Gravity-wave detection via correlated-spontaneous-emission lasers

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We show that, in principle, an active gravity-wave detector based on a correlated-spontaneous-emission laser has potential advantages over the more usual passive scheme.

I. INTRODUCTION

The current generation of laser interferometer gravity-wave (g-wave) detectors[1] operate as passive phase-measuring devices [see Fig. 1(a)]. In that figure a laser source drives two arms of an interferometer and the phase difference between the light propagating in these two arms is a measure of the intensity of the gravitational radiation. The shot-noise limit of such a passive system yields a minimum detectable gravity wave given by

$$h_{\min}^{(p)} \simeq \frac{\gamma}{\nu} \sqrt{\hbar\nu / P t_m} \quad \text{(passive device, shot-noise limit)}, \qquad (1)$$

where γ is the cavity decay rate, ν is the laser frequency, $\hbar\nu$ is the energy per photon, P is the laser power, and t_m is the measuring time.

On the other hand, putting the lasing medium inside the cavities as in Fig. 1(b) yields a much more favorable sensitivity to gravitational radiation in the shot-noise limit. Such a device is said to be active because of the presence of an active medium inside the cavity. The shot-noise limited sensitivity of such a device is given by

$$h_{\min}^{(a)} \simeq \frac{\omega_g}{\nu} \sqrt{\hbar\nu / P t_m} \quad \text{(active device, shot-noise limit)}, \qquad (2)$$

where ω_g is the gravity-wave frequency.

The shot-noise limit of the active system is interesting in that it is (1) potentially more sensitive than the passive device, since ω_g may be much smaller than γ; (2) the envisioned device may be of laboratory (rather than kilometer) dimensions. In this context it is worth noting that a similar situation is encountered in ring laser gyroscope interferometry.[2] There when one makes a passive laser gyro device based on measuring the phase difference between the counterpropagating waves (with the laser external to the Sagnac interferometer) long path lengths are required. This is accomplished by winding a fiber-optical wave guide many times around the perimeter of the optical circuit. On the other hand, gyros based on measuring the frequency difference (with the laser inside the optical cavity) between the counterpropagating laser beams do not require the many windings approach and achieve high sensitivity with modest optical paths.

However, as discussed below, the two-laser active interferometer device of Fig. 1(b) is limited not by shot noise, but by the spontaneous emission noise associated with each of the independent lasers of that figure. The phase fluctuations associated with spontaneous emission lead to a minimum detectable gravity wave of amplitude

$$h_{\min}^{(a)} \simeq \frac{\gamma}{\nu} \sqrt{\hbar\nu / P t_m} \quad \text{(active device, spontaneous-emission limit)}, \qquad (3)$$

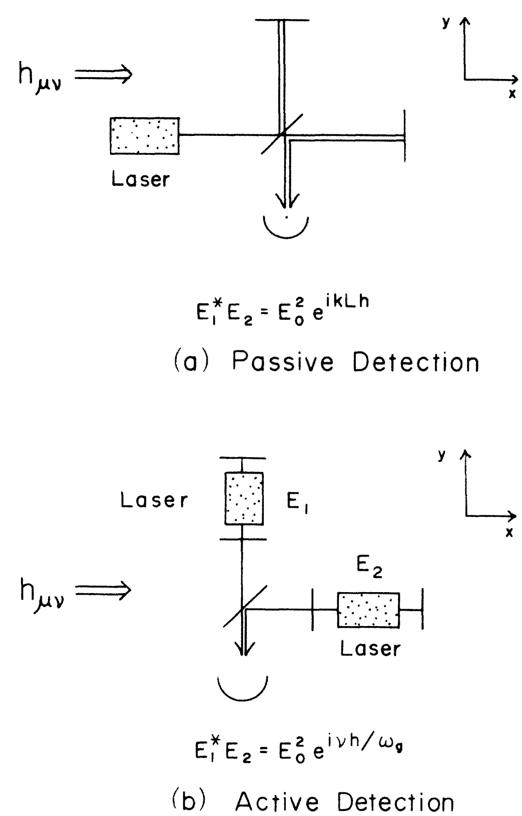


FIG. 1. (a) Passive system: an external laser drives a Michelson interferometer which is influenced by an incident g-wave denoted by $h_{\mu\nu}$, traveling in the positive x direction. (b) Active system: an incident g wave drives two independent lasers, which emit fields E_1 and E_2. These fields are heterodyned in order to observe the temporal beat note.

i.e., the same limit as that associated with the passive device, Eq. (1).

In recent research associated with correlated emission lasers (CEL), e.g., the quantum-beat laser,[7] it has been shown that uncertainty in the relative phase angle due to spontaneous emission can be eliminated. This device holds promise for a system in which the g-wave detector could operate as an active interferometer but without spontaneous emission noise. For such a system we find the sensitivity

$$h_{\min} \simeq \epsilon \frac{\gamma}{\nu} \sqrt{\hbar\nu / P t_m} \quad \text{(active detector, based on CEL)}, \tag{4}$$

where ϵ is a function of ω_g/γ. As discussed in Sec. IV, ϵ is around 10^{-2}, assuming reasonable values for ω_g and γ.

It is the purpose of this paper to inquire as to whether a "correlated-spontaneous-emission" g-wave detector might *in principle* improve the active system limit of Eq. (3). An example of such a scheme is offered herein. It is emphasized that there are many subtleties involving such a system, and the questions associated with measurement strategy for such a device require care in their analysis. It is hoped that this paper will provoke critical discussions, in particular concerning the extent to which such devices might be useful in future generation g-wave detectors. It is clearly worthwhile to investigate examples which challenge the accepted quantum-noise "limits" and sharpen our understanding of these limits.

II. PASSIVE VERSUS ACTIVE GRAVITY-WAVE INTERFEROMETERS

For a gravity wave propagating in the x direction the correction to the metric from flat space time may be written[3] as

$$h_{\mu\nu}(x,t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} h(x,t), \tag{5a}$$

where

$$h(x,t) = h_0 \cos(\omega_g t - k_g x). \tag{5b}$$

In the above, h_0 represents the strength of the gravity wave, ω_g its frequency, and k_g its wave vector. From the covariant Maxwell equations we obtain a modified wave equation for an electromagnetic signal propagating in the y and x directions of Fig. 1. Noting that the gravity-wave frequency is much smaller than the optical frequency we find

$$\frac{\partial^2 E_1}{\partial y^2} - \frac{1+h(x,t)}{c^2} \frac{\partial^2 E_1}{\partial t^2} = 0, \tag{6a}$$

$$\frac{\partial^2 E_2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E_1}{\partial t^2} = 0. \tag{6b}$$

Equations (6a) and (6b) lead to the following simple dispersion relations for the laser fields E_1 and E_2:

$$k_1^2 - \nu_1^2(1+h)/c^2 = 0, \tag{7a}$$

$$k_2^2 - \nu_2^2/c^2 = 0. \tag{7b}$$

We may view the above dispersion relations as implying a change in wave vector or frequency depending on whether the light source is taken as external or internal to the optical cavities under consideration. In the case of the laser external to the Michelson gravity wave detector [see Fig. 1(a)], Eqs. (7a) and (7b) yield

$$k_1 - k_2 = \frac{1}{2} \frac{\nu_1}{c} h_0 \cos(\omega_g t - k_g x_0), \tag{8}$$

while for the case of internal laser(s) [see Fig. 1(b)] the wave vector is viewed as constant and Eqs. (7a) and (7b) imply a frequency difference

$$\nu_1 - \nu_2 = \tfrac{1}{2} c k_1 h_0 \cos(\omega_g t - k_g x_0). \tag{9}$$

In the case of the passive device of Fig. 1(a), the signal due to a gravity wave translates into a phase shift obtained from Eq. (8) by multiplying by the effective path length $\tilde{L}$, which is essentially the number of bounces times the length of the arm L_1. In this case, the g-wave–induced phase shift is given by $\Delta\phi^{(p)} = 2\pi\tilde{L}h_0/\lambda$. In such an experiment the fundamental quantum limit is given by "photon shot noise." Denoting the average number of laser photons by $\bar{n}$, the power at the detector by P and assuming unit quantum efficiency for present purposes, one has the phase uncertainty due to shot noise for a measurement of duration t_m

$$\Delta\phi_n \simeq \frac{1}{\sqrt{\bar{n}}} = (\hbar\nu / P t_m)^{1/2}. \tag{10}$$

Equating $\Delta\phi^{(p)}$ to $\Delta\phi_n$ one finds the minimum detectable g-wave amplitude for such a passive system to be

$$h_{\min}^{(p)} \simeq \frac{\lambda}{2\pi\tilde{L}} \sqrt{\hbar\nu / P t_m} = \frac{\gamma}{\nu} \sqrt{\hbar\nu / P t_m}, \tag{11}$$

where we have introduced the cavity decay rate $\gamma = c/\tilde{L}$.

