Interference of two photons in parametric down conversion

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A theoretical treatment is given of the process in which the two photons produced simultaneously in the parametric frequency splitting of light are allowed to interfere. It is shown that, while there is no interference in the usual sense involving quantities that are of the second order in the field, fourth-order interference effects are present. These may be revealed by measuring the joint probability of detecting two photons at two points x,x' in the interference plane with photoelectric detectors as a function of the separation $x-x'$. The probability exhibits a cosine modulation with $x-x'$, with visibility that can approach 100%, even though the integration time in the experiment may greatly exceed the reciprocal bandwidth of the photons. The interference effect has a nonclassical origin and implies a violation of local realism in the highly correlated two-photon state.

I. INTRODUCTION

It has been known for many years that there exist explicitly quantum-mechanical effects in the interference of light, particularly when the atomic sources are independent and when very small numbers of atoms are involved.[1-8] The effects often show up more readily in quantities that are of the fourth order in the field amplitude than in second-order quantities. However, while the early calculations by Fano[2] already contained the essential quantum features relating to the interference of two photons, that treatment focused on understanding the Hanbury Brown—Twiss effect,[9] and any differences between classical and quantum theories remained unexplored. Violations of the laws of classical probability in interference have been emphasized only relatively recently.[8]

Fano appears to have been the first to point out that the probability of detecting two photons produced by two excited atoms with two detectors exhibits a cosine modulation with the separation of the two detectors.[2] However, he also concluded that the modulation would disappear in any measurement made in a time interval that is large compared with the reciprocal frequency spread of the light. This seems to suggest that with wide-band optical sources whose bandwidth $\Delta\omega$ is an appreciable function of the midfrequency ω_0, the cosine modulation would effectively integrate to zero in practice. Unfortunately, it is just in the possibility of achieving a relatively large depth of modulation that the quantum prediction differs from the classical one.

In the following we consider interference between the signal and the idler photon produced in the process of parametric down conversion.[10] The photons usually have a wide bandwidth, and they appear "simultaneously."[10-15] We calculate the probability of detecting both photons with two detectors in some measurable time interval that is much larger than the reciprocal bandwidth, and we show that the answer becomes effectively independent of the measurement interval. At the same time we show that the probability exhibits a cosine modulation with the separation of the two detectors that can be close to 100%. We point out that this carries implications for the existence of the same kind of nonlocal correlations that were first discussed by Einstein, Podolsky, and Rosen,[16] and were studied in recent experiments.[17-19]

II. THE TWO-PHOTON STATE

In the process of spontaneous parametric down conversion photons from an incident laser beam interact with a nonlinear medium, and split into two lower-frequency signal and idler photons, that we label 1 and 2 (see Fig. 1). We shall take the incident pump light beam to be in the form of an intense monochromatic plane wave

$$\mathbf{V}(\mathbf{r},t)=\mathbf{V}e^{i(\mathbf{k}_0\cdot\mathbf{r}-\omega_0 t)}, \tag{1}$$

that can be treated classically, and describe the interaction within the nonlinear medium parametrically through the second-order susceptibility χ. Then the interaction $\hat{H}_I(t)$ in the interaction picture is of the general form[12,14,20]

$$\hat{H}_I(t)=\int_{\mathscr{V}} d^3x\frac{1}{L^3}\sum_{\mathbf{k}_1,s_1}\sum_{\mathbf{k}_2,s_2}\chi_{ijl}(\omega_0,\omega_1,\omega_2)(\epsilon^*_{\mathbf{k}_1s_1})_i(\epsilon^*_{\mathbf{k}_2s_2})_j V_l\hat{a}^\dagger_{\mathbf{k}_1s_1}\hat{a}^\dagger_{\mathbf{k}_2s_2}e^{i[(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}-(\omega_0-\omega_1-\omega_2)t]}+\text{H.c.} \tag{2}$$

$\mathbf{k},s$ labels the plane-wave eigenmodes of a large cubical cavity of side L, with periodic boundary conditions, $\epsilon_{\mathbf{k}s}$ is a unit polarization vector, and the integral is to be taken over the volume $\mathscr{V}$ of the nonlinear medium. We shall take the initial state of the quantum field at time $t=0$ to be the vacuum state $|\psi_{\text{vac}}\rangle$.

The state $|\psi\rangle$ in the interaction picture after a time t is then given by

$$|\psi\rangle=\exp\left[-\frac{i}{\hbar}\int_0^t \hat{H}_I(t')dt'\right]|\psi_{\text{vac}}\rangle\ , \tag{3}$$

and to the lowest order, in which we limit ourselves to two-photon excitations, the state becomes

$$|\psi\rangle=|\psi_{\text{vac}}\rangle-\frac{i}{\hbar}\frac{1}{L^3}\sum_{\mathbf{k}_1,s_1}\sum_{\mathbf{k}_2,s_2}\chi_{ijl}(\omega_0,\omega_1,\omega_2)(\epsilon^*_{\mathbf{k}_1 s_1})_i(\epsilon^*_{\mathbf{k}_2 s_2})_j V_l e^{i(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{R}}$$
$$\times\prod_{m=1}^{3}\left[\frac{\sin[\frac{1}{2}(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)_m l_m]}{\frac{1}{2}(\mathbf{k}_0-\mathbf{k}_1-\mathbf{k}_2)_m}\right]e^{-(1/2)i(\omega_0-\omega_1-\omega_2)t}\frac{\sin[\frac{1}{2}(\omega_0-\omega_1-\omega_2)t]}{\frac{1}{2}(\omega_0-\omega_1-\omega_2)}|\mathbf{k}_1 s_1,\mathbf{k}_2 s_2\rangle\ . \tag{4}$$

