# Electromagnetic drift vortices

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Dipole vortex solutions are obtained for a set of nonlinear equations describing low-frequency electromagnetic turbulence in an inhomogeneous magnetized plasma.

Recently, it has been pointed out[1–3] that electromagnetic solitary dipole vortex solutions often contain discontinuities in the perturbed magnetic field **B** and the parallel (to the external magnetic field $B_0\hat{\mathbf{z}}$) current density $J$. The origin of the discontinuity lies in the oversimplified[4,5] relation

$$A = a\phi + bx\ , \tag{1}$$

where $a$ and $b$ are constants, between the scalar potential $\phi$ and the $z$ component $A$ of the vector potential. Because independent inner (regular at $r=0$) and outer (vanishes as $r\to\infty$) solutions must be obtained separately and matched at a certain radius (say $r_0$), a solution which is everywhere continuous in $\phi$ need not be continuous in $\mathbf{B}=-\hat{\mathbf{z}}\times\nabla A$ and $J=-(c/4\pi)\nabla^2 A$, since both $a$ and $b$ have different values inside and outside of $r=r_0$. It has been suggested[6] that sometimes **B** and $\nabla^2 A$ can still be made continuous at $r_0$ by allowing for specific amplitude, velocity, or angle of propagation, etc., by fixing the given parameters. More satisfactory solutions to this problem were recently reported by Mikhailovskii *et al.*,[2] Petviashvili and Pokhotelov,[3] and Liu and Horton.[7] These authors considered the drift-Alfvén modes and retained the exact relation between $\phi$ and $A$ instead of the ansatz (1), and solved an equivalent fourth-order linear partial differential equation. Their solutions are free from discontinuities up to the second derivatives, and therefore do not require invoking surface currents at $r=r_0$ in order to ensure continuity of the physical quantities. Using a similar approach, we show in this paper that the set of reduced magnetohydrodynamic equations[8] describing low- (compared to the ion cyclotron frequency $\Omega_i=eB_0/m_ic$) frequency electromagnetic turbulence also admits modon solutions which have continuous physical quantities.

We start with the following equations describing low-frequency electromagnetic turbulence:[4,5,8]

$$(\partial_t+\eta\kappa_i\partial_y)\nabla^2\phi+g\partial_y(p_e+\eta p_i) + \hat{\mathbf{z}}\times\nabla(\phi+\eta p_i)\cdot\nabla\nabla^2\phi+\hat{\mathbf{b}}\cdot\nabla\nabla^2 A=0\ , \tag{2}$$

$$(\partial_t-\kappa_e\partial_y)A+\hat{\mathbf{b}}\cdot\nabla\phi=\hat{\mathbf{b}}\cdot\nabla p_e\ , \tag{3}$$

$$(\partial_t+\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla)p_e-\kappa_e\partial_y\phi+\hat{\mathbf{b}}\cdot\nabla\nabla^2 A=0\ , \tag{4}$$

$$(\partial_t+\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla)p_i-\kappa_i\partial_y\phi=0\ , \tag{5}$$

where $\eta$ is the temperature ratio $T_i/T_e$, $\kappa_j=\rho_s\partial_x(\ln p_{j0})$ is the normalized drift velocity, $g=-2\rho_s\partial_x(\ln B_0)$ is the normalized curvature, and $\hat{\mathbf{b}}\cdot\nabla=\partial_z-\hat{\mathbf{z}}\times\nabla A\cdot\nabla$. In Eqs. (2)–(5), we have normalized $\phi$, $A$, $p_j$, $\nabla\equiv\nabla_\perp$, $\partial_z$, and $\partial_t$ by $T_e/e$, $\omega_{pi}T_e/e\Omega_i$, $p_{j0}$, $\rho_s^{-1}$, $\omega_{pi}/c$, and $\Omega_i$, respectively. Here, $\rho_s=(T_e/\Omega_i^2 m_i)^{1/2}$, and $\omega_{pi}=(4\pi n_0e^2/m_i)^{1/2}$. Equation (2) is from the conservation of the charge density, Eq. (3) follows from the Ohm's law, and Eqs. (4) and (5) represent the equations of state for the electrons and the ions. For the derivation of Eqs. (2) to (5), we refer the reader to Refs. 3 and 8. These equations describe drift-Alfvén waves, ballooning modes, electrostatic and electromagnetic drift waves, as well as nonlinear tearing modes. They have often been used for considering general problems of low-frequency turbulence in magnetized plasmas.