Consider next the active system of Fig. 1(a). The tiny frequency separation given by Eq. (9) implies a phase difference accumulated during a time t_m of magnitude

$$\Delta\phi_g \simeq \frac{\nu}{\omega_g} h_0 [\sin(\omega_g t_m - k_g x_0) - \sin(k_g x_0)]. \tag{12}$$

Now if we take the phase uncertainty to again be given by the shot-noise limit, equating (10) to (12) the minimum detectable gravity wave due to this active configuration is found to be[4]

$$h_{\min}^{(a)} \simeq \frac{\omega_g}{\nu} \sqrt{\hbar\nu / P t_m} \quad \text{(shot-noise limit)}. \tag{13}$$

This result differs from the passive result of Eq. (11) in two ways. First the sensitivity is improved by the factor $(\omega_g/\gamma)^{-1} \sim 10^3 - 10^4$. Furthermore we note that the sensitivity of the passive device goes as $\tilde{L}^{-1}$ and therefore large systems are implied in order to reach maximum sensitivity. No such dependence accrues for Eq. (13). This suggests that the active system may involve only laboratory (meter) dimensions.

However, in the analysis of an active device such as indicated in Fig. 1(b), we really have two independent lasers and there is therefore an additional source of frequency uncertainty due to the independent spontaneous emission fluctuations[5] of these two lasers. This implies an uncertainty in the frequency[6] between the two lasers of magnitude $\Delta\nu(\text{spont})\simeq\gamma\sqrt{\hbar\nu/Pt_m}$. Equating this frequency error to the g-wave—induced frequency difference given by Eq. (9) we find the spontaneous emission limit for the minimum detectable gravity wave signal

$$h_{\min}^{(a)}\simeq\frac{\gamma}{\nu}\sqrt{\hbar\nu/Pt_m}\quad\text{(spontaneous-emission limit)}\ . \tag{14}$$

We see that this result is the same as that given above in Eq. (11) and therefore the two systems have an identical sensitivity and dependence on cavity length.

In recent studies stimulated by the above considerations, we have investigated a method for quenching spontaneous emission noise in the relative phase angle between two laser signals.[7] We found that the spontaneous emission linewidth and associated uncertainty of the difference frequency between two laser modes could be eliminated by preparing the laser medium in a coherent superposition of two upper states (via, e.g., a microwave-field coupling $|a\rangle$ and $|b\rangle$) as in quantum-beat experiments, or by coherent pumping, as in the Hanle effect) as in Fig. 2. The heterodyne beat note of the radiation emitted from these states can, under the appropriate conditions, be freed from spontaneous-emission noise. We here apply these considerations to the detection of gravitational radiation. In the next section we develop the theory for such experiments and apply it to a possible experimental scheme.

III. THEORY OF A CORRELATED-SPONTANEOUS-EMISSION g-WAVE DETECTOR

Consider the g-wave detector of Fig. 2. There we depict a quantum-beat and/or Hanle laser driving a doubly resonant cavity having frequencies Ω_1 and Ω_2. The frequency Ω_2 is independent of the g wave, while the other cavity mode frequency Ω_1 is dependent on the gravitational radiation according to the relation

$$\Omega_1=\frac{n_1\pi c}{2L_1}[1-\tfrac{1}{2}h_0\cos(\omega_g t-k_g x_0)]\ . \tag{15}$$

Now the heterodyne cross term may be written in terms of amplitude ρ_i and phase $\theta_i(t)$ parameters as

$$\langle a_1^\dagger a_2\rangle=\rho_1\rho_2\langle e^{-i\theta(t)}\rangle e^{-i(\nu_1-\nu_2)t}\ , \tag{16}$$

where the relative phase difference $\theta(t)=\theta_1(t)-\theta_2(t)$ undergoes random fluctuations due to spontaneous emission and ν_i, $i=1,2$ is the actual lasing frequency of the ith mode having amplitude ρ_i. It is expression (16) which is of interest experimentally since the frequencies appearing therein are dependent on the g-wave amplitude through Eq. (15).

As in Ref. 7 we define the overall phase angle $\psi(t)$ as

$$\psi(t)=(\nu_1-\nu_2-\nu_3)t+\theta_1-\theta_2-\phi\ , \tag{17a}$$

$$\psi(t)=(\nu_1-\nu_2)t+\theta_1-\theta_2-\phi\ , \tag{17b}$$

where Eq. (17a) is appropriate to the quantum-beat laser (ϕ is the microwave-induced phase difference between atomic states $|a\rangle$ and $|b\rangle$ and ν_3 is the microwave frequency) and Eq. (17b) applies to the Hanle-effect laser, with ϕ determined by the choice of pump-radiation polarization. It follows directly from Ref. 7 that the amplitudes ρ_i, $i=1,2$ and the phase angle ψ obey the equations of motion

$$\dot\rho_1=\alpha_1\rho_1+\alpha_{12}\rho_2\cos\psi-\gamma_1\rho_1\ , \tag{18a}$$

$$\rho_2=\alpha_2\rho_2+\alpha_{21}\rho_2\cos\psi-\gamma_2\rho_2\ , \tag{18b}$$

$$\dot\psi=a-b\sin\psi\ . \tag{18c}$$

In the above the linear gain and cross-correlation coefficients are given by $\alpha_i=rg_i^2/\gamma_a^2$ and $\alpha_{12}=\alpha_{21}=rg_1g_2/\gamma_a^2$, where r is the rate of excitation to state $|a\rangle$, g_i is the atom-field coupling constant, and γ_a is the atomic decay rate which we take as the same for all atomic levels. The cavity decay rates are denoted by γ_i, a is given by Eq. (23), and the "locking" frequency b is given by

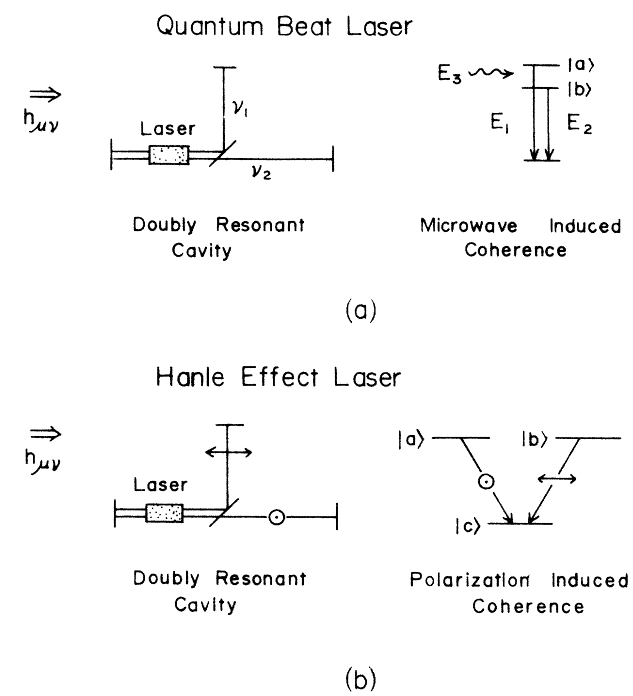


FIG. 2. (a) Quantum-beat-laser gravity-wave detector. Gravitational radiation perturbs frequency Ω_1 of doubly resonant cavity. Laser consisting of three level atoms coherently mixed by external microwave signal at frequency ν_3 drives cavities Ω_1 and Ω_2. Dichroic mirror causes light to be deflected in vertical direction for frequency ν_1 but transmits light at frequency ν_2. (b) Hanle-effect-laser gravity-wave detector, similar to quantum-beat system of (a) in that coherently excited atoms emit light of two polarizations which drives doubly resonant cavity via polarization sensitive mirror. Frequency of vertical arm is affected by gravitational radiation whereas horizontal laser radiation (copropagating with a gravitational wave) is not affected by gravitational radiation.

$$b=\alpha_{12}\frac{\rho_1}{\rho_2}+\alpha_{21}\frac{\rho_2}{\rho_1}\ . \tag{19}$$

Finally, in Ref. 7 it was shown that the ensemble average of the heterodyne signal (16) can, in the appropriate limit, be written as

$$\langle \hat a_1^\dagger(t)\hat a_2(t)\rangle=\rho_1\rho_2 e^{-D(\psi)t+i(\nu_1-\nu_2)t}\ , \tag{20}$$

where

$$D(\psi)=\tfrac{1}{8}[\alpha_1/\rho_1^2+\alpha_2/\rho_2^2 -(\alpha_{12}+\alpha_{21})e^{-i\psi}/\rho_1\rho_2]+\text{c.c.}\ , \tag{21}$$

(the numerical factor difference from the corresponding equation in Ref. 7 arises from the fact that we are using here amplitude, rather than intensity, gain coefficients).