Here $\mathbf{R}$ is the midpoint of the nonlinear medium which is assumed to be in the form of a rectangular parallelopiped of sides l_1,l_2,l_3, and $|\mathbf{k}_1 s_1,\mathbf{k}_2 s_2\rangle$ is a two-photon Fock state. If the interaction time t is sufficiently large, we can replace the sinc factor $\sin[\frac{1}{2}(\omega_0-\omega_1-\omega_2)t]/\frac{1}{2}(\omega_0-\omega_1-\omega_2)$ by $2\pi\delta(\omega_0-\omega_1-\omega_2)$ to a good approximation, and the oscillatory factor $\exp[-i(\omega_0-\omega_1-\omega_2)t/2]$ can then be discarded by a regularizing procedure.

Finally, if the two down-converted signal and idler waves are recombined at some distant point from which the pump beam is excluded, as shown in Fig. 2, and if the two photons arrive at $\mathbf{r}_1$ and $\mathbf{r}_2$ at time t_0, respectively, we may take the resulting two-photon state, which is the initial state for the interference experiment, to be expressible in the general form

$$|\psi\rangle=\frac{1}{L^3}\sum_{\substack{\mathbf{k}_1,\mathbf{k}_2\\(\mathbf{k}_1\neq\mathbf{k}_2)}}\sum\phi(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)e^{-i[(\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_2\cdot\mathbf{r}_2)-(\omega_1+\omega_2)t_0]}|\mathbf{k}_1 s_1,\mathbf{k}_2 s_2\rangle\ . \tag{5}$$

$\phi(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)$ is a weight function that is derivable from Eq. (4), which, by virtue of the sinc factors, is appreciably different from zero only when

$$\omega_0=\omega_1+\omega_2\ ,\qquad \mathbf{k}_0\approx\mathbf{k}_1+\mathbf{k}_2 \tag{6}$$

and the exponential factor has been pulled out for convenience of interpretation. Equations (6) will be recognized as the usual phase-matching conditions. We have taken both signal and idler photons to be in definite polarization states s_1,s_2, and have assumed that the directions of the signal and idler wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ do not overlap, so that $\mathbf{k}_1\neq\mathbf{k}_2$ in the double sum. Moreover, as we are particularly interested in interference effects, we shall suppose that we are dealing with the degenerate case in which both signal and idler waves are centered at the same frequency $\frac{1}{2}\omega_0$, although both have an appreciable frequency spread $\Delta\omega$. As a result of the spread of $\mathbf{k}_1,\mathbf{k}_2$, both signal and idler photons are pretty well localized in space and time, and this has been demonstrated experimentally.[10,15,21] Normalization of the state $|\psi\rangle$ requires that

$$\frac{1}{(2\pi)^6}\int d^3k_1\int d^3k_2\,|\phi(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)|^2=1\ . \tag{7}$$

It is not difficult to see, at least when the weight function $\phi(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)$ is real and non-negative, that the state $|\psi\rangle$ given by Eq. (5) corresponds to two photons having a spatial distribution with peaks at $\mathbf{r}_1$ and $\mathbf{r}_2$ at time t_0. For this purpose we consider the projection of $|\psi\rangle$ onto the two-photon "position state" in the Heisenberg picture

$$|\mathbf{r}'_1,t_1;\mathbf{r}'_2,t_2\rangle=\hat{\mathbf{V}}^\dagger(\mathbf{r}'_1,t_1)\hat{\mathbf{V}}^\dagger(\mathbf{r}'_2,t_2)|\psi_{\text{vac}}\rangle\ , \tag{8}$$

with

$$\hat{\mathbf{V}}(\mathbf{r},t)=\frac{1}{L^{3/2}}\sum_{\mathbf{k},s}\hat{a}_{\mathbf{k}s}\epsilon_{\mathbf{k}s}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\ . \tag{9}$$

It has been shown that the state $\hat{V}^\dagger(\mathbf{r},t)|\psi_{\text{vac}}\rangle$ is a one-photon state in which the photon is (more or less) localized at position $\mathbf{r}$ at time t,[22] provided no attempt is made to determine position to an accuracy better than a few wavelengths or the time to better than a few optical periods. From Eqs. (5), (8), and (9) together with the commutation relations for $\hat{a}_{\mathbf{k}s},\hat{a}^\dagger_{\mathbf{k}'s'}$, we readily find for a symmetric function

$$\phi(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)=\phi(\mathbf{k}_2 s_2,\mathbf{k}_1 s_1) \tag{10}$$

that in the limit $L\to\infty$

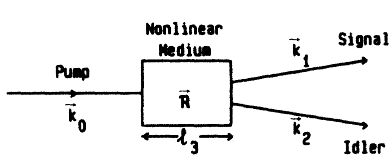

FIG. 1. Outline of the geometry for generating two down-converted photons.

