Čerenkov radiation and electromagnetic pulse produced by electron beams traversing a finite path in air

Fred R. Buskirk and John R. Neighbours
Physics Department, Naval Postgraduate School, Monterey, California 93943
and Hydrodynamics Group, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
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Coherent Čerenkov radiation has been investigated previously in the time domain for an infinite path. The present calculations for a finite path length show an effect analogous to diffraction (in the frequency domain) in which radiation fields appear both at Čerenkov angles and at other angles. The latter have previously been named electromagnetic pulse fields.

I. INTRODUCTION

Čerenkov radiation occurs when charges move faster than radiation in a medium. Most work[1-4] is concerned with optical radiation produced by a point charge, and involves the Fourier spectra of these fields. Here, in contrast, we explore the time dependence of the radiation fields, using our earlier[5] formulation to describe the radiation from a bunch of electrons passing through an infinite medium; however, now the medium has finite length, which causes diffraction so that the radiation is produced at angles other than the usual Čerenkov angle. This spreading by diffraction was investigated earlier in the Fourier-expansion approach,[6-8] but using the present time-dependent fields, new insights are developed, and for shorter paths the Čerenkov fields are related to other forms of radiation, namely, what is referred to as electromagnetic pulse[9] (EMP), transition, and ordinary dipole radiation. It is much easier to understand Čerenkov radiation from a finite-size charge than from a point charge; in the former case, the Čerenkov fields remain finite but become singular for a point charge.[10]

II. TIME DEPENDENCE OF FIELDS

Let all charges within a bunch move along the z axis with the same velocity v, which is larger than the velocity c of radiation in the medium. Let c_0 be the velocity of radiation in a vacuum, let $s^2=x^2+y^2$, and assume the volume charge density ρ_v has the form

$$\rho_v(\mathbf{r},t)=\rho_0(z-vt)\delta(x)\delta(y) , \tag{1}$$

where ρ_0 represents an arbitrary line density. Following our earlier work,[5] the vector potential is

$$\mathbf{A}(\mathbf{r},t)=\frac{\mathbf{v}}{c_0}\int R^{-1}\rho_0(u)dz' , \tag{2}$$

where the variable $u=z'-vt'$ becomes

$$u=z'-vt+\frac{v}{c}[s^2+(z-z')^2]^{1/2} . \tag{3}$$

Retardation is included by the form of u in Eq. (3). The magnetic field B has radiation terms resulting from taking the derivatives of ρ_0 in Eq. (2). This leads to B in the θ direction of cylindrical coordinates, with a magnitude

$$B=\frac{v^2}{cc_0}\int\frac{s}{R^2}\rho_0'(u)dz' . \tag{4}$$

An approximate evaluation of Eq. (4) proceeds as follows: $u(z')$ is plotted as a function of z'. For further calculation assume ρ_0' has linear rising and falling ramps, of width a, separated by a distance b. Then ρ_0' consists of two opposite polarity square pulses. For early times, the u curve is high (see Ref. 5, Fig. 1) and the integrand is zero for all values of z'. As time increases, the u curve drops and the minimum intersects the ρ_0' pulse, and contributes to the integral in Eq. (4). The field $\mathbf{E}$ may also be calculated; in Ref. 5 it was shown that $\mathbf{E}$ is perpendicular to $\mathbf{B}$ and to $\mathbf{R}_m$, and $\mathbf{R}_m$, the vector from the particle (at the retarded time) to the observer, is at an angle of θ_c to the z axis. Then $E/B=c/c_0$; both the fields fall off as $R^{-1/2}$, appropriate for radiation from a cylindrical source, and the total energy radiated agrees with the Fourier approach. The two opposite pulses of the radiation field have the same separation as the front and rear slopes of the current pulse.

We now calculate the radiation fields for the case of a finite path. The physical situation shown in Fig. 1 is one in which a beam emerges at $z'=0$ from an accelerator,

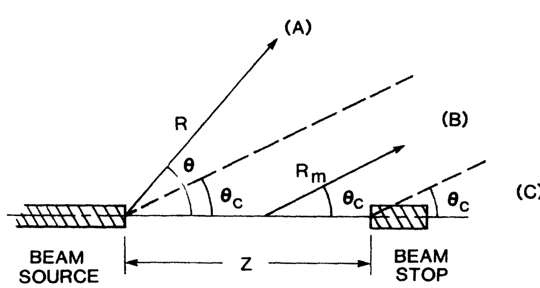

FIG. 1. For a finite path length Z, an observer in regions A or C finds EMP radiation, shown in Fig. 4 and developed in Secs. II A and II C. In region B, a Čerenkov field pulse described in Sec. II B occurs.

passes through a dielectric medium and at $z'=Z$, stops in an absorber. The path (usually air) from $z'=0$ to Z is the radiator. The beam bunch has a linear rising ramp of length a, is then constant and has a linear decrease of length a, with an effective length b, from the midpoint of the rise to the midpoint of the fall.