The equations of motion (18a), (18b), and (18c) and the phase diffusion coefficient $D(\psi)$ as given by Eq. (21) provide a description of our coupled laser system. Let us consider the physically reasonable case of $g_1=g_2$ and $\rho_1=\rho_2$; then all α's reduce to a single rate, α, and Eq. (21) assumes the simple form

$$D(\psi)=\frac{\alpha}{2\bar n}(1-\cos\psi)\ , \tag{22}$$

where $\bar n=\rho_1^2=\rho_2^2$.

The other essential ingredient in our analysis is the equation of motion for $\psi(t)$ provided by Eq. (18c). From Eqs. (15) and (17) we see that the frequency a appearing in Eq. (18c) is given by

$$a=\Delta+\tfrac{1}{2}\nu_1 h_0\cos(\omega_g t-k_g x_0)\ , \tag{23}$$

where $\Delta=\Omega_1-\Omega_2-\nu_3$ for the quantum-beat and $\Omega_1-\Omega_2$ for the Hanle-effect lasers. Note that $\Omega_1-\Omega_2\simeq\nu_3$ for the first and $\Omega_1-\Omega_2\simeq 0$ for the second.

IV. A MEASUREMENT STRATEGY

Now from Eq. (18c) we see that when $a=0$ the relative phase angle locks to the constant value $\psi=0$. In this case $D(\psi)$, as given by Eq. (22), vanishes. We must now address the problem of extracting g-wave information while taking advantage of the "noise quenched" configuration occurring when $D(\psi)=0$.

To this end consider the superheterodyne signal as per Fig. 3. Note that when we are locked the beat frequency contains no g-wave information but the relative phase angle $\delta\psi$ is given by (see Appendix A)

$$\delta\psi=[\Delta+\tfrac{1}{2}\nu_1 h_0\cos(\omega_g t-k_g x_0)]/b\ , \tag{24}$$

which does depend on the g wave. Thus when we arrange that $\Delta=0$, Eq. (24) yields

$$\delta\psi=\tfrac{1}{2}\frac{\nu_1}{b}h_0\cos(\omega_g t-k_g x_0)\ , \tag{25}$$

where we have noted that $\nu_1 h_0/b \ll 1$ and are considering times $t<\omega_g^{-1}$.

Equating (25) to the shot-noise error (10) we find

$$h_{\min}\simeq\frac{b}{\nu_1}\sqrt{\hbar\nu/Pt_m}\ , \tag{26}$$

and since in the above $b=2\alpha$ [(compare Eq. (19)] this reduces to the familiar result given by Eq. (11), since $\alpha\simeq\gamma$.

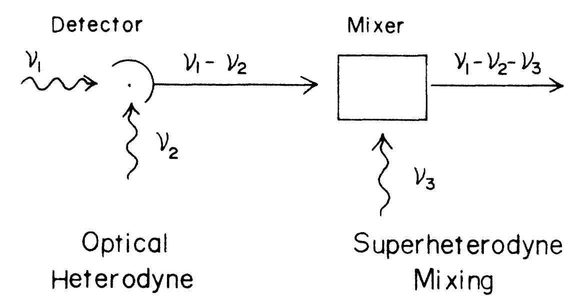
In order to improve the sensitivity, we introduce a mode-mode coupling designed to "reduce" b. In other words, we want a mode coupling that will result in the addition of a term $c\sin\psi$ to the phase equation (18c). We do this by injecting some light from one of the cavities into the other. To this end it is convenient to replace the resonator of Fig. 2 by two coupled ring cavities as in Figs. 4 and 5. The outside loops are necessary to be able to control separately the injection phases. Other schemes lead to a coupled-cavity problem which does not produce the $c\sin\psi$ term in the phase equation [see Eq. (35c) below]. This point is discussed further in Appendix B.

Consider first the Hanle-laser system of Fig. 4. There we couple the (running wave) fields by taking the light which is transmitted through a mirror of one cavity, changing the polarization, and injecting it into the other cavity. This we do with an efficiency κ.

For the case of the quantum beat laser, we remove light from one cavity and inject it into the other while using a nonlinear element as a parametric frequency converter, see Fig. 5. The parametric process is described[8] by the interaction Hamiltonian

$$H_p=g\hat a_1^\dagger\hat a_2\alpha_3+g^*\hat a_1\hat a_2^\dagger\alpha_3^*\ , \tag{27}$$

where α_3 is the (classical) microwave-field strength and g



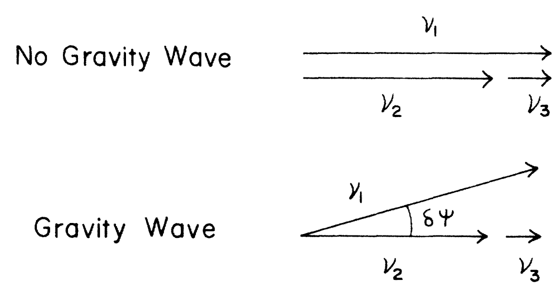


FIG. 3. Schematic illustration of experimental setup for quantum-beat laser. Radiation at ν_1 and ν_2 from quantum-beat laser influenced by g wave is heterodyne detected by photomultiplier. The current at the beat-note frequency $\nu_1-\nu_2$ is superheterodyned with the external microwave signal at frequency ν_3. The signal exiting the mixer is at frequency $\nu_1-\nu_2-\nu_3$. When gravity wave is present a small phase angle develops, as indicated in the figure.

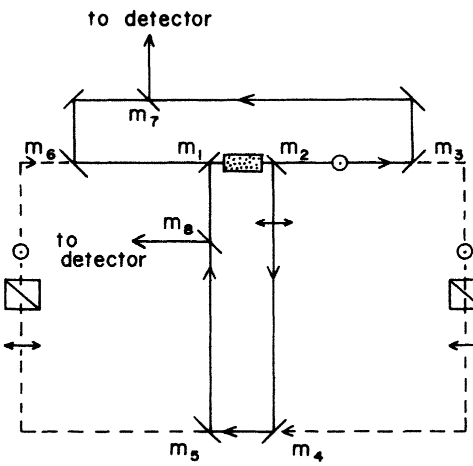


FIG. 4. Running-wave Hanle-laser gravity-wave detector consisting of two coupled ring cavities. Lasing medium is common to both cavities. Mirrors m_1 and m_2 totally reflect polarization of cavity 1 and totally transmit polarization of cavity 2. Gravitational radiation incident from left influences frequency of cavity 1 whereas cavity 2 remains essentially unaffected. Running waves propagate in the indicated directions. Light of polarization 1 leaves cavity 1 at m_5, its polarization is changed, and it is then partly coupled into cavity 2 via the mirror m_6. Similarly, light is extracted from cavity 2 at the mirror m_3, polarization rotated, and partly coupled into cavity 1 at mirror m_4. Mirrors m_7 and m_8 remove a small fraction of laser radiation for purposes of heterodyne detection.

is the purely imaginary coupling constant $g=i\,|g|$. The equation of motion for the conversion process[9] is then given by

$$\frac{\partial \hat{a}_1}{\partial t}=|g|\,\alpha_3\hat{a}_2 \ , \tag{28a}$$

$$\frac{\partial \hat{a}_2}{\partial t}=-\,|g|\,\alpha_3^*\hat{a}_1 \ ; \tag{28b}$$

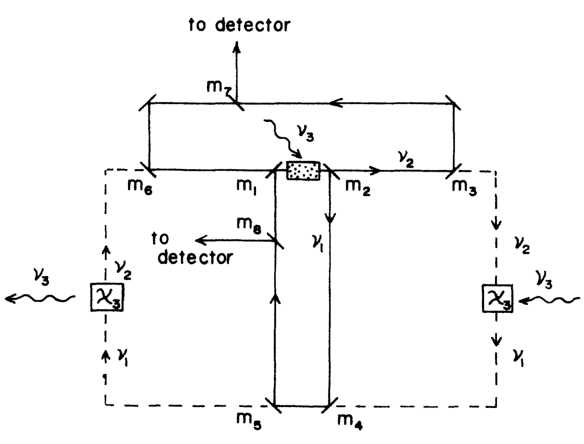


FIG. 5. Figure illustrating the coupling of E_1 and E_2 via parametric frequency conversion in order to increase system sensitivity in quantum-beat system. Analogous to Fig. 4 except that now light is frequency upshifted or downshifted in the outside loops, and the mirrors m_1 and m_2 discriminate between frequencies rather than polarizations.

again we denote the conversion efficiency by κ.