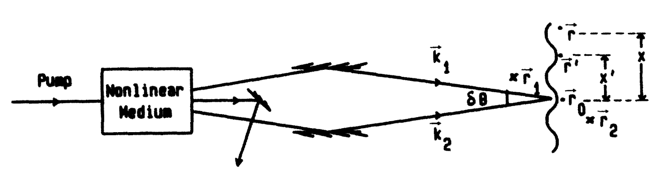

FIG. 2. The geometry for the two-photon interference experiment.

$$\langle \mathbf{r}_2',t_2;\mathbf{r}_1',t_1 \mid \psi\rangle=\frac{1}{(2\pi)^6}\int d^3k_1 \int d^3k_2 \phi(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)\big(\epsilon_{\mathbf{k}_1 s_1}\epsilon_{\mathbf{k}_2 s_2} e^{i[\mathbf{k}_1\cdot(\mathbf{r}_1'-\mathbf{r}_1)+\mathbf{k}_2\cdot(\mathbf{r}_2'-\mathbf{r}_2)-\omega_1(t_1-t_0)-\omega_2(t_2-t_0)]} + \epsilon_{\mathbf{k}_2 s_2}\epsilon_{\mathbf{k}_1 s_1} e^{i[\mathbf{k}_2\cdot(\mathbf{r}_1'-\mathbf{r}_2)+\mathbf{k}_1\cdot(\mathbf{r}_2'-\mathbf{r}_1)-\omega_2(t_1-t_0)-\omega_1(t_2-t_0)]}\big) . \tag{11}$$

The square of this probability amplitude gives the probability of locating one photon at $\mathbf{r}_1',t_1$ and one at $\mathbf{r}_2',t_2$ in the foregoing restricted sense. Now if $\phi(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)$ is a real non-negative weight function, then the double Fourier transform in Eq. (11) is a (tensor) correlation function in the variables $\mathbf{r}_1',t_1,\mathbf{r}_2',t_2$, and the first term has its greatest absolute value when

$$\begin{aligned} &\mathbf{r}_1'=\mathbf{r}_1 \text{ at time } t_1=t_0 , \\ &\mathbf{r}_2'=\mathbf{r}_2 \text{ at time } t_2=t_0 , \end{aligned} \tag{12}$$

whereas $\mathbf{r}_1$ and $\mathbf{r}_2$ are interchanged in the second term. Hence the two-photon wave packet represented by $|\psi\rangle$ is peaked at $\mathbf{r}_1$ and $\mathbf{r}_2$ at time t_0. Naturally, in performing an interference experiment one would strive to make both photons arrive at the same locality at the same time.

III. THE PHOTON-DETECTION PROBABILITY

We now calculate the probability of detecting a photon out of the superposed signal and idler waves at some point $\mathbf{r}$ at time t. This probability $P(\mathbf{r},t)$ is given by the expectation value of $\hat{\mathbf{V}}^\dagger(\mathbf{r},t)\cdot\hat{\mathbf{V}}(\mathbf{r},t)$, where $\hat{\mathbf{V}}(\mathbf{r},t)$ is the detection operator in the Heisenberg picture, which we take to be defined by Eq. (9). Thus

$$\begin{aligned} P(\mathbf{r},t)&=K\langle\psi \mid \hat{\mathbf{V}}^\dagger(\mathbf{r}\cdot t)\cdot\hat{\mathbf{V}}(\mathbf{r},t)\mid\psi\rangle \\ &=K\frac{1}{L^3}\sum_{\mathbf{k}',s'}\sum_{\mathbf{k}'',s''}\langle\psi\mid \hat{a}^\dagger_{\mathbf{k}'s'}\hat{a}_{\mathbf{k}''s''}\mid\psi\rangle \\ &\qquad\times \epsilon^*_{\mathbf{k}'s'}\cdot\epsilon_{\mathbf{k}''s''}e^{i[(\mathbf{k}''-\mathbf{k}')\cdot\mathbf{r}-(\omega''-\omega')t]} , \end{aligned}$$

where K is a proportionality factor. In order to evaluate the matrix element we make use of Eq. (5), and we observe that

$$\langle \mathbf{k}_2 s_2,\mathbf{k}_1 s_1 \mid \hat{a}^\dagger_{\mathbf{k}'s'}\hat{a}_{\mathbf{k}''s''}\mid \mathbf{k}_1' s_1',\mathbf{k}_2' s_2'\rangle$$

is nonzero only when

$$\mathbf{k}'',s''=\mathbf{k}_1',s_1',\ \ \mathbf{k}',s'=\mathbf{k}_1,s_1,\ \ \mathbf{k}_2',s_2'=\mathbf{k}_2,s_2$$

or

$$\mathbf{k}'',s''=\mathbf{k}_2',s_2',\ \ \mathbf{k}',s'=\mathbf{k}_2,s_2,\ \ \mathbf{k}_1',s_1'=\mathbf{k}_1,s_1 .$$