The calculation of the fields is based on Eq. (4) for B; corresponding results will hold for E. We again assume that s/R^2 is about constant in the range of integration and may be factored out. Then $s/R=\sin\theta$ and we have

$$B=\frac{v^2}{cc_0}\frac{\sin\theta}{R}\int \rho_0'(u)dz' . \tag{5}$$

The main difference between the calculations below and those done previously[5] is that the range of z' is finite, from 0 to Z, to represent a finite length (Z) of radiation source. The results of the integration depend on the relation of various parameters.

A. Outside the Čerenkov cone, Z long

Figure 2 represents the situation involved in evaluating Eq. (5). The $u(z')$ curve moves down in time [Eq. (3)]. For early times, the integrand is always zero. Later the u curve moves down and intersects the dotted rectangle representing the region of u and z' where the integrand is constant. In the situation shown, the integral builds up to a peak value in a time interval a/v, and the integral saturates at the value $\rho_0\Delta z=\rho_0 a$ divided by the slope of the u curve. From Eq. (3), the slope is

$$\frac{\partial u}{\partial z'}=1-\frac{v}{c}\frac{z-z'}{[s^2+(z-z')^2]^{1/2}}$$
$$=1-\frac{v}{c}\cos\theta . \tag{6}$$

Furthermore $\rho_0' a$ is the peak value of ρ_0 which is $I_0 c_0/v$, where I_0 is the peak current.

Thus the integral saturates at

$$B_{\max}=\frac{v}{c}\frac{\sin\theta}{R}I_0\frac{1}{1-\frac{v}{c}\cos\theta} , \tag{7}$$

where R is measured from the start of the source, $z'=0$, to the observer, and it makes an angle θ to the z axis.

The rise time was a/v, a similar fall time occurs, and the duration of the pulse is given by Z times the slope of the u curve divided by v, or $Z[1-(v/c)\cos\theta]/v$. Because this combination appears often, we define the effective length

$$Z_e=Z\left[1-\frac{v}{c}\cos\theta\right] . \tag{8}$$

The significance of Z_e is that it yields the time difference for two signals emitted by a given charge at two points separated by a distance Z in the laboratory. Note that $Z_e=0$ at the Čerenkov angle, as expected, because signals emitted at θ_c from all parts of the path reach a distant observer at the same time.

Thus we have, for the leading ramp of the current pulse, a field at the observer of value given by Eq. (7) with lengths shown in Fig. 3(a). Here the lengths are times multiplied by v. Also shown is the negative field pulse caused by the back ramp of the current pulse. The two pulses combine to give the symmetric pulse of Fig. 3(b), which a separation that is the larger of Z_e or b, and a duration that is the smaller of Z_e or b.

B. On the Čerenkov cone

If the observer is in the Čerenkov region (region B in Fig. 1), the rectangular region of integration in Fig. 1 is centered about the minimum in the $u(z')$ curve. If Z is large, the integral for the field is the same as in our earlier paper. The pulse starts when the $u(z')$ curve, as it advances down in time, becomes tangent to the rectangular integration region. The integral increases as $t^{1/2}$ until the u curve is tangent to the lower boundary, and then decreases, again proportional to $t^{1/2}$. The result is

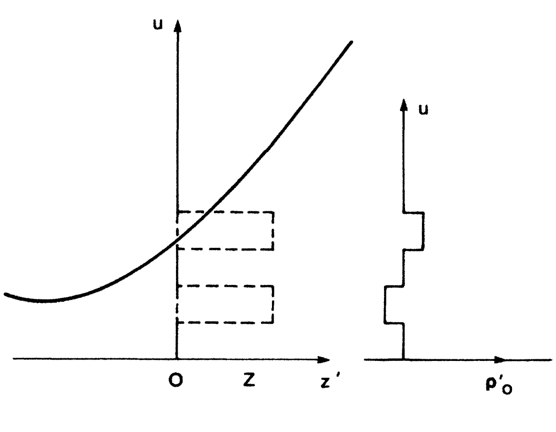

FIG. 2. Variable u plotted as a function of z'. As time increases, the curves are displaced downward. The derivative of the current pulse is shown. When the $u(z')$ first intersects the ρ_0' pulse, the integrand of Eq. (4) becomes nonzero, and the Čerenkov pulse starts (Sec. II).

