Generating antibunched light from the output of a nondegenerate frequency converter

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The signal and idler photons generated by a parametric down converter are highly correlated. We show that the information obtained from a photodetector monitoring the idler port can be used to open and close a shutter in the signal port in such a manner that an antibunched photon beam is produced.

In the parametric down-conversion process, an incoming pump photon interacting with a nonlinear medium having a second-order polarizability decays into a pair of lower-frequency photons. In the nondegenerate case when the two photons are emitted into modes that can be distinguished due to spatial or frequency separation, it is conventional to refer to one mode as the signal mode and the other as the idler mode. The frequencies ω_p, ω_s, and ω_i of the pump, signal, and idler modes are not independent, but are related through $\omega_p = \omega_s + \omega_i$. The pairs of signal and idler photons generated in the parametric down-conversion process are highly correlated[1] and consequently, as pointed out by Graham,[2] a measurement of the number of idler photons generated determines the number of signal photons that have been generated. Recently Hong and Mandel[3] have readdressed the issue of time correlation between signal and idler photons. They found that the two-photon correlation function has a range set by the reciprocal bandwidth of the down converted photons.

Here it is shown how this strong time correlation between signal and idler photons can be exploited to generate an antibunched light beam.[4,5] The device to be considered is depicted in Fig. 1. It has a configuration similar to a device proposed by Saleh and Teich[6] for generating weakly sub-Poissonian light from a green/violet cascade in ^{40}Ca but it is operated somewhat differently. Jakeman and Walker[7] have described a related scheme for generating sub-Poissonian light in which a shutter in the pump of a parametric down converter is controlled by a photodetector in the idler beam. Hong and Mandel have recently performed experiments in which signal photons generated in a parametric down converter are used to gate a photodetector observing the corresponding idler photon.[8] They demonstrate that in this manner an ideal one-photon state is produced. The calculations performed here are for a class of experiments including those of Hong and Mandel.

In operation the parametric down converter F of the device depicted in Fig. 1 is pumped with the laser P. The signal and idler input ports a_1 and a_2, respectively, are terminated with blackbody absorbers B. The signal and idler photons exit the parametric down converter via ports b_1 and b_2, respectively. The photoemissive detector D monitors the photons leaving the idler output port.

An electronic box E examines the signal generated by the photodetector and decides whether to open the shutter S placed in the signal beam path. Consider the case when D has unit quantum efficiency. The electronic box E can then be programmed to search for the arrival of n photons within a coherence time interval T_c by examining the output of D. When such an event occurs, E opens and closes the shutter in such a manner that only the corresponding n photons in the signal beam are allowed to pass through.

In order for this feed-forward scheme to work, the decision whether or not to open the shutter must be made before the n-photon packet arrives at the shutter. This can be arranged by making the signal beam path from F to the mirror M to the shutter S sufficiently long. The shutter must open or close on a time scale fast compared to the coherence time T_c. The coherence time can be lengthened by means of optical cavities supporting the

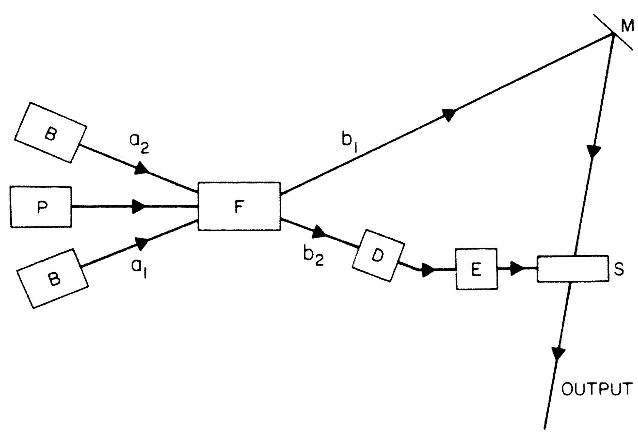

FIG. 1. A scheme for generating antibunched light from a nondegenerate down converter. Cold blackbodies B terminate the input ports a_1 and a_2 of the down converter F. Photons from the pump P decay into correlated pairs of photons which leave the signal port b_1 and idler port b_2. The photodetector D counts the idler port photons. The electronic box E is programmed to open shutter S when a particular number of photons is counted by D in a coherence time.

signal and idler modes.[9] Cavities with linewidths of the order of 1 MHz can readily be constructed. Hence coherence times as long as 10^{-6} s can be achieved. Kerr switches can easily switch on the time scale of 10^{-8} s.