In either the Hanle or quantum-beat case, we note that in coupling the field out of, say, cavity 2 we have

$$E_2(\text{out})=t_3E_2(\text{in}) \ , \tag{29a}$$

where t_3 is the transmission of mirror m_3, and passing through the polarization or frequency converter yields

$$\Delta E_1(\text{out})=\kappa t_3E_2(\text{in})e^{-i\phi-i\delta_1} \ , \tag{29b}$$

where ΔE_1 is the field with the appropriate polarization or frequency generated with an efficiency κ. The phase shift along the outside loop is written as $\phi+\delta_1$, with ϕ as in Eqs. (17). Finally we note that this couples into cavity 1 through mirror 2 to yield

$$\Delta E_1(\text{in})=t_4't_3\kappa\rho_2(\text{in})e^{-i\theta_2-i\phi-i\delta_1} \ , \tag{30}$$

where t_4' is the transmission coefficient of mirror 2 (going in). Now light is emitted from cavity 2 according to the round trip time $\Delta t=p_2/c$, where p_2 is the perimeter of the second ring, so that

$$\frac{\Delta E_1}{\Delta t}=\frac{ct_4't_3}{p_2}\kappa\rho_2e^{-i(\theta_2+\phi)-i\delta_1}$$
$$=\gamma_{12}\rho_2e^{-i(\theta_2+\phi)-i\delta_1} \ , \tag{31}$$

where we have defined

$$\gamma_{12}=\frac{ct_4't_3}{p_2}\kappa \ . \tag{32}$$

Similarly removal of field from cavity 1, frequency converting and injecting into cavity 2 leads to

$$\frac{\Delta E_2}{\Delta t}=\gamma_{21}\rho_1e^{-i(\theta_1-\phi)-i\delta_2} \ , \tag{33}$$

where now

$$\gamma_{12}=\frac{ct_5t_6'}{p_1}\kappa \ . \tag{34}$$

The injection rates as given by Eqs. (31) and (33) alter the time evolution of the cavity amplitude and phases so that when $\delta_1=\delta_2=\pi$ Eqs. (18a), (18b), and (18c) become

$$\dot{\rho}_1=\alpha_1\rho_1+\alpha_{12}\rho_2\cos\psi-\gamma_1\rho_1-\gamma_{12}\rho_2\cos\psi \ . \tag{35a}$$

$$\dot{\rho}_2=\alpha_2\rho_2+\alpha_{21}\rho_1\cos\psi-\gamma_2\rho_2-\gamma_{21}\rho_1\cos\psi \ , \tag{35b}$$

$$\dot{\psi}=a-b\,\sin\psi+c\,\sin\psi \ . \tag{35c}$$

Here γ_1 represents the total loss rate of cavity 1, and γ_2 that of cavity 2. The coupling mirrors m_4 and m_5 introduce losses in cavity 1 at a rate $(1-r)c/p_1$ (where $r=r_4,r_5$), so that one may write

$$\gamma_1=(1-r_4)\frac{c}{p_1}+(1-r_5)\frac{c}{p_1}+\gamma_{\text{other}} \ , \tag{36}$$

where γ_{other} represents any other losses for cavity 1. Equation (36) assumes that r_4 and r_5 are real and positive. In this case, the reflection coefficients r_4', r_5', for light of frequency or polarization 1 incident on the mirrors from the other side, must be equal in magnitude, but negative. The various transmitted and reflected waves at mirror 4 are shown in Fig. 6, assuming $t'_4=t_4$, real and positive, and taking into account the fact that δ_1 must equal π in order to have the correct sign for c in Eq. (35c). Note that, then, the light leaving the system at that mirror has amplitude

$$\begin{aligned} E_{\text{out}} &= -r'_4 t_3 E_2 e^{-i\phi} + t_4 E_1 \\ &= r_4 t_3 E_2 e^{-i\phi} + t_4 E_1 \end{aligned} \tag{37}$$

(and frequency or polarization 1). This is important, as will be discussed later.

Let us now for simplicity assume that all the coupling mirrors have the same transmission and reflection coefficients, and that $p_1=p_2$. We have then

$$c = 2\kappa\gamma_c \ , \tag{38}$$

when $\rho_1=\rho_2$, where γ_c, the loss rate for each cavity associated with the coupling mirrors (m_4 and m_5 for cavity 1, m_3 and m_6 for cavity 2) equals

$$\gamma_c = 2(1-r)\frac{c}{p} \simeq \frac{t^2 c}{p} = \frac{\gamma_{12}}{\kappa} = \frac{\gamma_{21}}{\kappa} \tag{39}$$

[compare with Eqs. (32) and (34)]. The total loss rate γ, assumed again to be the same for both cavities, is then simply

$$\gamma = \gamma_c + \gamma_{\text{other}} \ , \tag{40}$$

as in Eq. (36).

Now we can see that Eq. (35c) gives the g-wave–induced phase angle (see Appendix A)

$$\delta\psi = \frac{1}{2}\frac{\nu_1}{b-c} h_0 \cos(\omega_g t - k_g x_0) \ , \tag{41}$$

and the shot-noise limit (26) now reads[10]

$$h_{\min} \simeq \frac{\alpha-\gamma_c}{\nu}\sqrt{\hbar\nu/Pt_m} \ , \tag{42}$$

since $b-c=2(\alpha-\gamma_c)$ when $\rho_1=\rho_2$ and $\kappa=1$.

Now the question naturally arises "how small can $\alpha-\gamma_c$ be?" It may be easily seen that nothing substantial is gained by letting it be smaller than about $\omega_g/2$, if the measurement time $t_m<\omega_g^{-1}$ [in this case, Eq. (41) is no longer valid when $\alpha-\gamma_c<\omega_g/2$]. If indeed $\alpha-\gamma_c\simeq\omega_g/2$, Eq. (42) becomes once again the active-device shot-noise limit of Eq. (2).

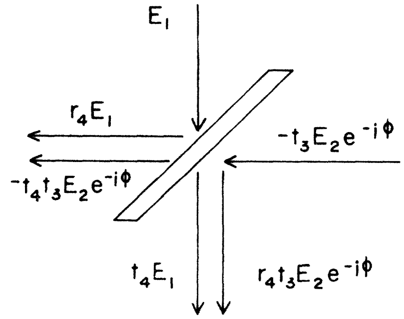


FIG. 6. Incident, reflected, and transmitted waves at mirror m_4. Note that one must have $r'_4=-r_4$. A similar picture is obtained at mirror m_6 in Figs. 4 and 5.

A problem arises, however, because, as a result of the phase-matching necessary to achieve our purposes, the light leaving the system (at mirrors m_2 and m_8 in Fig. 4) is in a superposition of modes 1 and 2 with a relative phase such that the interference term they yield is proportional to $\cos\psi$ [see Eq. (37) and Fig. 6]. This is a quantity that depends on the gravity-wave amplitude only in second order. Thus, it is not suitable for detection purposes. This point is discussed further in Appendix C.

To obtain a heterodyne signal proportional to $\sin\psi\simeq\psi$ [first order in h, see Eq. (41)] we need to extract some additional light from the cavity (indicated at mirrors m_7 and m_8 in Figs. 4 and 5) so that we can combine it with the appropriate relative phase. But any additional loss (beyond that associated with γ_c) is going to require larger gain to bring the system above threshold, so that $\alpha-\gamma_c$ will increase. Clearly, then, if we still want to have $\alpha-\gamma_c\simeq\omega_g/2$, the losses introduced by the extraction of a "signal" beam must be kept small enough that one still has

$$\gamma-\gamma_c \simeq \omega_g/2 \ , \tag{43}$$

where γ is the total loss rate for each cavity (including γ_c and the signal-beam extraction losses). As a result of this we find that the power in the signal beams P_s (coupled out at m_7, m_8) is related to the actual power of the laser P_0 (e.g., the power escaping at mirror m_4) by

$$P_s = (\omega_g/2\gamma)P_0 \ . \tag{44}$$

It is this P_s that must be used for P in Eq. (42). The result is then seen to be of the form (4) with an ϵ given by $\epsilon=\sqrt{\omega_g/2\gamma}$; i.e.,

$$h_{\min} \simeq \sqrt{\omega_g/2\gamma}\,\frac{\gamma}{\nu}\sqrt{\hbar\nu/P_0 t_m} \ . \tag{45}$$

Thus if we assume $\omega_g\sim10^2$ and $\gamma\sim10^6$ we see that $\epsilon\sim10^{-2}$. At this point we do not discount the possibility that a more efficient coupling mechanism might be found to reduce ϵ even further. The important point is that ϵ can be substantially less than unity. Some insight into the origin of the new limit (45) is provided in the following section.[11]

V. INTERPRETATION OF THE RESULTS

The new limit (45) has a simple interpretation in terms of the fundamental physics involved in the correlated spontaneous emission laser. Such a laser may be understood as actually operating in a single generalized mode of the radiation field when the relative phase angle ψ is locked to zero, i.e., in the absence of a gravitational wave. The gravitational wave may then be envisioned as exciting another mode (orthogonal to the original one), and detection of the gravity wave requires the detection of at least one photon in the new mode. This condition leads to the limit (45).