Then $P(\mathbf{r},t)$ reduces to

$$P(\mathbf{r},t)=K\frac{1}{L^9}\sum_{\substack{\mathbf{k}_1,s_1\\(\mathbf{k}_1\neq\mathbf{k}_2)}}\sum_{\mathbf{k}_2,s_2}\phi^*(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)\left[\sum_{\substack{\mathbf{k}_1',s_1'\\(\mathbf{k}_1'\neq\mathbf{k}_2)}}\phi(\mathbf{k}_1' s_1',\mathbf{k}_2 s_2)(\epsilon^*_{\mathbf{k}_1 s_1}\cdot\epsilon_{\mathbf{k}_1' s_1'})e^{i[(\mathbf{k}_1'-\mathbf{k}_1)\cdot(\mathbf{r}-\mathbf{r}_1)-(\omega_1'-\omega_1)(t-t_0)]} + \sum_{\substack{\mathbf{k}_2',s_2'\\(\mathbf{k}_2'\neq\mathbf{k}_1)}}\phi(\mathbf{k}_1 s_1,\mathbf{k}_2' s_2')(\epsilon^*_{\mathbf{k}_2 s_2}\cdot\epsilon_{\mathbf{k}_2' s_2'})e^{i[(\mathbf{k}_2'-\mathbf{k}_2)\cdot(\mathbf{r}-\mathbf{r}_2)-(\omega_2'-\omega_2)(t-t_0)]}\right] . \tag{13}$$

Because the wave vectors $\mathbf{k}_1,\mathbf{k}_1'$ both belong to the signal wave, and $\mathbf{k}_2,\mathbf{k}_2'$ both belong to the idler wave, there is no $\cos[(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r}]$ modulation with position $\mathbf{r}$, and $P(\mathbf{r},t)$ does not exhibit interference fringes. This simply reflects the absence of a phase relation between the signal and idler waves. We shall see, however, that there are higher-order interference effects.

IV. TWO-PHOTON-DETECTION PROBABILITY

Next we suppose that the superposed signal and idler waves are detected by two photoelectric detectors located at two points $\mathbf{r},\mathbf{r}'$ in a plane approximately perpendicular to $\mathbf{k}_1+\mathbf{k}_2$, in the neighborhood of $\mathbf{r}_1,\mathbf{r}_2$ (see Fig. 2). Then the joint probability $P_2(\mathbf{r},t;\mathbf{r}',t')$ of detecting one photon at $\mathbf{r}$ at time t and another at $\mathbf{r}'$ at time t' is expressible in the form[23]

$$\begin{aligned} P_2(\mathbf{r},t;\mathbf{r}',t')&=K\langle\psi\mid \hat{V}_i^\dagger(\mathbf{r},t)\hat{V}_j^\dagger(\mathbf{r}',t')\hat{V}_j(\mathbf{r}',t')\hat{V}_i(\mathbf{r},t)\mid\psi\rangle \\ &=K\frac{1}{L^6}\sum_{\mathbf{k}s}\sum_{\mathbf{k}'s'}\sum_{\mathbf{k}''s''}\sum_{\mathbf{k}'''s'''}\langle\psi\mid\hat{a}^\dagger_{\mathbf{k}s}\hat{a}^\dagger_{\mathbf{k}'s'}\hat{a}_{\mathbf{k}''s''}\hat{a}_{\mathbf{k}'''s'''}\mid\psi\rangle(\epsilon^*_{\mathbf{k}s})_i(\epsilon^*_{\mathbf{k}'s'})_j(\epsilon_{\mathbf{k}''s''})_j(\epsilon_{\mathbf{k}'''s'''})_i \\ &\qquad\times e^{i[(\mathbf{k}'''-\mathbf{k})\cdot\mathbf{r}+(\mathbf{k}''-\mathbf{k}')\cdot\mathbf{r}'-(\omega'''-\omega)t-(\omega''-\omega')t']} . \end{aligned}$$

On introducing the expansion (5) for $|\psi\rangle$, and observing that the matrix element

$$\langle \mathbf{k}_1 s_1, \mathbf{k}_2 s_2 | \hat{a}^\dagger_{\mathbf{k}s} \hat{a}^\dagger_{\mathbf{k}'s'} \hat{a}_{\mathbf{k}''s''} \hat{a}_{\mathbf{k}'''s'''} | \mathbf{k}'_1 s'_1, \mathbf{k}'_2 s'_2 \rangle$$

is nonzero only for the four combinations

$$\mathbf{k}'',s''=\mathbf{k}'_1,s'_1,\ \ \mathbf{k}''',s'''=\mathbf{k}'_2,s'_2,\ \ \mathbf{k},s=\mathbf{k}_1,s_1,\ \ \mathbf{k}',s'=\mathbf{k}_2,s_2\ ,$$

or

$$\mathbf{k}'',s''=\mathbf{k}'_2,s'_2,\ \ \mathbf{k}''',s'''=\mathbf{k}'_1,s'_1,\ \ \mathbf{k},s=\mathbf{k}_1,s_1,\ \ \mathbf{k}',s'=\mathbf{k}_2,s_2\ ,$$

or

$$\mathbf{k}'',s''=\mathbf{k}'_1,s'_1,\ \ \mathbf{k}''',s'''=\mathbf{k}'_2,s'_2,\ \ \mathbf{k},s=\mathbf{k}_2,s_2,\ \ \mathbf{k}',s'=\mathbf{k}_1,s_1\ ,$$