To obtain the modon solutions, we first define a two-dimensional quasistationary coordinate system $(x,\xi)$, with $\xi=y+\alpha z-Mt$, where $\alpha$ and $M$ are constants. Furthermore, we introduce the Kadomtsev scalar potential[9] $\psi=\phi-MA/\alpha$ and let[4]

$$p_e=\psi-\kappa_e(\phi-\psi)/M\ , \tag{6}$$

and

$$p_i=-\kappa_i\phi/M\ , \tag{7}$$

so that Eqs. (3) and (5) are satisfied exactly. Equations (2) and (4) can then be written in terms of the Poisson brackets

$$[\phi-Mx,\psi+G(\gamma-1)\phi-\gamma\nabla^2\phi]=0\ , \tag{8}$$

$$[\phi-\psi-Mx,\psi-(\alpha^2\delta/M^2)\nabla^2(\phi-\psi)]=0\ , \tag{9}$$

where $G\equiv g/M\ll 1$ has been used. We have also defined $\delta=(1+\kappa_e/M)^{-1}$ and $\gamma=\delta(1-\eta\kappa_i/M)$. The Poisson bracket is given by

$$[f_1,f_2]\equiv\partial_x f_1\partial_\xi f_2-\partial_x f_2\partial_\xi f_1\ .$$

Clearly, Eqs. (8) and (9) are satisfied if

$$\psi-\gamma\nabla^2\phi=[G(1-\gamma)+C/M]\phi-CX\ , \tag{10}$$

and

$$\psi=(\alpha^2\delta/M^2)\nabla^2(\phi-\psi)\ , \tag{11}$$

 

where $C$ is a constant. We note that Eq. (11) is the simplest choice, such that if $\psi$ and $\nabla^2\phi$ are continuous, $\nabla^2\psi$ is also continuous. It is clear from Eq. (9) that more general choices are possible.

Eliminating $\psi$ in (10) and (11), one obtains

$$\nabla^4\phi+\beta_1\nabla^2\phi+\beta_2\phi=\beta_3 x \ , \tag{12}$$

where

$$\beta_1=(a-1)/\gamma+M^2/\alpha^2\delta \ ,$$

$$\beta_2=a\beta_3/C=aM^2/\alpha^2\delta\gamma \ ,$$

and

$$a=G(1-\gamma)+C/M \ .$$

Equation (12) is a fourth-order linear inhomogeneous partial differential equation for which no uniformly valid well-behaved analytic representation of the solution is known. However, piecewise well-behaved analytic solutions can be obtained in an inner region containing $r=0$ and an outer region containing $r\rightarrow\infty$, where $r^2=x^2+\xi^2$, and then joined at some intermediate location $r=r_0$, under the condition that all physical quantities must be continuous.

Accordingly, we proceed to obtain the solutions of Eq. (12) in the inner ($r<r_0$) and the outer ($r>r_0$) regions. In the outer region, the localization condition requires $\beta_3=C=0$, so that Eq. (12) becomes homogeneous. The appropriate solution is

$$\phi_{\text{out}}=[B_1K_1(\lambda_1 r)+B_2K_1(\lambda_2 r)]\cos\theta \ , \tag{13}$$

where $B_1$ and $B_2$ are constants, and $\cos\theta=x/r$. We have also defined

$$\lambda_{1,2}^2=\tfrac{1}{2}[-\beta_1\pm(\beta_1^2-4\beta_2)^{1/2}]$$

for $\beta_1<0$, $\beta_1^2>4\beta_2>0$, and $C=0$. Thus, the outer solution decays very rapidly as $r\rightarrow\infty$.

For the inner region $r<r_0$, we have $C\neq 0$. The corresponding solution of (12) is

$$\phi_{\text{in}}=[B_3J_1(\lambda_3 r)+B_4I_1(\lambda_4 r)+(\beta_3/\beta_2)r]\cos\theta \ , \tag{14}$$

where

$$\lambda_{3,4}^2=\tfrac{1}{2}[(\beta_1^2-4\beta_2)^{1/2}\pm\beta_1] \tag{15}$$

for $\beta_2<0$, and $C\neq 0$. The Kadomstev potential $\psi$ then follows from Eqs. (10), (13), and (14).

The five integration constants $B_1$, $B_2$, $B_3$, $B_4$, and $C$ can be determined from the condition that $\phi$, $\partial_r\phi$, $\nabla^2\phi$, $\psi$, and $\partial_r\psi$ must be continuous at $r=r_0$. The procedure is straightforward but tedious.[2,7] The results are cumbersome and do not yield any further physically interesting points, and shall thus be omitted here. Note, however, that the current density ($\propto\nabla^2 A$) is also continuous at $r_0$ because of Eq. (11) and the definition of $\psi$.

The solutions given by Eqs. (13) and (14) represent a well-localized vortex structure with $\phi\rightarrow r^{-1/2}\exp(-\lambda_2 r)$ as $r\rightarrow\infty$.