For definiteness, consider the Hanle-effect laser (similar results obtain for the quantum-beat laser). As before, we define annihilation operators associated with the modes of the two cavities as $\hat{a}_1$ and $\hat{a}_2$ (corresponding, for instance, to orthogonal linear polarizations), but when the locking condition $\psi=0$ is satisfied, the laser is effectively operating in the single mode

$$\hat{a}_+=\frac{1}{\sqrt{2}}(\hat{a}_1+e^{-i\phi}\hat{a}_2) , \tag{46}$$

(which may correspond to circularly polarized light, for example). In fact, from the semiclassical equations

$$\dot{E}_1=(\alpha-\gamma)E_1+\alpha e^{-i\phi}E_2 , \tag{47a}$$

$$\dot{E}_2=(\alpha-\gamma)E_2+\alpha e^{+i\phi}E_1 , \tag{47b}$$

and the definitions,

$$E_\pm=\frac{1}{\sqrt{2}}(E_1\pm e^{-i\phi}E_2) , \tag{48}$$

one proceeds, by multiplying Eq. (47b) by $e^{-i\phi}$ and adding (substracting) the resulting equation to (47a), to obtain

$$\dot{E}_+=(2\alpha-\gamma)E_+ , \tag{49a}$$

$$\dot{E}_-=-\gamma E_- \tag{49b}$$

[note that ϕ is time independent, being determined solely by the pumping mechanism to levels $|a\rangle$ and $|b\rangle$, as mentioned in connection with Eq. (17b)].

Equations (49) are decoupled, and in fact it may be shown that this decoupling holds also when nonlinear terms are taken into account, (i.e., when α is the saturated gain). This will be shown in a later publication. Equations (49) also show that the mode E_+ ($\hat{a}_+$ in the quantum theory) is above threshold, while the mode E_- ($\hat{a}_-$) is damped. The damping of E_- turns out to be equivalent to the locking of the phases $\theta_1(t)$ and $\theta_2(t)$ of E_+ and E_- to the particular value $\theta_1-\theta_2-\phi=0$. This may be seen as follows: from Eq. (48), we have

$$E_1=(E_++E_-)/\sqrt{2} , \tag{50a}$$

$$E_2=e^{i\phi}(E_+-E_-)/\sqrt{2} . \tag{50b}$$

Then when $E_-\to 0$ we are left with just

$$E_1(t)=E_+(t)/\sqrt{2} , \tag{51a}$$

$$E_2(t)=e^{i\phi}E_+(t)/\sqrt{2} , \tag{51b}$$

so that $E_1(t)=E_2(t)e^{-i\phi}$. This implies $\rho_1=\rho_2$ and $\theta_1(t)=\theta_2(t)+\phi$, or $\psi=0$.

Also, one can see from this argument why when $E_-=0$ (i.e., $\psi=0$) the relative phase diffusion due to spontaneous emission is zero. Equations (49) show that the atomic medium contributes gain to the E_+ mode only. Therefore the spontaneous emission takes place only in this mode. The spontaneous E_+ photons have of course a random phase, and will cause a random change in the phase of $E_+(t)$, but when Eqs. (51) apply this random phase change will be common to $E_1(t)$ and $E_2(t)$; hence the *relative* phase $\theta_1(t)-\theta_2(t)$ will not change.

Consider now what happens in the presence of a gravitational wave. It will modify the frequency of mode 1, which may be represented by adding a term $-ih(t)\nu E_1/2$ to the right-hand side of Eq. (47a). This leads to the coupling

$$\dot{E}_+=[2\alpha-\gamma-ih(t)\nu/2\sqrt{2}]E_+-ih(t)\nu E_-/2\sqrt{2} , \tag{52a}$$

$$\dot{E}_-=-[\gamma+ih(t)\nu/2\sqrt{2}]E_--ih(t)\nu E_+/2\sqrt{2} . \tag{52b}$$

Since $h_0\nu\ll\alpha,\gamma,\omega_g$, and $|E_-|\ll|E_+|$ if we are never very far from the locked-in regime, the modifications to Eq. (49a) may be neglected, but Eq. (52b) will read

$$\dot{E}_-=-\gamma E_--ih(t)\nu E_+/2\sqrt{2} , \tag{53}$$

which shows that the gravity wave acts as a source of photons in the "E_-" mode. What is more, the signal we want to detect, as discussed above, is proportional to $\sin\psi=\sin(\theta_1-\theta_2-\phi)$, i.e., to

$$e^{i\phi}E_1E_2^*-E_1^*E_2e^{-i\phi}=E_-E_+^*-E_-^*E_+ . \tag{54}$$

Hence, in order to detect any signal at all, at least one photon of the E_- type must reach the detector. Suppose that all losses are transmission losses, so that the γ in Eq. (53) gives the rate at which the E_- photons are leaving the cavity. Then we have

$$\dot{n}'_-=2\gamma n_- , \tag{55}$$

where n'_- is the number of photons outside, and n_- the number of photons inside the cavity. If E_+ in Eq. (53) is taken as constant, and the observation time is large enough compared to γ, one will get the steady-state result

$$E_-=\frac{-ih\nu}{2\sqrt{2}\gamma}E_+ , \tag{56}$$

or

$$n_-=\frac{h^2\nu^2}{8\gamma^2}n_+ . \tag{57}$$

When substituted in Eq. (55) this gives, over a measurement time t_m,

$$n'_-=\frac{h^2\nu^2}{4\gamma}n_+t_m , \tag{58}$$

for the number of E_- photons reaching the detector. The condition that this number equals at least one gives

$$h_0\simeq\frac{1}{\nu}\sqrt{\gamma/n_+t_m} . \tag{59}$$

Now we note that we may arrange for the E_+ and E_- modes to have different decay rates, given by γ_+ and γ_-. The nominal power of the laser is $\gamma_+n_+\hbar\nu$, so we have

$$h_0\simeq\frac{1}{\nu}\sqrt{\gamma_+\gamma_-}\sqrt{\hbar\nu/P_0t_m} , \tag{60}$$

where we have noted that the γ that appears in Eq. (59) is now γ_-. Note that if $\gamma_-\sim\omega_g$, Eq. (60) gives the limit (45).

It is in fact possible to show (see Appendix E), that the maximum signal is obtained when $\gamma_- \sim \omega_g$ (the analysis above is an approximation valid only when $\gamma_- \gg \omega_g$). To do this, one must actually integrate Eqs. (53) and (55), assuming a definite form for $h(t)$; for instance, Eq. (5b). E_+ in Eq. (53) may still be treated as approximately constant in steady state. Then $E_-(t)$ is obtained by integrating Eq. (53), and used to obtain $n_-(t)$; this is in turn substituted into Eq. (55) which is then integrated between $t=0$ and $t=t_m$ to give the total number of E_- photons leaving the cavity (and reaching the detector) in a time t_m. The calculation is straightforward, but lengthy; the result for $n'_-(t_m)$ is

$$n'_- = \frac{\gamma_-}{4}\frac{h_0^2\nu^2 n_+}{\gamma_-^2+\omega_g^2}\left\{\frac{t_m}{2} + [\sin(2\omega_g t_m + 2\chi) - \sin(2\chi)]/4\omega_g - [\cos\delta - e^{-\gamma_- t_m}\cos(\omega_g t_m + \delta)]/(\gamma_-^2+\omega_g^2)^{1/2} + (1-e^{-2\gamma_- t_m})\cos^2(\chi)/2\gamma_-\right\}, \tag{61}$$

where δ and χ are unimportant phases. It is easy to see that, for given t_m and ω_g, the factor $\gamma_-(\gamma_-^2+\omega_g^2)^{-1}$ is maximum when $\gamma_-=\omega_g$, and that the factor $\gamma_-(\gamma_-^2+\omega_g^2)^{-3/2}$, occurring in the third term, is maximum when $\gamma_-=\omega_g/\sqrt{2}$. As for the last term, it goes as $(1-e^{-2\gamma_- t_m})/(\gamma_-^2+\omega_g^2)$, and it is maximum also around $\gamma_- \sim \omega_g$ or $1/t_m$. Hence we see that n' is maximum when $\gamma_- \sim \omega_g$, in which case its order of magnitude is

$$n'_- \sim \frac{h_0^2\nu^2 n_+ t_m}{\omega_g}, \tag{62}$$

which is like Eq. (58) with $\gamma(\gamma_-)=\omega_g$. The limit (60), then, with $\gamma_-=\omega_g$, is seen to coincide with the result (45).