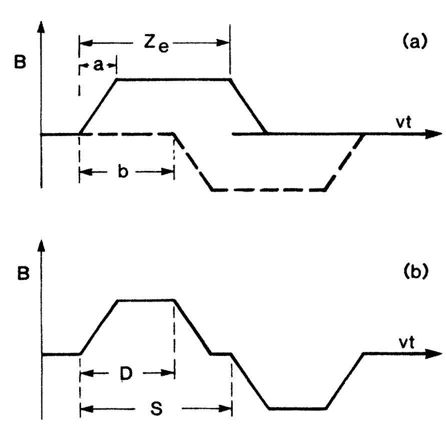

FIG. 3. Field pulse outside the Čerenkov cone for finite path length. (a) shows the positive and negative pulses separately, whereas (b) shows the composite field. The separation S is the larger of Z_e or b, whereas the duration D is the smaller (Secs. II A and II C).

$$B_{\max}=\frac{v^2}{c^2}\frac{1}{R_m}\frac{\sin\theta_c}{\tan^{3/2}\theta_c}I_0 2\sqrt{2}\sqrt{s/a} \ . \tag{9}$$

The positive and negative pulses have rise and fall times of a/v and are separated by b/v. The field falls as $1/R_m$ as one proceeds outward at a fixed angle with $s=R_m\sin\theta_c$.

If the path is short, the B field pulse, as a function of time, increases as $t^{1/2}$ as noted above, but the maximum value of the integral for Eq. (5) occurs when the $u(z')$ curve first intersects the vertical limits (0 and Z) of integration of the rectangular region in the u-z' plane. Then the maximum magnetic field becomes

$$B_{\max}=\frac{v^2}{cc_0}\frac{\sin\theta_c}{R_m}\rho_0'(u)Z$$
$$=\frac{v^2}{cc_0}\frac{\sin\theta_c}{R_m}\rho_0'a\frac{Z}{a} \ . \tag{10}$$

Eliminating ρ_0 in terms of I_0 yields

$$B_{\max}=\frac{v}{c}\frac{\sin\theta_c}{R_m}I_0\frac{Z}{a} \ . \tag{11}$$

In this case, positive and negative pulses of width a have a separation b, which is the same as for the long radiator, but the pulses have flat tops.

If the observer moves out along the Čerenkov cone, the long-path case changes into the short-path case. In the former, the source is long and the observer sees a field associated with cylindrical symmetry. But as the observer moves out, the radiator of length Z appears to be short, leading to the R^{-1} field of Eq. (11) instead of the $R^{-1/2}$ (cylindrical) field of Eq. (9). The transition from the cylindrical wave to the spherical wave occurs when $2\Delta z$ [defined in Eq. (18) of Ref. 5] is equal to Z. This yields for the value of s, denoted by s_s,

$$as_s\frac{v^2}{c^2}\frac{2}{\tan^3\theta_c}=Z^2 \ . \tag{12}$$

That is, for $s>s_s$ [where s_s satisfied Eq. (12)] the wave becomes spherical and decreases as R^{-1} or s^{-1}.

C. Outside the Čerenkov cone, Z short

In the Secs. II A and II B cases were considered in which $Z_e>b>a$, but other situations are possible. Because a is the rise and fall length of the current pulse, and b is the width measured between the half-maximum points, we must have $b\geq a$. We could then have $b>Z_e>a$ and $b>a>Z_e$ cases. For the latter the field is again obtained by evaluating Eq. (5). The horizontal range of integration is short so that the range of the variable z' is always Z, if we are off the Čerenkov cone. The result is

$$B=\frac{v^2}{cc_0}\frac{\sin\theta}{R}\rho_0'Z \ . \tag{13}$$

Noting that $v/c_0\rho_0 a=I_0$, the peak current in the pulse, we find

$$B=\frac{v}{c}\frac{\sin\theta}{R}I_0\frac{Z}{a} \ . \tag{14}$$

The field pulse at the observer will have a rise and fall time Z_e/v, the pulse length is a/v, and it will be followed by a similar negative pulse at a time b/v later.