When E is programmed in the manner described above, the output light leaving the shutter will consist of a series of n-photon wave packets interspersed with vacuum wave packets. The intensity correlation function measured via the prescription of Hanbury-Brown and Twiss[10] will exhibit antibunching. It will be shown here that even when an inefficient photodetector ($\eta < 1$) is used to monitor the idler beam the output beam can still exhibit a significant amount of antibunching.

It is worth noting that the device F, which is referred to as a parametric down converter, could also be a four-wave mixer. The four-wave mixing process in which two pump photons are converted into a signal and an idler photon via $\omega_p + \omega_p = \omega_s + \omega_i$ is not significantly different from the parametric down-conversion process at least as far as the dynamics of the signal and idler photons is concerned.[9,11–13] In fact, any device which performs a mode transformation which to a good approximation is of the form

$$\begin{bmatrix} b_1 \\ b_2^\dagger \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2^\dagger \end{bmatrix}, \tag{1}$$

where

$$\begin{aligned} |S_{11}|^2 - |S_{12}|^2 &= 1, \\ |S_{22}|^2 - |S_{21}|^2 &= 1, \\ S_{11}S_{21}^* &= S_{12}S_{22}^*, \end{aligned} \tag{2}$$

can be used for F provided $|S_{11}|^2 > 1$ and $|S_{22}|^2 > 1$. Caves and Schumaker[14] have developed mathematical machinery for conveniently treating the quantum-mechanical behavior of devices performing mode transformations of the type Eq. (1).

For the transformation Eq. (1) one can show that

$$b_1^\dagger b_1 - b_2^\dagger b_2 = a_1^\dagger a_1 - a_2^\dagger a_2 . \tag{3}$$

Hence the difference between the number of signal photons and idler photons is conserved under the action of the down converter. When the input modes derive from cold blackbody absorbers, the input state is an eigenstate of $a_1^\dagger a_1 - a_2^\dagger a_2$ with zero eigenvalue. Equation (3) then implies that the number of photons exiting port b_1 is equal to the number leaving port b_2.

When the incoming light a_1 and a_2 consists of vacuum fluctuations the phases of the S_{ij} become inconsequential and, without loss of generality, Eq. (1) can be taken to be

$$\begin{bmatrix} b_1 \\ b_2^\dagger \end{bmatrix} = \begin{bmatrix} \cosh\left[\dfrac{\beta}{2}\right] & \sinh\left[\dfrac{\beta}{2}\right] \\ \sinh\left[\dfrac{\beta}{2}\right] & \cosh\left[\dfrac{\beta}{2}\right] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2^\dagger \end{bmatrix} . \tag{4}$$

It can then be shown[15] that the joint probability that n_1 photons are emitted from the signal port and n_2 photons are emitted from the idler port is

$$p(n_1,n_2) = \delta_{n_1,n_2} \operatorname{sech}^2\left[\frac{\beta}{2}\right]\left[\tanh^2\left[\frac{\beta}{2}\right]\right]^{n_1} . \tag{5}$$

In order to determine the statistics of the photons emerging from the shutter it is necessary to determine the statistics of the photodetector's response to light coming from the idler port. If an ideal photodetector (unit quantum efficiency) is used for D, and E is programmed to open the shutter for a coherence time T_c when D registers m photons, then the light leaving the shutter will consist of a series of m-photon wave packets interspersed with vacuum wave packets.

Consider now the case when the quantum efficiency η of the photodetector D is less than unity. In this case there is a finite probability that D will register m photons whenever $n \geq m$ are emitted from the idler port. The probability that m photons are detected by the photodetector, given that n photons were emitted from the idler output port, is

$$P_d(m \mid n) = \begin{cases} 0 \ \text{ if } n < m \\ \eta^m (1-\eta)^{n-m} \begin{bmatrix} n \\ m \end{bmatrix} & \text{otherwise} . \end{cases} \tag{6}$$

Equation (6) characterized the random deletion noise of an inefficient photodetector and has the same form as the probability that m photons will pass through a partly silvered mirror with a transmission coefficient η, given that n photons were directed to the mirror.[5,16]

The probability $P_s(n,m)$ that n photons will be emitted from the idler port while m are detected by the photodetector is from Eqs. (5) and (6)

$$P_s(n,m) = P(n,n) P_d(m \mid n) . \tag{7}$$

This is also the probability that the shutter will open and let an n-photon wave packet $|n\rangle$ pass,

$$P_s(n,m) = \begin{cases} 0 \ \text{ for } n < m \\ \eta^m (1-\eta)^{n-m} \begin{bmatrix} n \\ m \end{bmatrix} \operatorname{sech}^2\left[\dfrac{\beta}{2}\right]\left[\tanh^2 \dfrac{\beta}{2}\right]^n & \text{otherwise} . \end{cases} \tag{8}$$