or

$$\mathbf{k}'',s''=\mathbf{k}'_2,s'_2,\ \ \mathbf{k}''',s'''=\mathbf{k}'_1,s'_1,\ \ \mathbf{k},s=\mathbf{k}_2,s_2,\ \ \mathbf{k}',s'=\mathbf{k}_1,s_1\ ,$$

we arrive at

$$\begin{aligned}
P_2(\mathbf{r},t;\mathbf{r}',t') &= K\frac{1}{L^{12}} \sum_{\mathbf{k}_1,s_1} \sum_{\mathbf{k}_2,s_2} \sum_{\mathbf{k}'_1,s'_1} \sum_{\mathbf{k}'_2,s'_2} \phi^*(\mathbf{k}_1 s_1,\mathbf{k}_2 s_2)\phi(\mathbf{k}'_1 s'_1,\mathbf{k}'_2 s'_2) e^{i[(\mathbf{k}_1-\mathbf{k}'_1)\cdot\mathbf{r}_1+(\mathbf{k}_2-\mathbf{k}'_2)\cdot\mathbf{r}_2-(\omega_1+\omega_2-\omega'_1-\omega'_2)t_0]} \\
&\quad\times[(\boldsymbol{\epsilon}^*_{\mathbf{k}_1 s_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_1 s'_1})(\boldsymbol{\epsilon}^*_{\mathbf{k}_2 s_2}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_2 s'_2}) e^{i[(\mathbf{k}'_1-\mathbf{k}_1)\cdot\mathbf{r}+(\mathbf{k}'_2-\mathbf{k}_2)\cdot\mathbf{r}'-(\omega'_1-\omega_1)t-(\omega'_2-\omega_2)t']} \\
&\quad+(\boldsymbol{\epsilon}^*_{\mathbf{k}_2 s_2}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_2 s'_2})(\boldsymbol{\epsilon}^*_{\mathbf{k}_1 s_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_1 s'_1}) e^{i[(\mathbf{k}'_2-\mathbf{k}_2)\cdot\mathbf{r}+(\mathbf{k}'_1-\mathbf{k}_1)\cdot\mathbf{r}-(\omega'_2-\omega_2)t-(\omega'_1-\omega_1)t']} \\
&\quad+(\boldsymbol{\epsilon}^*_{\mathbf{k}_1 s_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_2 s'_2})(\boldsymbol{\epsilon}^*_{\mathbf{k}_2 s_2}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_1 s'_1}) e^{i[(\mathbf{k}'_2-\mathbf{k}_1)\cdot\mathbf{r}+(\mathbf{k}'_1-\mathbf{k}_2)\cdot\mathbf{r}'-(\omega'_2-\omega_1)t-(\omega'_1-\omega_2)t']} \\
&\quad+(\boldsymbol{\epsilon}^*_{\mathbf{k}_2 s_2}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_1 s'_1})(\boldsymbol{\epsilon}^*_{\mathbf{k}_1 s_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}'_2 s'_2}) e^{i[(\mathbf{k}'_1-\mathbf{k}_2)\cdot\mathbf{r}+(\mathbf{k}'_2-\mathbf{k}_1)\cdot\mathbf{r}'-(\omega'_1-\omega_2)t-(\omega'_2-\omega_1)t']}]\ . \qquad (14)
\end{aligned}$$

In the limit $L\rightarrow\infty$ the sums over the wave vectors $\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}'_1,\mathbf{k}'_2$ can be replaced by integrals, and if the polarizations and the directions of the waves are taken to be well defined, each three-dimensional integral with respect to d^3k reduces to a one-dimensional integral $\omega^2 d\omega$ with respect to frequency.

Finally, we integrate $P_2(\mathbf{r},t;\mathbf{r}',t')$ over the time interval T for which the measurement proceeds, which we shall take to be much longer than the reciprocal bandwidth $1/\Delta\omega$. Then the measured probability becomes, with $\boldsymbol{\kappa}=\mathbf{k}/k$,