It is of interest to point out that if the curvature vanishes ($g\rightarrow 0$), Eq. (12) for the outer region becomes

$$\nabla^4\phi-\nu_1\nabla^2\phi=0 \ , \tag{16}$$

where $\nu_1=1/\gamma-M^2/\alpha^2\delta$. On the other hand, for the inner region, we have

$$\nabla^4\phi+\nu_2\nabla^2\phi+\nu_3(\phi-Mx)=0 \ , \tag{17}$$

where

$$\nu_2=(C/M+M^2\gamma/\alpha^2\delta-1)/\gamma \ ,$$

and

$$\nu_3=CM/\alpha^2\delta\gamma \ .$$

Thus, the outer and inner solutions are given by[2,7]

$$\phi_{\text{out}}=[A_1K_1(\sqrt{\nu_1}r)+A_2/r]\cos\theta \ , \tag{18}$$

$$\phi_{\text{in}}=[A_3J_1(\lambda_5 r)+A_4I_1(\lambda_6 r)+Mr]\cos\theta \ , \tag{19}$$

where for $\nu_1>0$ and $\nu_3<0$,

$$\lambda_{5,6}^2=\tfrac{1}{2}[(\nu_2^2-4\nu_3)^{1/2}\pm\nu_2] \ .$$

We note that the constant $C$ is related to $\lambda_5$ and $\lambda_6$ by

$$C/M=(\lambda_5^2-\lambda_6^2)\gamma+1-M^2\gamma/\delta\alpha^2 \ . \tag{20}$$

The Kadomtsev potential $\psi$ can be obtained from Eqs. (10) with $G=0$, (18) and (19). Again, continuity of $\phi$, $\partial_r\phi$, $\nabla^2\phi$, $\psi$, and $\partial_r\psi$ at $r=r_0$ determines the five constants $A_1$, $A_2$, $A_3$, $A_4$, and $C$. Here, we have $\phi\rightarrow r^{-1}$ as $r\rightarrow\infty$, so that the vortex is not well localized. The present solution ($g=0$) is a generalization of Liu and Horton[7] to include finite ion temperature effects.[8,9] On the other hand, for $g=0$, $T_i=0$, and $\kappa_j\rightarrow\infty$, one finds that our basic system of equations (10) and (11) describe nonlinear shear Alfevn waves[10] in a uniform plasma. Note that Ref. 10 has briefly addressed the issue of the solitary Alfvén vortices without presenting any specific results for the vortex profiles.

In this paper, we have shown that the set of nonlinear reduced magnetohydrodynamic equations describing low-frequency plasma motion admits dipole vortex solutions without discontinuities in the magnetic field and the current density. The drift-Alfvén vortices in a warm plasma are found to be not well localized. Inclusion of magnetic curvature effects gives rise to spatially well-localized electromagnetic ballooning vortices. In contrast to our previous studies,[4,5] the present solutions are free from discontinuities in the physical quantities, such as the vorticity, the perturbed magnetic field, and the parallel component of the current density. The electromagnetic vortices discussed here satisfy the conditions $\beta_1<0$ and $\beta_1^2>4\beta_2>0$ for the ballooning mode case, and $\nu_1>0$ and $\nu_3<0$ for the drift-Alfvén case. These conditions are considerably different from those for the corresponding electromagnetic vortices with discontinuities in the magnetic field and current density perturbations.[4,5] The latter vortices are based on an ad hoc $\phi-A$ relation such that $\phi$ satisfies a second-order (rather than fourth-order) differential equation, similar to that of the hydrodynamic Rossby[11] and the electrostatic Hasegawa-Mima vortices.[12–15]

The present calculation makes use of the local approximation which is valid for vortices whose dimension is much smaller than the density gradient scalelength and the radius of curvature of the inhomogeneous magnetic field.

The stability of the vortex discussed here, as well as the statistical behavior of an ensemble of such electromagnetic vortices, are important for the understanding of anomalous particle and heat transport in a magnetically confined fusion plasma. However, these topics are outside the scope of this Brief Report.

At present, there do not exist any experimental observations which conclusively verify the existence of dipolar vortex motion in plasmas. However, there have been a few numerical experiments[15,16] which support the existence of vortex motion in a magnetized plasma. First, Horton[15] has carried out numerical investigation of the Hasegawa-Mima equation[13] and his results indicate the existence of drift wave monopolar and dipolar vortices. Secondly, Bekki and Kaneda[16] have numerically analyzed the basic system of Eqs. (2)–(4) assuming $g=0$ and $p_i=0$. The results of their numerical simulation show the formation of three-dimensional electromagnetic vortices. In view of the above studies, we conclude that the present as well as earlier investigations[1–7,10–14] related to vortex motion involving various kinds of wave motion are necessary in order to understand their existence region and propagation characteristics. Furthermore, the existence of large-scale motion in both laboratory and space plasmas is also an indication of vortex interaction, which often leads to coalescence of the vortices.[17]

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