In effect, then, we may say that what the extraction-reinjection technique discussed here accomplishes is to make γ_+ different from γ_-, and $\gamma_- \sim \omega_g$. This can also be seen directly from Eqs. (35), specialized to the case $\alpha_1=\alpha_2=\alpha_{12}=\alpha_{21}$, $\gamma_1=\gamma_2=\gamma$, $\gamma_{12}=\gamma_{21}=\gamma_c$; for the complex amplitudes E_1 and E_2 one then has (in the absence of the gravity wave)

$$\dot E_1 = \tfrac{1}{2}(\alpha-\gamma)E_1 + \tfrac{1}{2}(\alpha-\gamma_c)e^{-i\phi}E_2, \tag{63a}$$

$$\dot E_2 = \tfrac{1}{2}(\alpha-\gamma)E_2 + \tfrac{1}{2}(\alpha-\gamma_c)e^{i\phi}E_1. \tag{63b}$$

Then, using the definition (48) again, one finds the equations for E_+ and E_-,

$$\dot E_+ = \tfrac{1}{2}[2\alpha-(\gamma+\gamma_c)]E_+, \tag{64a}$$

$$\dot E_- = -\tfrac{1}{2}(\gamma-\gamma_c)E_-, \tag{64b}$$

so that $\gamma_+ \equiv \gamma+\gamma_c$, and $\gamma_- \equiv \gamma-\gamma_c=\gamma_{\text{other}} \sim \omega_g$, as per Eqs. (40) and (43).

VI. CONCLUSIONS AND FINAL COMMENTS

In this paper we have discussed the passive- and active-cavity approaches to interferometric detection of very small displacements. We have shown how, with an appropriate measurement scheme, a correlated spontaneous emission laser may be used to achieve a sensitivity surpassing (in principle) the conventional schemes.

We note in conclusion that the "nonlocal" or "tidal" nature of the g-wave interaction with the detector takes a very different form in passive and active devices. In the passive case the signal goes as

$$\Delta\phi^{(p)} = \frac{\tilde L}{\lambda}h_0, \tag{65}$$

where $\tilde L$ equals the number of bounces times the cavity length, whereas in the active scheme we have

$$\Delta\phi^{(a)} = \frac{\nu}{\omega_g}h_0. \tag{66}$$

Thus, for a passive device, it is clear that gravity-wave detection is a nonlocal effect and measures deviations from Lorentzian space-time only when $\tilde L$ is appreciable. For an active detector the factor of $\lambda/\tilde L$ is replaced by ν/ω_g. In this case the nonlocality is to be understood as a comparison of the g-wave amplitude at different points in spacetime, i.e., comparing the g-wave–induced phase difference between detectors at different points in space, or comparing the phase shift of a single detector at different times. In this sense we are viewing the effect of the g wave [according to Eq. (6a)] as leading to a kind of time-dependent "red shift." We recall that red-shift experiments as carried out by Pound and Rebka involved the emission of radiation from nuclei localized at one point in space and the subsequent comparison with an absorber at another point in space-time. In their experiment, we note that the atoms have vanishing physical extent as compared with the gravitational parameters, just as in our case the size of our "quantum-beat g-wave detector" may be small compared to the gravitational wavelength λ_g.

Finally, we note that the largest useful $\tilde L$, as it appears in Eq. (65), is λ_g, and in that case

$$\Delta\phi^{(p)} \simeq \frac{\lambda_g}{\lambda}h_0 \text{ (passive)}. \tag{67}$$

But, since $\nu=c/\bar\lambda$ and $\omega_g=c/\bar\lambda_g$, Eq. (66) may be written as

$$\Delta\phi^{(a)} = \frac{\bar\lambda_g}{\bar\lambda}h_0 = \frac{\lambda_g}{\lambda}h_0 \text{ (active)} \tag{68}$$

so that the active and passive phase shifts are seen to be the same in this limit.

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APPENDIX A: CEL RESPONSE IN THE LOCKED REGIME

Consider the locking Eq. (18c) with a given by Eq. (23). By Eq. (19), b is of the order of magnitude of α_{12}, which will be equal to α. the gain rate; this in turn will be of the order of magnitude of γ, the cavity loss rate. Assuming that both the detuning Δ and $h_0\nu$ are much smaller than γ we have $a \ll b$ (typical numbers might be $\gamma \sim 10^6$ Hz for the cavity loss rate, $\nu \sim 10^{15}$ Hz for the laser frequency, and $h_0 \sim 10^{-21}$ for the gravity wave). Then we expect the solution of Eq. (18c) to be very close to the stable steady-state solution that one obtains when $a=0$, namely, $\psi=0$. Deviations of ψ from 0 will be of the order of a/b, so one may expand the sine function to first order, and obtain

$$\dot{\psi}=a(t)-b\psi \ . \tag{A1}$$

With the initial condition that $\psi=0$ (the steady-state solution in the absence of the gravitational wave) the solution to Eq. (A1) is

$$\psi=\int_0^t e^{-b(t-t')}a(t')dt' \ . \tag{A2}$$

Substitution of Eq. (23) for $a(t')$ gives

$$\psi(t)=\frac{\Delta}{b}(1-e^{-bt})+\frac{1}{2}\frac{\nu h_0}{b^2+\omega_g^2}\{b\cos(\omega_g t-k_g x_0)+\omega_g \sin(\omega_g t-k_g x_0)-e^{-bt}[b\cos(k_g x_0)-\omega_g \sin(k_g x_0)]\} \ . \tag{A3}$$

If the cavity decay rate γ is much larger than the frequency ω_g of the gravitational wave we will have $b \gg \omega_g$, and it is easy to see then that Eq. (A3) approaches Eq. (24) after a time $t \gg 1/b$ (which may still be much smaller than one gravitational wave period). In this case, the phase of the field inside the cavity follows "instantaneously" the gravitational wave. The case in which the effective b is of the order of ω_g is treated in detail in Sec. V.

APPENDIX B: RING CAVITIES VERSUS LINEAR CAVITIES

The transition from the system illustrated in Fig. 2 to the one presented in Figs. 4 and 5 may appear to involve a "quantum jump" in complexity, for which the motivation provided in the body of the paper may seem insufficient. We have explained how the system may be unlocked by extracting some light from one cavity and injecting it into the other one, and shown in detail how this works for our proposed cavity arrangement. What we wish to address in this Appendix is why we have chosen this particular arrangement (specifically, two ring cavities) to illustrate our scheme, rather than the linear "doubly resonant" cavities of Fig. 2. The main reasons are as stated in Sec. IV, namely, to avoid feedback along the injection loops, and to retain control over the phase difference between the extracted and the injected signal [the δ's in Eqs. (29)–(33), which as discussed in the text, should be equal to π]. We will elaborate on these points here.