$$\begin{aligned}
P_2(\mathbf{r},\mathbf{r}') &= \text{const}\times\int_{t_0-T/2}^{t_0+T/2}\int P_2(\mathbf{r},t;\mathbf{r}',t')dt\,dt' \\
&= \text{const}\times\int\int\int\int d\omega_1 d\omega_2 d\omega'_1 d\omega'_2 \omega_1^2\omega_2^2\omega_1'^2\omega_2'^2 \phi^*(\omega_1,\omega_2)\phi(\omega'_1,\omega'_2) \\
&\quad\times\Bigg\{ e^{i\boldsymbol{\kappa}_1\cdot(\mathbf{r}-\mathbf{r}_1)(\omega'_1-\omega_1)/c} e^{i\boldsymbol{\kappa}_2\cdot(\mathbf{r}'-\mathbf{r}_2)(\omega'_2-\omega_2)/c} \frac{\sin[(\omega'_1-\omega_1)T/2]}{(\omega'_1-\omega_1)/2}\frac{\sin[(\omega'_2-\omega_2)T/2]}{(\omega'_2-\omega_2)/2} \\
&\quad+e^{i\boldsymbol{\kappa}_2\cdot(\mathbf{r}-\mathbf{r}_2)(\omega'_2-\omega_2)/c} e^{i\boldsymbol{\kappa}_1\cdot(\mathbf{r}'-\mathbf{r}_1)(\omega'_1-\omega_1)/c} \frac{\sin[(\omega'_2-\omega_2)T/2]}{(\omega'_2-\omega_2)/2}\frac{\sin[(\omega'_1-\omega_1)T/2]}{(\omega'_1-\omega_1)/2} \\
&\quad+|\boldsymbol{\epsilon}^*_{\mathbf{k}_1 s_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}_2 s_2}|^2 e^{i[\boldsymbol{\kappa}_2\omega'_2\cdot(\mathbf{r}-\mathbf{r}_2)/c-\boldsymbol{\kappa}_1\omega_1\cdot(\mathbf{r}-\mathbf{r}_1)/c]} \\
&\quad\times e^{i[\boldsymbol{\kappa}_1\omega'_1\cdot(\mathbf{r}'-\mathbf{r}_1)/c-\boldsymbol{\kappa}_2\omega_2\cdot(\mathbf{r}'-\mathbf{r}_2)/c]} \frac{\sin[(\omega'_2-\omega_1)T/2]}{(\omega'_2-\omega_1)/2}\frac{\sin[(\omega'_1-\omega_2)T/2]}{(\omega'_1-\omega_2)/2} \\
&\quad+|\boldsymbol{\epsilon}^*_{\mathbf{k}_1 \mathbf{s}_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}_2 s_2}|^2 e^{i[\boldsymbol{\kappa}_1\omega'_1\cdot(\mathbf{r}-\mathbf{r}_1)/c-\boldsymbol{\kappa}_2\omega_2\cdot(\mathbf{r}-\mathbf{r}_2)/c]} e^{i[\boldsymbol{\kappa}_2\omega'_2\cdot(\mathbf{r}'-\mathbf{r}_2)/c-\boldsymbol{\kappa}_1\omega_1\cdot(\mathbf{r}'-\mathbf{r}_1)/c]} \\
&\quad\times\frac{\sin[(\omega'_1-\omega_2)T/2]}{(\omega'_1-\omega_2)/2}\frac{\sin[(\omega'_2-\omega_1)T/2]}{(\omega'_2-\omega_1)/2}\Bigg\}\ . \qquad (15)
\end{aligned}$$

The integrals can be simplified if we substitute $\omega_1'-\omega_1=\omega_1'',\omega_2'-\omega_2=\omega_2''$ in the first two terms, and integrate with respect to ω_1'',ω_2''. We shall make use of the fact that over a small frequency range of order $1/T$ the factors outside the large parentheses do not vary much, whereas

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\, e^{i\omega x}\frac{\sin(\omega T/2)}{\omega/2}=\theta(T/2-x)\,, \tag{16}$$

where $\theta(x)$ is the unit step function that vanishes for negative argument. Provided T is large enough, the ω_1'',ω_2'' integrals within the large parentheses then yield unity. We handle the third and fourth terms in Eq. (15) in a similar manner, by substituting $\omega_2'-\omega_1=\omega_2'',\omega_1'-\omega_2=\omega_1''$, integrating with respect to ω_2'',ω_1'' and using Eq. (16). We then obtain for sufficiently large T, with the assumption that $\phi(\omega_1,\omega_2)$ is symmetric with respect to ω_1,ω_2,

$$\begin{aligned}P_2(\mathbf{r},\mathbf{r}')=\text{const}\times\int\int d\omega_1 d\omega_2\omega_1^4\omega_2^4\,|\phi(\omega_1,\omega_2)|^2\\ \times\Big(1+\tfrac{1}{2}|\epsilon^*_{\mathbf{k}_1s_1}\cdot\epsilon_{\mathbf{k}_2s_2}|^2 e^{i[\omega_1(\boldsymbol\kappa_2-\boldsymbol\kappa_1)\cdot(\mathbf{r}-\mathbf{r}_0)/c+\omega_2(\boldsymbol\kappa_1-\boldsymbol\kappa_2)\cdot(\mathbf{r}'-\mathbf{r}_0)/c]}e^{i[\boldsymbol\kappa_2\cdot(\mathbf{r}_2-\mathbf{r}_0)-\boldsymbol\kappa_1\cdot(\mathbf{r}_1-\mathbf{r}_0)](\omega_2-\omega_1)/c}\\ +\tfrac{1}{2}|\epsilon^*_{\mathbf{k}_1s_1}\cdot\epsilon_{\mathbf{k}_2s_2}|^2 e^{i[\omega_2(\boldsymbol\kappa_1-\boldsymbol\kappa_2)\cdot(\mathbf{r}-\mathbf{r}_0)/c+\omega_1(\boldsymbol\kappa_2-\boldsymbol\kappa_1)\cdot(\mathbf{r}'-\mathbf{r}_0)/c]}e^{i[\boldsymbol\kappa_2\cdot(\mathbf{r}_2-\mathbf{r}_0)-\boldsymbol\kappa_1\cdot(\mathbf{r}_1-\mathbf{r}_0)](\omega_2-\omega_1)/c}\Big)\,.\end{aligned} \tag{17}$$

We have introduced a reference point $\mathbf{r}_0$ in the detector plane on the line containing the points $\mathbf{r},\mathbf{r}'$ (see Fig. 2). We note immediately that the probability is independent of the integration time T, as one would expect, provided T is long enough to detect the localized photons. Moreover, the interference terms have not integrated to zero, despite the fact that we have taken the measurement time T to be much longer than the coherence time $1/\Delta\omega$. Because of the symmetry of $\phi(\omega_1,\omega_2)$ under the exchange $\omega_1\leftrightarrow\omega_2$, the second interference term in Eq. (17) is just the complex conjugate of the first.

