Čerenkov line radiation

Jun-han You, Fu-hua Cheng, and Fu-zhen Cheng
Center for Astrophysics, University of Science and Technology of China, Hefei, Anhui Province, China

T. Kiang
Dunsink Observatory, Castleknock, Dublin 15, Republic of Ireland
(Received 15 October 1985)

Čerenkov radiation in the vicinity of atomic spectral lines in a gaseous medium was numerically calculated in an earlier paper of ours [J. H. You and F. H. Cheng, Acta Phys. Sinica **29**, 927 (1980)]. The results showed that a new line-emission mechanism, the Čerenkov line mechanism, exists. The main characteristics are as follows: asymmetric profile, a small red shift of line, polarization (if the velocities of electrons are anisotropically distributed), and a large linewidth. The basic theory of Čerenkov line emission was then developed [J. H. You, T. Kiang, F. H. Cheng, and F. Z. Cheng, Mon. Not. R. Astron. Soc. **211**, 667 (1984)]. In the present paper, the main formulas have been revised and simplified, and a systematic physical discussion is given. Furthermore, the causes of Čerenkov lines, and their basic characteristics are clarified.

I. INTRODUCTION

It is well known that the Čerenkov effect in a solid or liquid produces a continuum spectrum. It was shown in our previous work,[1,2] that relativistic electrons, when moving through a dense gas, will produce Čerenkov emission lines. In the neighborhood of an atomic spectral line (wavelength λ_{ij}), the refractive index n can be appreciably greater than unity (Fig. 1). This fact suggests that Čerenkov radiation over such limited ranges may exist when there is a sufficiently high density of atoms and relativistic electrons. However, so far this effect has not been noticed by physicists due to difficulties in practical measurements. Another possible reason may be that people have been under the impression that, where n is large, the extinction coefficient κ will also be large (Fig. 1), which causes the absorption of Čerenkov radiation.[3] However, it will be shown in the present paper that the Čerenkov emissivity near λ_{ij} varies approximately as $J_\lambda \propto \Delta\lambda^{-1}$, $\Delta\lambda$ representing the displacement from the given atomic line λ_{ij}, but $\kappa \propto \Delta\lambda^{-2}$, i.e., the absorption decreases with wavelength shift more rapidly than the refractive index. Therefore there is a net Čerenkov radiation in spite of the extinction. The shaded region in Fig. 1 represents the actual range of Čerenkov radiation. It will be shown that, under the normal conditions, the width of this region is of the order 10–100 Å for lines in the optical region. Therefore this emission looks more like a line than a continuum. We call it the "Čerenkov emission line." Clearly this terminology must be used with care because it is not a real line in the exact meaning—it is not at the precise position of λ_{ij}, but displaced to the red side.

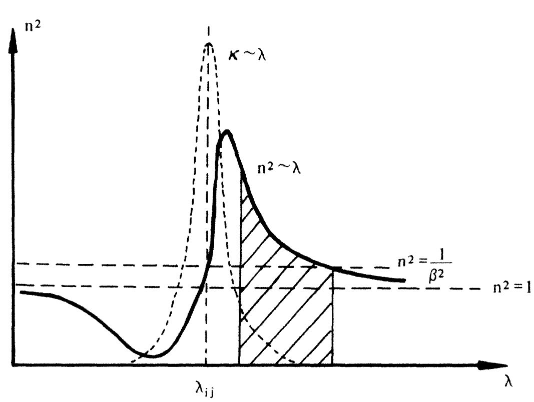

FIG. 1. Relation between refractive index n and wavelength, and relation between extinction coefficient and wavelength.

Obviously, the Čerenkov effect is the main mechanism of producing line emissions by relativistic electrons, because for the latter the collisional cross sections for excitation and ionization are all very small. The Čerenkov emission line has three remarkable features: (1) it is broad (10–100 Å), (2) the profile of the line is asymmetric, being steep on the blue side and flat on the red side, and (3) its peak is not precisely at λ_{ij}, but red-shifted; we call it "Čerenkov red shift" so as to distinguish it from other types of red-shift mechanisms (Doppler, gravitational, etc.).

An elegant experimental confirmation of Čerenkov lines in O_2, Br_2, and Na vapor using a ^{90}Sr β-ray source has been obtained by Xu *et al.* in the laboratory.[4] By use of the fast coincidence technique, they found line emission at the expected directions, wavelengths, and plane of polarization. Potential applications of the mechanism in physics and astrophysics is conceivable.