It may at first seem that the extraction-reinjection technique proposed in Sec. IV to reduce the locking might be used with the linear cavity of Fig. 2. In fact, with reference to, e.g., Fig. 2(b), assume that the top mirror is used to extract an amount of light from cavity 1 which is then rotated in polarization and injected into cavity 2 via the mirror on the right. If the length of the path between the mirrors equals a half-integer number of wavelengths, the term added to the field E_2 in cavity 2 is

$$\Delta E_2=-t^2E_1 \ , \tag{B1}$$

every round trip, if both mirrors have amplitude transmission coefficients t. This may be seen by writing the standing-wave fields in both cavities as a sum of traveling waves and looking at how these traveling waves are coupled by the extraction and reinjection of light. In the same way, the same circuit would lead to an amount $\Delta E_1=-t^2E_2$ being added to the field in cavity 1. This is actually more than is needed to unlock the system, since the cavity losses per round trip are only

$$(\Delta E)_{\text{loss}}=-(1-r)E\simeq\frac{t^2}{2}E \ . \tag{B2}$$

This reasoning, however, fails for the following reason. When light from cavity 1 reaches the right-hand side mirror in Fig. 2, it is partly reflected and sent back on itself together with the transmitted field from cavity 2. What is then added to cavity 1 is not just some field from cavity 2, but a superposition

$$-t(tE_2+rtE_1) \ . \tag{B3}$$

Again, some of this light will be reflected at the top mirror, and sent back to cavity 2. We find then that there is no simple way to estimate the magnitude—and, most importantly, the phase—of the field added to either cavity. In effect, we have set up a coupled-cavity problem, the solution of which is not trivial; and it is not obvious that when equations of motion for E_1 and E_2 are written, that take into account the multiple reflections within the outside circuit (now a cavity in its own right), they will have the form that we want them to have.

Compared to this, we believe, it may be appreciated that the scheme illustrated in Figs. 4 and 5 has a greater conceptual simplicity, since (1) there is no possibility of feedback along the injection loops and (2) the phase-shift of the injected light may be easily controlled by choosing the optical path length of the injection loop.

Note that feedback appears to be unavoidable as long as we use the same mirror for extraction from, and injection into, a given cavity. One could easily think of schemes, based on the linear cavities of Fig. 2, with two separate injection loops, involving beam splitters, and one-way isolators to prevent feedback. The simplest of these schemes do not work, because they lead to losses larger than the coupling they provide; this is understandable intuitively, since light must be absorbed or otherwise lost at the one-way isolators.

We are still considering the coupled-cavity problem, and do not rule out the possibility that a useful coupling might be achieved in that case, but we believe that at the present time the scheme presented in this paper is the simplest "proof-of-principle" demonstration of our ideas. This point is argued further in Appendix D, which considers an alternative solution to the problem of how to extract the signal beams.

APPENDIX C: THE OUTPUT FIELD

We mention in the text that the field leaving the system at the mirrors m_4 and m_6 is not in a state useful for detection purposes. Since this is an important point it should be made as clear as possible.

Recall that the success of the extraction-reinjection "unlocking" technique requires that the injected field be out of phase by π radians relative to the phase difference with which the system tends to lock. That is, if the system, as indicated by Eq. (18c) tends to lock at $\psi=0$, the injected light should tend to lock the system at $\psi=\pi$. This results in phase relationships between the waves at, e.g., mirror 4 as shown in Fig. 6. There the phase of the cavity field (proportional to E_1) relative to the injected wave is $\theta_1-\theta_2-\phi-\pi=\psi-\pi$. As a result of this, however, the output waves (bottom part of the beam splitter) have a phase difference of ψ. As shown in Eq. (37), the output field goes as

$$E_{\text{out}}=r_4t_3E_2e^{-i\phi}+t_4E_1 \, , \tag{C1}$$

where r_4, t_4, and t_3 are assumed real and positive for definiteness. The basic result discussed here does not arise from any particular choice of phase shifts at the beam splitter, but from general properties of (nonabsorptive) beam splitters; in particular, time reversal invariance requires $r_4=-r_4'$.

The g-wave information is entirely contained in the phase difference ψ. We normally measure ψ by combining the fields E_1 and E_2 and looking at the interference term. The output field [Eq. (C1)] contains both E_1 and E_2, but their relative phase is such that they add almost in phase (since $\psi \ll 1$). That is, on the outside of the mirrors m_4 and m_6 the beams interfere to give a "bright fringe." We emphasize that this results solely from the fact, pointed out above, that on the opposite side of the beam splitter they must add "almost out of phase," i.e., with a phase difference $\psi+\pi$, for the unlocking mode coupling to be effective.

Essentially, then, to try to measure ψ by looking at the output of mirror 4 (or mirror 6) would be to look at a bright fringe, which is an unfavorable condition. Favorable conditions are at a dark fringe (beams combined with a phase difference $\psi\pm\pi$) or halfway between bright and dark (beams combined with a phase difference $\psi\pm\pi/2$). In the first case (dark fringe) the power at the detector would go like

$$P_{\text{det}}=2P_0(1-\cos\psi)\simeq P_0\psi^2 \, , \tag{C2}$$

(assuming equal power in both beams, for simplicity) while halfway between bright and dark one would have

$$P_{\text{det}}=2P_0\pm 2P_0\sin\psi\simeq 2P_0\pm 2P_0\psi \, . \tag{C3}$$

The first case has smaller photon counting noise and a smaller signal, the second one larger photon counting noise and a larger signal. If we write the equivalent photon counting power as

$$P_{\text{noise}}=\sqrt{P_{\text{det}}(\hbar\nu/t_m)} \, , \tag{C4}$$

[if $n=P_{\text{det}}t_m/\hbar\nu$ is the total number of photons reaching the detector in the measurement time t_m, Eq. (C4) is simply $P_{\text{noise}}=\sqrt{n}\,\hbar\nu/t_m$], we see that for the dark-fringe detector $P_{\text{det}}\equiv P_0\psi^2$, so

$$P_{\text{noise}}=\psi\sqrt{P_0\hbar\nu/t_m}; \quad P_{\text{signal}}=P_0\psi^2 \, , \tag{C5}$$

while for the detector halfway between bright and dark fringes $P_{\text{det}}\simeq 2P_0$ (the dominant term) and so

$$P_{\text{noise}}=\sqrt{2P_0(\hbar\nu/t_m)}; \quad P_{\text{signal}}=2P_0\psi \, . \tag{C6}$$

We see that Eqs. (C5) and (C6) give the same signal-to-noise ratio (aside from a factor $\sqrt{2}$). On the other hand, at a bright fringe

$$P_{\text{det}}=2P_0(1+\cos\psi)=4P_0-P_0\psi^2 \, , \tag{C7}$$

so that

$$P_{\text{noise}}=2\frac{P_0\hbar_\nu}{t_m}, \quad P_{\text{signal}}=P_0\psi^2 \, , \tag{C8}$$

which gives a much smaller signal-to-noise ratio than either Eq. (C5) or Eq. (C6), since $\psi \ll 1$.

The only way to combine E_1 and E_2 to obtain either a dark fringe or a half-fringe is then to extract some additional light from both cavities, since the light leaving the two output mirrors m_4 and m_6 is a superposition with the wrong phase difference (ψ, bright fringe). Hence the introduction of the beam splitters m_7 and m_8. A possible alternative arrangement to operate on a dark fringe is discussed in Appendix D.

APPENDIX D: OPERATION AT A DARK FRINGE?

We have considered the possibility, for the Hanle-effect laser, of extracting the field from the cavity in a way that would not make use of the beam splitters m_7 and m_8 and would lead to operation "at a dark fringe" (see Appendix C). In this scheme (see Fig. 7), mirror m_1 would be an ordinary 50/50 beam splitter. The polarization of the field in cavity 1 would be rotated just before it reaches m_1, to align it with the field in cavity 2, and the path lengths of both cavities would be chosen so that in the absence of a gravity wave one would have destructive interference (a dark fringe) between E_1 and E_2 at the "out" port of m_1; that is, no net field would leave the cavity at m_1, and the field inside cavity 1 would change from E_1 to

$$\frac{1}{\sqrt{2}}E_1+\frac{1}{\sqrt{2}}E_2e^{-i\phi} . \quad \text{(D1)}$$

Immediately behind m_1, a quarter-wave plate would convert this field to the appropriate circular polarization to interact with the gain medium. After passage through the gain medium, the beam would once again be split into its x and y polarization components at the dichroic mirror m_2.

With this arrangement no light would leave the cavity unless the phases of E_1 and E_2 were mismatched, so that the interference at the out port of the beam splitter was no longer entirely destructive. If a relative phase difference ψ developed, the output field would be proportional to

$$E_1-E_2e^{-i\phi}=\rho_1e^{-i\theta_1}-\rho_2e^{-i\theta_2-i\phi}\simeq -i\rho\psi e^{-i\theta_1} , \quad \text{(D2)}$$

if $\rho_1=\rho_2=\rho$ and $\psi=\theta_1-\theta_2-\phi$. The total output power would therefore be proportional to ψ^2, as in the dark-fringe detection schemes discussed in Appendix C.

