d}{dx} \left[\frac{g}{n^2-\epsilon} \right] \right] E_y = 0 , \tag{3}$$

$$E_x = -\frac{ig}{n^2-\epsilon} E_y - \frac{in}{(n^2-\epsilon)k_0} \frac{d}{dx} E_y , \tag{4}$$

where $n = k/k_0$ and

$$g = \frac{\Omega}{\omega} \frac{\omega_p^2}{\omega^2 - \Omega^2} ,$$

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2} .$$

The $\pm$ waves then correspond to opposite signs of k and, hence, n.

Our aim is to find the in-vacuum amplitude of the electric field

$$E_{2y} = \frac{e^{ik_2 x}}{2ik_2} \int_{-\infty}^{\infty} R(x) e^{-ik_2 x} dx \tag{5}$$

associated with the second harmonic emission out of the plasma.

III. THEORY

Since a dc magnetic field H_0 present in the critical region is assumed to be weak ($u \ll 1$), the second harmonic response (2) of the plasma to the field $\mathbf{E},\mathbf{H}$ at the fundamental frequency can be substantially simplified to the form

$$R(x)\simeq -\frac{2e}{mc^2}\left\{\left[E_y^+\frac{d}{dx}\left[\frac{N}{N_c}E_x^-\right]+E_y^-\frac{d}{dx}\left[\frac{N}{N_c}E_x^+\right]\right]\right.$$
$$\left.+k_0O(u)(E_y^+E_x^-+E_y^-E_x^+)\right\}, \qquad (6)$$

where $O(u)\approx u \ll 1$. In absence of the dc magnetic field H_0 the second harmonic currents corresponding to the two counterpropagating waves are exactly in opposite phase, i.e., both terms in square and round brackets vanish and $R(x)|_{H_0=0}=0$ (see also, e.g., Ref. 3). However, it can be shown that these terms contain contributions of the first order of magnitude in the small parameter u.

Consequently, the last term in (6) is neglected being of second order of magnitude in $O(u^2)$. Equation (3) is solved by using the Wentzel-Kramers-Brillouin (WKB) approximation far from the turning point $x \ll x_R$, then in terms of the Airy functions in the intermediate region[4] $x<0$ and, finally, by an iterative procedure[5] (with a small parameter $ka \ll 1$) in the transition region $0<x<a$. The solution thus obtained is matched to an evanescent wave for $x \geq a$ and to the incident wave at $x \to -\infty$. Thus, in the transition region $0<x<a$, where the source of the second harmonic emission is localized,

$$E_y(x)=E_y(0)\left[1-k_0 n\int_0^x \frac{g}{\epsilon}dx + k_0^2\frac{\epsilon_1}{\kappa_1}\int_0^x\frac{n^2-\epsilon}{\epsilon}dx\right]+F_1 , \qquad (7)$$

$$E_x(x)=-\frac{n\epsilon}{\epsilon^2-g^2}H(x)$$
$$=-i\frac{n\epsilon}{\epsilon^2-g^2}k_0\frac{\epsilon_1}{\kappa_1}E_y(0)\times\left[1+nk_0\int_0^x\frac{g}{\epsilon}dx-\frac{\kappa_1}{\epsilon_1}\int_0^x\frac{g^2}{\epsilon}dx\right]+F_2 , \qquad (8)$$

where $\kappa_1=k_0(n^2-\epsilon_1)^{1/2}$ and

$$E_y(0)=-\frac{1}{D}2ib\phi\left[k_0^2\frac{\epsilon_1}{\kappa_1}\int_0^a\left[1-\frac{n\kappa_1 g}{k_0\epsilon_1(n^2-\epsilon)}\right]dx-1\right]. \qquad (9)$$

Here

$$b=\exp\left[\int_0^{x_R}\kappa\, dx\right]$$

is the factor originating from the WKB approximation, $\kappa=k_0(n^2-\epsilon)^{1/2}$,

$$\phi=\left[\frac{\kappa_1}{k_0\epsilon_1}\cos\theta\right]^{1/2}E_0 ,$$

E_0 is the amplitude of the incident wave and

$$D=D_0+i(D_l+D_r) \qquad (10)$$

is the dispersion function of surface waves. The real part

$$D_0=1+\frac{\epsilon_1\kappa_2}{\epsilon_2\kappa_1} , \qquad (11)$$

with $\kappa_2=k_0(n^2-\epsilon_2)^{1/2}$ represents the dispersion function of a looseless surface wave propagating along a plasma density step. The imaginary terms represent losses of surfaces waves and, in particular,

$$D_l=\frac{b^2}{2}(2-D_0)$$

corresponds to leakage of energy from the surface wave into vacuum while

$$D_r=Imk_0^2\frac{\epsilon_1}{\kappa_1}\int_0^a\frac{n^2-\epsilon}{\epsilon}\left[1-\frac{n\kappa_1 g}{k_0\epsilon_1(n^2-\epsilon)}\right]^2dx$$

corresponds to resonance absorption, i.e., to a linear conversion of the surface modes into the upper hybrid modes in the vicinity of the critical point $\epsilon=0$.

The terms F_1 and F_2 in (7) and (8) are proportional to $b\phi$; however, they do not contain the dispersion function D. That means that our expressions for E_x and E_y split in two parts. One corresponding to the directly driven field by tunneling beyond the turning point and the other $\propto D^{-1}$ corresponding to resonant excitation of surface waves. Since we are interested in the case when the second harmonic is generated by coalescence of two counterpropagating surface waves we will keep only those parts of $E_x(x)$ and $E_y(x)$ which are $\propto D^{-1}$.