III. SEMI-INFINITE PATH

Let the beam emerge from the accelerator window at $z=0$ and traverse a path, which is idealized to be infinitely long. Starting from the point where the beam emerges, define a cone with apex angle θ_c relative to the beam. If the observer is anywhere inside the cone, the minimum of the $u(z')$ curve will intersect the $\rho_0'(u)$ pulses, and the radiation field will be the same as found in Ref. 5 and described in Sec. II B. The radiation fields will have a positive and negative pulse separated by a distance b. This region is dominated by Čerenkov radiation.

Now let the observer be outside the Čerenkov cone. The situation is somewhat like that described in Sec. II except that the horizontal range of integration is infinite in the positive direction. The rising part, or head, of the current pulse leads to fields which are essentially constant, whereas the following tail gives an opposite field delayed by a time b/v. We thus have a field pulse in the region outside the Čerenkov cone of length b but only of one sign. This is EMP, and in the above model both EMP and Čerenkov radiation exist. The field lines are shown qualitatively in Fig. 4.

IV. JUSTIFICATION OF MODEL

The model using a finite path is supported to represent radiation from a bunch of electrons, emerging from an accelerator system at $z'=0$, and stopped by some means at $z'=Z$. No specific account has been made for these boundaries; radiation by return currents has been neglected. Consider the following model: the charge that suddenly appears at $z'=0$ in all the cases is furnished by a source electron pulse of lower velocity v_s which moves in the region $z'<0$ and meets the previously specified pulse

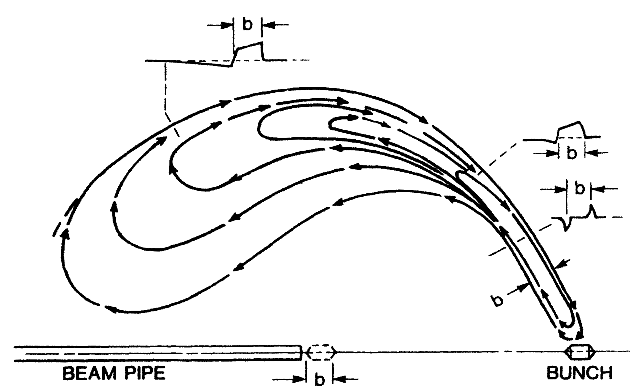

FIG. 4. Qualitative representation of field lines for an electron bunch traversing a semi-infinite path. As the position of the observer changes, the pulse form changes as shown in the inset. Čerenkov radiation occurs at the lower right; EMP pulses occur to the upper left.

at $z'=0$. To conserve charge at all times at $z'=0$, $b_s=bv_s/v$, $a_s=av_s/v$, and $I_{0s}=I_0$, where I_{0s}, a_s, and b_s are the current, rise, and width parameters for the slow pulse approaching the boundary from negative z'.

The radiation pulse from the slow incoming electrons is given by Eq. (7) with the appropriate lower velocity v_s substituted. By inserting a very low value of v_s the field becomes small, and because b_s becomes small, the time in which the field pulse occurs becomes short, so that the radiated energy becomes very small. Thus we conclude that inclusion of source and return currents, required to conserve charge, contribute little to radiation fields calculated, and may be neglected.

V. DISCUSSION

Čerenkov radiation has been described above and in Ref. 5 in terms of time dependence of the radiation fields caused by finite charge distributions such as are realized by bunches emitted by an accelerator. In the usual Fourier-expansion formalism, either a finite size of the charge or dispersion in the medium limits the radiated power at the high-frequency end of the spectrum, and a finite length of path in the medium produces diffraction of the angle of the emitted radiation about the Čerenkov angle.

The present time-dependent field formulation reveals the following properties of Čerenkov radiation. (a) The radiation is associated with dI/dt at the leading and trailing parts of the pulse. (b) The Čerenkov radiation (for an infinite path) consists of positive and negative pulses separated by a distance, which is the pulse length. (c) For a semi-infinite path, Čerenkov radiation appears within a cone of angle θ_c, whose apex is at the start of the path. An EMP pulse appears outside that cone. (d) For a finite path of length Z, both Čerenkov and EMP appear, the latter dominating as Z becomes smaller. For Z short and β small, the $\sin\theta/(1-\beta\cos\theta)$ dependence of the EMP pulse becomes $\sin\theta$; thus the fields at low β becomes essentially a single pulse of dipole radiation.

Finally, it should be noted that Čerenkov radiation is not a different radiation to be added on to other forms of radiation when $v>c$ (medium) but should appear naturally in a correct calculation of the radiation. If $v<c$, radiation also occurs but without the characteristic shock-wave-like character of Čerenkov radiation.

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