Notice that the sum of $P_s(n,m)$ over n from $n=m$ to $n=\infty$ is not unity. There is a probability

$$\bar{P}_s = 1 - \sum_{n=m}^{\infty} P_s(n,m) \tag{9}$$

that the down converter may emit fewer than m photons or the photodetector reports less in the counting interval T_c. In this case the shutter remains closed and only the vacuum state $|0\rangle$ emerges from the shutter. The filtering process performed by the shutter can thus be described by a mixed state represented by the density matrix

$$\rho_m = \left[1 - \sum_{n=m}^{\infty} P_s(n,m)\right] |0\rangle\langle 0| + \sum_{n=m}^{\infty} P_s(n,m)\,|n\rangle\langle n| \ . \quad (10)$$

Using ρ_m one can now determine the statistics of the photons passed by the shutter. The number operator $\hat{n}_s$ for the number of photons in the beam leaving the shutter is

$$\hat{n}_s = \sum_{n=0}^{\infty} n\,|n\rangle\langle n| \ . \quad (11)$$

From Eqs. (10) and (11) one can show that the mean number of photons leaving the shutter

$$\langle \hat{n}_s \rangle = \mathrm{Tr}(\rho_m \hat{n}_s) \quad (12)$$

and its second moment $\langle \hat{n}_s^2 \rangle$ is given by

$$\langle \hat{n}_s \rangle = \frac{C_m \gamma^m (m+\gamma)}{(1-\gamma)^{m+2}} \quad (13)$$

and

$$\langle \hat{n}_s^2 \rangle = \frac{C_m \gamma^m [m^2 + (3m+1)\gamma + \gamma^2]}{(1-\gamma)^{m+3}} , \quad (14)$$

where C_m and γ are defined by

$$C_m \equiv \left[\frac{\eta}{1-\eta}\right]^m \mathrm{sech}^2\left[\frac{\beta}{2}\right] \quad (15)$$

and

$$\gamma \equiv (1-\eta)\tanh^2\left[\frac{\beta}{2}\right] . \quad (16)$$

In deriving the above equations use was made of the following sums:

$$\sum_{n=m}^{\infty} n \binom{n}{m} x^{n-m} = \frac{m+x}{(1-x)^{m+2}} \quad (17)$$

and

$$\sum_{n=m}^{\infty} n^2 \binom{n}{m} x^{n-m} = \frac{m^2 + (3m+1)x + x^2}{(1-x)^{m+3}} . \quad (18)$$

From Eqs. (13) and (14) $\Delta\hat{n}_s$ can be computed and one can further calculate the Mandel's Q parameter[17]

$$Q \equiv \frac{(\Delta\hat{n}_s)^2 - \langle \hat{n}_s \rangle}{\langle \hat{n}_s \rangle} , \quad (19)$$

which determines the degree of sub-Poissonian behavior. The correlation function $g_2(0)$ is related to the Q parameter via $Q = \langle n \rangle [g_2(0) - 1]$. Since $g_2(0)$ measures the degree of antibunching, Q for a single mode also measures the degree of antibunching. In fact $\Delta = \langle n \rangle Q$ was first introduced by Stoler as a measure of the degree of antibunching of a single-mode field.[4] Antibunching occurs when $Q < 0$. Using Eqs. (13) and (14) we find

$$Q = \frac{m^2 + 4m\gamma + 2\gamma^2}{(1-\gamma)(m+\gamma)} - \langle \hat{n}_s \rangle . \quad (20)$$

The Q parameter computed using Eqs. (15) and (16) is plotted versus β for several values of η in Fig. 2. The figure clearly indicates a regime of β and η values for which the beam emerging from the shutter will be antibunched. The curves in Fig. 2 are plotted for $m=1$, i.e., for the case when one photon is registered by the detector during a coherence time T_c. From Eqs. (13) and (20) it can be shown that no antibunching will occur in the shutter output for $m \geq 2$. The detector's efficiency η must exceed 0.8 in order to produce antibunching even for $m=1$.

We have up to now been considering the statistics of the steady-state beam emerging from the shutter. This beam consists, as we have seen, of m-photon pulses interspersed with vacuum pulses.