Let us examine the interference pattern in more detail. In practice one would like $\mathbf{r}_1$ and $\mathbf{r}_2$ to coincide with $\mathbf{r}_0$. However, $(\omega_2-\omega_1)/c$ is bounded by the reciprocal coherence length of each photon, so that even in the worst case the exponential factor.

$$e^{i[\boldsymbol\kappa_2\cdot(\mathbf{r}_2-\mathbf{r}_0)-\boldsymbol\kappa_1\cdot(\mathbf{r}_1-\mathbf{r}_0)](\omega_2-\omega_1)/c}\approx 1\,,$$

provided $|\mathbf{r}_2-\mathbf{r}_0|$ and $|\mathbf{r}_1-\mathbf{r}_0|$ are both much smaller than the coherence length $c/\Delta\omega$. Henceforth we assume that this is the case.

If the points $\mathbf{r},\mathbf{r}',\mathbf{r}_0$ all lie in a plane perpendicular to the vector $\boldsymbol\kappa_1+\boldsymbol\kappa_2$ characterizing the direction of the incident light, then from Fig. 2 we have, since $|\boldsymbol\kappa_1-\boldsymbol\kappa_2|=\delta\theta$,

$$\begin{aligned}(\boldsymbol\kappa_1-\boldsymbol\kappa_2)\cdot(\mathbf{r}-\mathbf{r}_0)&=x\delta\theta\,,\\ (\boldsymbol\kappa_2-\boldsymbol\kappa_1)\cdot(\mathbf{r}'-\mathbf{r}_0)&=-x'\delta\theta\,,\\ (\boldsymbol\kappa_1-\boldsymbol\kappa_2)\cdot(\mathbf{r}-\mathbf{r}')&=(x-x')\delta\theta\,.\end{aligned} \tag{18}$$

Now two light waves of wavelength $2\lambda_0$ or frequency $\omega_0/2$ traveling in the directions $\boldsymbol\kappa_1,\boldsymbol\kappa_2$, which are inclined to each other at some small angle $\delta\theta$, give rise to interference fringes with spacing

$$S=2\lambda_0/\delta\theta=4\pi c/\omega_0\delta\theta\,. \tag{19}$$

If the frequency density function $\phi(\omega_1,\omega_2)$ has each frequency centered on $\omega_0/2$, and we substitute $\omega_1=\omega_0/2+\omega'$, $\omega_2=\omega_0/2+\omega''$ in Eq. (17) and make use of Eqs. (18) and (19), we can write

$$\begin{aligned}P_2(\mathbf{r},\mathbf{r}')=\text{const}\times\int\int_{-\infty}^{\infty}d\omega' d\omega''(\omega_0/2+\omega')^4(\omega_0/2+\omega'')^4|\phi(\omega_0/2+\omega',\omega_0/2+\omega'')|^2\\ \times(1+\tfrac{1}{2}|\epsilon^*_{\mathbf{k}_1s_1}\cdot\epsilon_{\mathbf{k}_2s_2}|^2 e^{2\pi i(x-x')/S}e^{i(\omega'x-\omega''x')\delta\theta/c}+\text{c.c.})\,.\end{aligned} \tag{20}$$

The factor $\exp[i(\omega'x-\omega''x')\delta\theta/c]$ represents an effective spread in phase difference between the field at $\mathbf{r},\mathbf{r}'$. We can interpret its significance by introducing the two-dimensional Fourier transform

$$\begin{aligned}F(\tau_1,\tau_2)=\int\int_{-\infty}^{\infty}d\omega' d\omega''(\omega_0/2+\omega')^4(\omega_0/2+\omega'')^4\\ \times|\phi(\omega_0/2+\omega',\omega_0/2+\omega'')|^2\\ \times e^{-i(\omega'\tau_1-\omega''\tau_2)}\,.\end{aligned} \tag{21}$$

As the integrand, apart from the Fourier kernel, is real and non-negative, $F(\tau_1,\tau_2)$ is a correlation function. Then Eq. (20) can be expressed in the more compact form

$$\begin{aligned}P_2(\mathbf{r},\mathbf{r}')=\text{const}\times F(0,0)\{1+|\epsilon^*_{\mathbf{k}_1s_1}\cdot\epsilon_{\mathbf{k}_2s_2}|^2\\ \times|f(x'\delta\theta/c,x\delta\theta/c)|\\ \times\cos[2\pi(x-x')/S+\alpha]\}\,,\end{aligned} \tag{22}$$

where $f(\tau_1,\tau_2)$ is the normalized correlation function

$$f(\tau_1,\tau_2)=F(\tau_1,\tau_2)/F(0,0)\ , \tag{23}$$

and α is the phase of $F(\tau_1,\tau_2)$.