We have concluded, however, that this approach is not satisfactory for the following reason. Equation (D1) implies a very strong coupling between the modes. The change in E_1 in one round trip due to the beam splitter m_1 would be

$$\Delta E_1=\frac{1}{\sqrt{2}}E_2e^{-i\phi}-\left[1-\frac{1}{\sqrt{2}}\right]E_1 . \quad \text{(D3)}$$

The first term leads to an equivalent mode-coupling coefficient

$$b'=\sqrt{2}\frac{c}{L} , \quad \text{(D4)}$$

after dividing by the round trip time L/c. This is larger than the usual mode-coupling coefficient b [Eq. (19)] which, as discussed in the text, would be of the order of magnitude of $2\gamma \sim t^2c/L$, where t is the mirror transmission. The phase difference between the modes E_1 and E_2 would lock to the value

$$\psi\sim \tfrac{1}{2}h\nu\frac{L}{c} , \quad \text{(D5)}$$

which is smaller, by a factor of t^2, than the usual locked result $h\nu/2\gamma$. Note that $1/t^2$ is essentially the number of bounces. The strong coupling between the modes therefore reduces the size of the signal to that which would be obtained with just a single-pass cavity. Also, the locking coefficient is so large that our extraction-reinjection technique would be ineffective in trying to unlock it. We therefore conclude that this approach is not useful for our system, at least in the simple form discussed here.

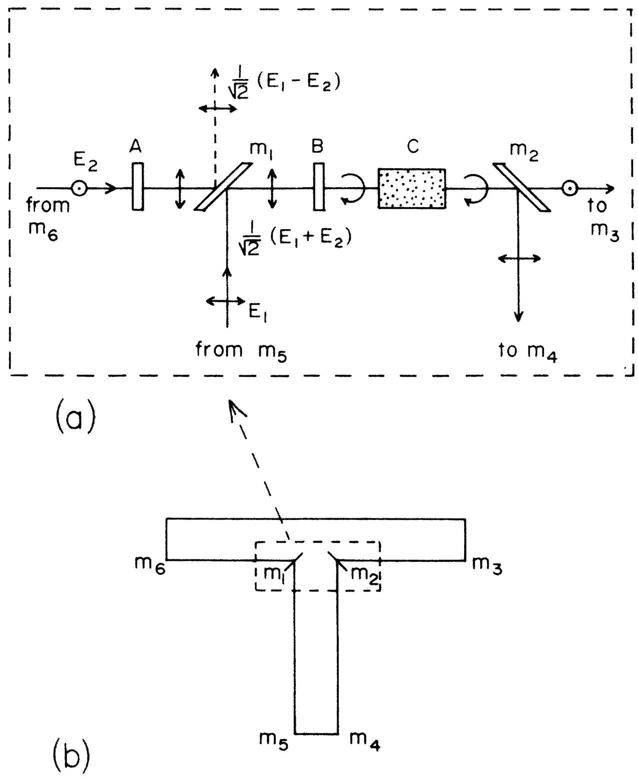

FIG. 7. A possible scheme for operation at a dark fringe. Part (a) of the figure shows the arrangement to be used in the inset (dashed box) shown in (b). The optical elements in (a) are as follows: A, polarization rotator by 90°; m_1, ordinary 50/50 beam splitter; B, quarter-wave plate; C, gain medium; m_2, polarization-sensitive mirror.

APPENDIX E

Here we show explicitly the algebra leading to Eq. (61), for the number n'_- of photons leaving the cavity in mode E_-. We begin with Eq. (53), where we treat E_+ as a constant, since we assume that it is well stabilized and well above threshold. For $h(t)$ we use Eq. (5b). Introducing the integrating factor $e^{\gamma_- t}$, we find that E_- is given by

$$\begin{aligned} E_-(t)&=-i\frac{h_0\nu E_+}{2\sqrt{2}}e^{-\gamma_- t}\int_0^t e^{\gamma_- t'}\cos(\omega_g t'-k_g x)dt' \\ &=-i\frac{h_0\nu E_+}{4\sqrt{2}}\left[\left[\frac{e^{i\omega t}-e^{-\gamma t}}{\gamma+i\omega}\right]e^{i\phi}+\text{c.c.}\right] , \end{aligned} \quad \text{(E1)}$$

where, in the second line, the subscripts have been dropped from γ_- and ω_g and $-k_g x$ is written simply as a constant phase ϕ. Next, noticing that

$$\frac{1}{\gamma \pm i\omega}=\frac{1}{(\gamma^2+\omega^2)^{1/2}}e^{\pm i\delta}\,, \tag{E2}$$

where

$$\delta=-\tan^{-1}(\omega/\gamma) \tag{E3}$$

we can rewrite Eq. (E1) as

$$E_-(t)=-i\frac{h_0\nu}{2\sqrt{2}}\frac{E_+}{(\gamma^2+\omega^2)^{1/2}}[\cos(\omega t+\phi+\delta) - e^{-\gamma t}\cos(\phi+\delta)]\,, \tag{E4}$$

from which the number of E_- photons inside the cavity follows immediately [squaring $E_-(t)$ and multiplying by a constant which will transform E_+ on the right-hand side into n_+]:

$$n_+(t)=\frac{1}{8}\frac{h_0^2\nu^2 n_+}{\gamma^2+\omega^2}[\cos^2(\omega t+\phi+\delta)+e^{-2\gamma t}\cos^2(\phi+\delta) - 2e^{-\gamma t}\cos(\omega t+\phi+\delta)\cos(\phi+\delta)]\,. \tag{E5}$$

Equation (E5) is the result that needs to be substituted into Eq. (55), and integrated between 0 and t_m to obtain the total number of E_- photons leaving the cavity. The result is Eq. (61), when the subscripts are restored and the definition

$$\chi=\phi+\delta=-k_g x-\tan^{-1}(\omega_g/\gamma_-) \tag{E6}$$

is used.

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[1]For an early discussion of laser interferometer detection of gravitational radiation, see R. Weiss, Massachusetts Institute of Technology Report No. 105, 1972, p. 54 (unpublished). For general background we have found the article by K. Thorne, Rev. Mod. Phys. **52**, 285 (1980) to be helpful. A more up to date review of work at the California Institute of Technology and at the University of Glasgow is described in the article of Drever *et al.* and that of the Max-Planck Gravity Wave Group in the paper by H. Billing *et al.* may be found in *Quantum Optics, Experimental Gravitation and Measurement Theory,* Vol. 94, *NATO ASI series,* edited by P. Meystre and M. Scully (Plenum, New York, 1983). For a review of recent optical g-wave detection work see the articles by C. Borde and co-workers, in Ann. Phys. (Paris) **10**, 201 (1985).

[2]See, for example, W. W. Chow, J. Gea-Banacloche, L. M. Pedrotti, V. E. Sanders, W. Schleich, and M. O. Scully, Rev. Mod. Phys. **57**, 61 (1985).

[3]See for example, J. Weber, in *General Relativity and Gravitational Waves* (Interscience, New York, 1961), or in *Gravitation* edited by C. Misner, K. Thorne, and J. Wheeler (Freeman, San Francisco, 1973).

[4]Considerations along these lines involving active detection have been put forth by N. Chebotayev and co-workers, see S. Bagayev *et al.*, Appl. Phys. **25**, 161 (1981).

[5]See for example, M. Sargent III, M. O. Scully, and W. E. Lamb, Jr. *Laser Physics* (Addison-Wesley, Reading, Mass., 1974).

[6]J. Gea-Banacloche, M. Scully, and D. Anderson, Opt. Commun. **57**, 67 (1986).

[7]M. Scully, Phys. Rev. Lett., **55**, 2802 (1985).

[8]See for example A. Yariv, *Quantum Electronics* (Wiley, New York, 1967).

[9]It has long been recognized that frequency conversion via parametric processes is free of quantum fluctuations. See, for example, W. Louisell, A. Yariv, and A. Siegman, Phys. Rev. **124**, 1646 (1961). For a more recent treatment see R. Graham, in *Quantum Optics,* edited by S. Kay and A. Maitland (Academic, New York, 1970).

[10]If we consider the quantization of the microwave field, then noise will accrue in the parametric process as well. In this case the noise in the microwave maser driving the quantum-beam laser and the parametric converters would go as (maser intensity)$^{-1}$ and this could be made very small since the energy per photon is small. Reasonable estimates indicate that this noise could be made negligible on the scale of the present "experiment." On the other hand, the polarization converter used in the Hanle-laser case would clearly be noise-free.

[11]In further related work we have found our result to be similar to one obtained by Drever and co-workers (article cited in Ref. 1) using a different method known as light recycling. We shall explore the relationship between these methods in a subsequent publication. We wish to thank Carlton Caves for very helpful conversations regarding this issue.