It is also of interest to examine the statistics of the photons within a single pulse emerging from the shutter. If the photodetector were perfect, i.e., $\eta = 1$, then each emerging pulse would have exactly m photons. In that case the collection of emitted pulses would constitute a field in a photon number eigenstate $|m\rangle$. When $\eta < 1$ the photon number in the pulses emerging from the shutter is a random variable $n \geq m$ which is distributed according to the probability $P_s(n,m)$. The density matrix describing the state of the field produced by stacking all the emergent pulses into a series of contiguous time intervals of duration T_c is a filtration of $\hat{\rho}_m$ given by Eq. (10). We can think of it as a subensemble obtained by systematically deleting all the vacuum pulses. Defining the projection operator

$$\hat{P} = \sum_{n=1}^{\infty} |n\rangle\langle n| = 1 - |0\rangle\langle 0| , \quad (21)$$

we can write the density matrix describing the single-pulse statistics by

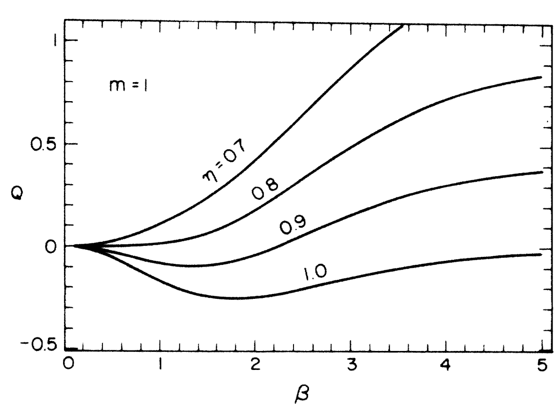

FIG. 2. The Q parameter for the light leaving the shutter. The Q parameter is plotted as a function of the parametric gain parameter β for various photodetector efficiencies η for the case when the photodetector opens the shutter only when one photon is counted ($m=1$) during a coherence time T_c. Note that the photodetector must have an efficiency better than 80% if antibunching is to be observed.

$$\tilde{\rho}_m = \hat{P}\hat{\rho}_m\hat{P}/\mathrm{Tr}[\hat{P}\hat{\rho}_m] = \hat{P}\hat{\rho}_m\hat{P}\Big/\sum_{n=m}^{\infty} P_s(n,m) . \tag{22}$$

The conditional density matrix $\tilde{\rho}_m$ is just $\hat{\rho}_m$ with its vacuum component deleted and then suitably renormalized.

The photon probability distribution for the single-pulse statistics is given by

$$P_m(n) = \langle n \mid \tilde{\rho}_m \mid n \rangle = \begin{cases} \dfrac{(1-\gamma)^{m+1}}{\gamma^m} \begin{bmatrix} n \\ m \end{bmatrix} \gamma^n & \text{for } n \geq m \\ 0 & \text{for } n < m . \end{cases} \tag{23}$$

Using Eq. (23) along with Eqs. (17) and (18) we find the following single-pulse photon number moments:

$$\langle \hat{n}_p \rangle_m = \frac{m+\gamma}{1-\gamma} , \tag{24}$$

$$\langle \hat{n}_p^{\,2} \rangle = \frac{m^2+(3m+1)\gamma+\gamma^2}{(1-\gamma)^2} . \tag{25}$$

The number variance is given by

$$(\Delta\hat{n}_p)^2 = \frac{(m+1)\gamma}{(1-\gamma)^2} . \tag{26}$$

The Q parameter for this case is given by

$$Q_p = \frac{\gamma^2+2m\gamma-m}{(1-\gamma)(m+\gamma)} . \tag{27}$$

It is interesting to consider various limiting cases for Eq. (27). When $\eta \to 1$ we have $\gamma \to 0$ in which case the Q parameter is equal to (-1). This indicates that we are producing pulses of exactly m photons, i.e., photon number eigenstates. This can also be seen from Eq. (26), i.e., the number variance vanishes for $\gamma \to 0$.

From Eq. (27) we can see that $\partial Q_p/\partial\beta \geq 0$ with the equality holding for $\beta = 0, \infty$. So Q_p is monotonically increasing with β for all finite β. Therefore Q_p reaches its maximum value as a function of β for $\beta = \infty$. Since $\gamma \to (1-\eta)$ when $\beta \to \infty$ we find the limit

$$\lim_{\beta\to\infty} Q_p = \frac{(1-\eta)^2+(1-2\eta)m}{\eta(1+m-\eta)} . \tag{28}$$

From Eq. (27) we see that as long as $\gamma < \frac{1}{2}$, we will get antibunched shutter output for all m values greater than $\gamma^2/(1-2\gamma)$. Provided that $\eta > \frac{1}{2}$ we can see from Eq. (28) that the shutter output will be antibunched for all m values greater than $(1-\eta^2)/(2\eta-1)$. These results illustrate the possible trade offs between detection efficiency and parametric gain. Our results indicate that if we can find a way to make β suitably large we can produce an intense source of antibunched photons.