V. DISCUSSION OF THE INTERFERENCE PATTERN

It is clear from Eq. (22) that the joint probability $P_2(\mathbf{r},\mathbf{r}')$ exhibits a cosine modulation with the separation $x-x'$ of the two detectors. This is a form of interference, although it involves a correlation function that is of the fourth order in the field. If the spectral density $(\omega_0/2+\omega')^4(\omega_0/2+\omega'')^4\,|\phi(\omega_0/2+\omega',\omega_0/2+\omega'')|^2$ is unchanged under the transformation $\omega'\to-\omega'$, $\omega''\to-\omega''$, then $F(\tau_1,\tau_2)$ is real and the phase α is zero. The relative depth of modulation is determined by the polarization factor $|\boldsymbol{\epsilon}^*_{\mathbf{k}_1s_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}_2s_2}|^2$ and by the magnitude $|f|$. For two photons that are similarly polarized $|\boldsymbol{\epsilon}^*_{\mathbf{k}_1s_1}\cdot\boldsymbol{\epsilon}_{\mathbf{k}_2s_2}|^2$ will be close to unity. Let us examine the magnitude $|f|$ more closely.

As $\phi(\omega_0/2+\omega',\omega_0/2+\omega'')$ has a spread of order $\Delta\omega$ in both frequencies ω',ω'', it is clear that $f(\tau_1,\tau_2)$ has a range of order $1/\Delta\omega$ in both variables τ_1,τ_2. Moreover, $|f(\tau_1,\tau_2)|$ is close to $f(0,0)=1$ when $\tau_1,\tau_2\ll1/\Delta\omega$. It follows that the "visibility" of the interference pattern will be close to unity when x,x' are sufficiently small, despite the fact that the measurement or integration time T is much longer than the coherence time $1/\Delta\omega$, because the two photons are so briefly localized in time.

It is convenient to express x,x' in multiples N,N' (not necessarily integral) of the fringe spacing S given by Eq. (19). Then

$$\begin{aligned}x&=N4\pi c/\omega_0\delta\theta\ ,\\ x'&=N'4\pi c/\omega_0\delta\theta\ ,\end{aligned} \tag{24}$$

so that

$$\begin{aligned}x\delta\theta/c&=4\pi N/\omega_0\ ,\\ x'\delta\theta/c&=4\pi N'/\omega_0\ .\end{aligned} \tag{25}$$

The condition for $|f(x'\delta\theta/c,x\delta\theta/c)|\approx1$, which requires the delay times to be much less than $1/\Delta\omega$, can be expressed in the form

$$N,N'\ll\omega_0/4\pi\Delta\omega\ . \tag{26}$$

In other words, the number of fringes separating $\mathbf{r}$ from $\mathbf{r}_0$ and $\mathbf{r}'$ from $\mathbf{r}_0$ must be much less than the reciprocal of 2π times the relative bandwidth. This is easy to satisfy when the relative bandwidth $2\Delta\omega/\omega_0$ is small, but may be difficult to satisfy when the bandwidth is appreciable. Still it seems that at least one or two interference maxima and minima, or interference fringes, should be resolvable when $2\Delta\omega/\omega_0$ is 10% or less.

Let us examine the case $2\Delta\omega/\omega_0\ll1$ a little more closely. Under these conditions the visibility of the interference pattern should be close to 100% over many fringes. It then follows from Eq. (22) that the joint detection probability $P_2(\mathbf{r},\mathbf{r}')$ vanishes when

$$x-x'=(n+\tfrac{1}{2})S,\quad n=0,\pm1,\pm2,\ldots\ . \tag{27}$$

In other words, two photons can never be detected at two points separated by an odd number of half interference fringes, despite the fact that one photon can be detected anywhere, because there is no interference pattern in the second-order sense. This conclusion is without classical analogy. It can be shown that with two independent classical sources the maximum realizable visibility of the modulation would be 50%, so that $P_2(\mathbf{r},\mathbf{r}')$ can never vanish.[8]

The vanishing of $P_2(\mathbf{r},\mathbf{r}')$ for two photons at widely separated points $\mathbf{r},\mathbf{r}'$ confronts us with another example of quantum-mechanical nonlocality, for the outcome of a photoelectric measurement at $\mathbf{r}$ appears to be influenced by where we have chosen to place the $\mathbf{r}'$ detector. At certain positions $\mathbf{r}'$ we can never hope to get a count at $\mathbf{r}$ when there is a count at $\mathbf{r}'$, whereas at other positions $\mathbf{r}'$ it is possible. This conclusion is related to the fact that in quantum mechanics we cannot associate an objective physical reality with the two photons that is independent of the measurement we choose to make. The phenomenon involves the same violation of local realism that was recently tested in the experiments of Aspect and his collaborators,[18,19] and was first discussed by Einstein, Podolşky, and Rosen.[16]

We conclude that two photons produced in the process of spontaneous parametric down-conversion should exhibit interference effects, despite the fact that the phase of each interfering component is undefined and the measurement or integration time greatly exceeds the coherence time. These are clearly nonclassical interference effects.

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