Once the amplitudes of the counterpropagating surface waves have been evaluated one can calculate the source function $R(x)$ for generation of the second harmonic. Since

$$\frac{1}{N_c}\frac{dN}{dx} \gg \frac{1}{E_{x,y}}\frac{d}{dx}E_{x,y} ,$$

the source term can be even further simplified to the following form:

$$R(x)\simeq -\frac{2e}{mc^2}(E_x^-E_y^+ + E_x^+E_y^-)\frac{1}{N_c}\frac{dN}{dx} . \qquad (12)$$

The term in the brackets does not vanish due to the fact that both the dispersion properties and the damping rates of counterpropagating surface wave differ[5] in the presence of a dc magnetic field H_0. Inserting (7) and (8) into (12) one obtains

$$R(x)=\frac{4e}{mc^2}\frac{1}{N_c}\frac{dN}{dx}E_y^+(0)E_y^-(0)\frac{i\epsilon}{\epsilon^2-g^2}\frac{\epsilon_1}{\kappa_1}k_0^2|n|^3\times\left[2-\frac{\kappa_1}{\epsilon_1}\int_0^x\frac{g^2}{\epsilon}dx-k_0^2\frac{\epsilon_1}{\kappa_1}\int_0^x\frac{n^2-\epsilon}{\epsilon}dx\right]\times\int_0^x\frac{g}{\epsilon}dx . \qquad (13)$$

One can immediately see that the source of the second harmonic is localized within the region where the dc magnetic field is present ($g\neq 0$). We repeat that it is the magnetic field that provides necessary asymmetry of the second harmonic currents.

The quantity of physical interest is the in-vacuum intensity of the emitted second harmonic radiation

$$I_{2x}=\frac{c}{8\pi}\mathrm{Re}(E_{2y}H_2^*) .$$

Inserting (13) into (5) one, after some manipulation, obtains

$$I_{2x}=-\frac{1}{2}\frac{(4\pi)^5e^2}{m^2c^3\omega^2}(k_0L)^2\left[\frac{\Omega_0}{\omega}\right]^2\frac{b^4}{|D^+D^-|^2}I_{0x}^2\ , \quad (14)$$

where we have retained only the lowest-order term. Here $L=N_c(dN/dx)^{-1}|_{N=N_c}$ and $\Omega_0=\Omega$ ($\epsilon=0$). I_{0x} is the component of the Poynting vector of the incident radiation along the plasma density gradient.

At optimum, i.e., when the counterpropagating surface waves are resonantly excited,

$$D_0=0\ ,$$

i.e.,

$$n^2=\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}\ .$$

Then,

(a) if $D_l \gg D_r$ ($D_l=b^2$),

$$I_{2x}\simeq-\frac{1}{2}\frac{(4\pi)^5e^2}{m^2c^3\omega^2}(k_0L)^2\left[\frac{\Omega_0}{\omega}\right]^2 I_{0x}^2\ , \quad (15)$$

i.e., the intensity of the second harmonic emission increases with increasing size of region of its generation ($L\propto a$);

(b) if $D_l \ll D_r$,

$$|D^+D^-|^2\simeq\left[k_0\frac{\epsilon_1}{\kappa_1}n^2\pi k_0L\right]^2[1+O((\Omega_0/\omega)^2)]\ .$$

That means that the case $D_r \gg D_l$ is equivalent to

$$k_0n^2\frac{\epsilon_1}{\kappa_1}k_0L \gg b^2\ .$$

Then

$$I_{2x}\simeq-\frac{1}{2}\frac{4^5\pi e^2}{m^2\omega^2c^3}\left[\frac{\epsilon_1+\epsilon_2}{\epsilon_1^2\epsilon_2^2}\right]^2\left[\frac{\Omega_0}{\omega}\right]^2\frac{b^4}{(k_0L)^2}I_{0x}^2\ . \quad (16)$$

The fact that now $I_{2x}\propto(k_0L)^{-2}$ has the following physical background: resonance absorption increases with increasing L as long as the distance between the turning point x_R and the point $x=0$ where $\epsilon=\epsilon_1$ remains constant ($b=$const).

To demonstrate the feasibility of this mechanism for generating second harmonic emission we will consider some real experimental conditions. For surface waves to exist a relatively high plasma density step across the critical is necessary. Such a step has been observed by, for example, Fedosejevs *et al.*[6] with characteristic parameters such as $\epsilon_2\simeq-8$, $\epsilon_1\simeq0.7$, and $I_{0x}\simeq10^{14}$ W/cm^2 (CO_2 laser). Since, according to Ref. 6, resonance absorption is assumed to be the dominant absorption mechanism, one can expect that $\Omega_0/\omega\simeq0.1$, (i.e., $H_0\simeq1$ MG). Further, taking $L\simeq\lambda_0/6$, i.e., $k_0L\simeq1$ and $b\simeq0.5$ one obtains a value for the conversion coefficient of

$$K_2\equiv I_{2x}/I_{0x}\simeq7\times10^{-5}\ .$$

This considerably exceeds that corresponding to generation of the second harmonic by coalescence of two counterpropagating surface waves in metallic films on a nonlinear substrate (see, e.g., Ref. 7).

At very high intensities the increasingly steep plasma density gradient in the critical region results in suppression firstly of the parametric processes and then of resonance absorption. On the other hand, the mechanism proposed here will increase in importance because firstly $K_2\propto I_{0x}$ and, secondly, D_r decreases (with increasing plasma density gradient at critical region).

IV. CONCLUSION

We have described a new mechanism for the generation of the second harmonic in a laser-produced plasma. This mechanism is an alternative to known mechanisms which are associated with the parametric processes and/or resonance absorption. The mechanism proposed is of increasing importance as the intensity of the heating radiation increases and eventually can even become dominant.

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