In Fig. 3 we plot the Q parameter Q_p given in Eq. (27) versus β for $m=1$ for several values of η. It is notable that in the single-pulse case one finds antibunching even for very small detector efficiencies. This is due to the fact that the deleterious effect of vacuum fluctuations has been eliminated by looking only at the single-pulse statistics.

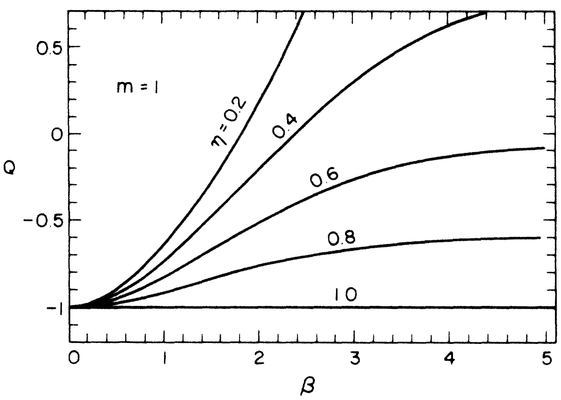

FIG. 3. The Q parameter for the individual light pulses emitted by the shutter. The Q parameter is plotted for the case when the shutter is opened when the photodetector counts a single photon ($m=1$) in the coherence time T_c. The Q parameter is plotted as a function of the parametric gain parameter β for various values of photodetector efficiency η. Note that large amounts of antibunching ($Q<0$) can be achieved even when the photodetector's efficiency is poor.

Figure 4 contains a plot of Q_p for a variety of m values and for $\eta = \frac{1}{2}$. Note that for a given value of η and β the degree of antibunching increases with increasing m. This is because the numbers of photons that actually get through the shutter are closer, in a relative sense, to the number for which the shutter has been set.

CONCLUSION

In conclusion we have shown that it may be possible to generate an antibunched photon beam from the light emitted from the signal port of a parametric down converter by exploiting the strong correlation between signal and idler photons. One uses the idler photons in a feed-

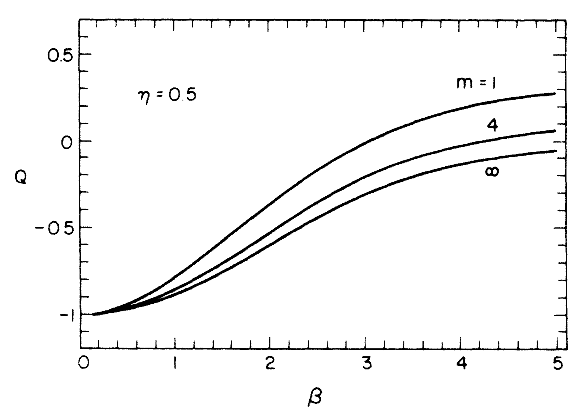

FIG. 4. The Q parameter for individual light pulses emitted by the shutter. The Q parameter is plotted as a function of the parametric gain parameter β for the case when the shutter is opened when m-photon pulses are counted during a coherence time. The cases $m=1$, 4, and ∞ are plotted for the case when the photodetector efficiency η is 0.5.

forward scheme to activate a shutter in the signal beam in order to tailor the signal beam photon statistics.

It should be pointed out that the analysis performed here is a single-mode analysis and consequently a number of effects which may degrade the performance of the device described here have not been taken into account. In particular, since the signal and idler photons are only correlated to within T_c there is the possibility that when one photon is detected in the idler and the shutter is opened for a time interval T_c about the idler photon's arrival time the corresponding signal photon may lie outside this interval and may not be passed. A proper treatment of this mode spillover problem requires a nontrivial wideband analysis. We are currently undertaking such an analysis. Further, it may be possible to devise a strategy for opening the shutter which will minimize the effects of mode spillover. In particular, the box E of Fig. 1 may be programmed to look for photons in the idler beam which are isolated from nearby photons by many coherence times so that the shutter can be safely opened for several coherence times in order to guarantee the corresponding signal photon's passage through the shutter. It is hoped that the single-mode results presented here are sufficiently enticing to encourage experimental work.

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