A possible field is a rediscussion of the broad emission lines of quasars or quasistellar objects (QSO's). The strange properties of emission lines of QSO's have over the years attracted wide attention,[5] for example, their abnormal intensity ratios [small value of ratio $I(\text{Ly-}\alpha)/I(H_\beta)$,[6] steep Balmer decrement,[7] etc.], the obviously asymmetric profile, the large line width, the different widths of various lines,[8–10] and different red shifts,[11–13] etc. We suggest that some, if not all, of these lines are to be identified with the Čerenkov lines generated by relativistic electrons. At present, it is well known that there are abundant relativistic electrons and dense gas in QSO's. This is just the sufficient condition to produce the Čerenkov line emission. The advantage of this new mechanism is that the large linewidth and the asymmetry of profile are all intrinsic properties. More importantly, with this new emission mechanism we can now make a fresh attack on some difficult problems of emission lines of QSO's. As an example, the very steep Balmer decrement (the relative strength ratios for H_α, H_β, etc.), one of the most important puzzles in QSO's, has been satisfactorily explained by us in terms of this new mechanism.[14] The theoretically expected values are precisely in agreement with the observed ones. Another advantage of the Čerenkov mechanism is that only one free parameter has to be fixed, i.e., if the first ratio $I(H_\alpha)/I(H_\beta)$ is just taken to be the observed mean value, then other ratios $I(H_\gamma)/(H_\beta)$, $I(H_\delta)/I(H_\beta)$, $I(H_\epsilon)/I(H_\beta)$ will be determined uniquely by the Čerenkov line-emission formulas. It seems to be a support for the new mechanism which must now be taken into account more seriously when explaining the emission lines of QSO's.

With this purpose, we shall, in the present paper, set out the basic theory of Čerenkov line radiation. Some of the formulas in the paper have been given in our previous ones,[1,2] but the main part of them have been revised. Physical discussions are given in detail. Some of the formulas are given for spectral astrophysical purpose because our main interest is in this field. It will be noticed that with a suitable modification of the expression for the photoelectric absorption k_2, our formulas for hydrogen may also be used for other atoms and ions. Forbidden lines can not be produced by the Čerenkov effect because the corresponding dipole matrix element $\overleftrightarrow{D}_{ij}$ is zero.

II. BASIC FORMULAS AND PHYSICAL ANALYSIS

The cgs system of units will be used throughout. This means, in particular, that all wavelengths will be in centimeters unless explicitly stated otherwise.

A. The refractive index n and the extinction coefficient κ

The core of the calculation of Čerenkov radiation is the evaluation of the refractive index n of the gaseous medium. We begin with the rigorous formula

$$(\tilde{n}^2-1)/(\tilde{n}^2+2)=\frac{4\pi}{3}N\alpha \ , \tag{1}$$

where

$$\tilde{n}=n-i\kappa \tag{2}$$

is the complex refractive index, N is the number density of the atoms, and α is the polarizability per atom. When the atoms are distributed over various energy levels, with densities N_a, $N\alpha$ will be replaced by a double summation over all pairs of levels a and b,

$$N\alpha=\sum_a N_a\alpha_a=\sum_a N_a \sum_{b(\neq a)} \frac{e^2}{2\pi m}\frac{f_{ab}}{2\pi(\nu_{ab}^2-\nu^2)+i\Gamma_{ab}\nu} \ , \tag{3}$$

where e and m are the charge and the mass of an electron, and f_{ab} and Γ_{ab} are the oscillator strength and damping constant for the atomic line of frequency ν_{ab}. Because we shall always be concerned with the neighborhood of a given atomic line λ_{ij}, it follows that we need only keep two terms in the above summation, Eq. (3) becomes

$$N\alpha\approx N_i\alpha_{ij}+N_j\alpha_{ji} \tag{4}$$

where

$$\alpha_{ij}=\frac{1}{2\pi}(e^2/m)f_{ij}/[2\pi(\nu_{ij}^2-\nu^2)+i\Gamma_{ij}\nu] \tag{5}$$

and α_{ji} is obtained from Eq. (5) by replacing f_{ij} by f_{ji}. We shall always use i for the lower level and j for the upper level. f_{ij} and f_{ji} are related to each other through the statistical weights g_i and g_j,[15] $g_if_{ij}=-g_jf_{ji}$. The relation between f_{ij} and Einstein's spontaneous emission coefficient A_{ji} is

$$f_{ij}=\frac{mc^3}{8\pi^2e^2\nu_{ij}^2}(g_j/g_i)A_{ji}$$

and

$$\Gamma_{ij}=\Gamma_i+\Gamma_j=\sum_{i'(<i)}A_{ii'}+\sum_{j'(<j)}A_{jj'} \ .$$

Using these relations, Eq. (1) becomes

$$(\tilde{n}^2-1)/(\tilde{n}^2+2)=b/(z+ig) \tag{6}$$

with

$$b=\frac{c^3}{12\pi^2}\nu_{ij}^{-2}A_{ji}g_j\left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right] \ , \tag{7}$$

$$z=2\pi(\nu_{ij}^2-\nu^2) \ , \tag{8}$$

$$g=\Gamma_{ij}\nu \ . \tag{9}$$

From Eqs. (2) and (6), we have

$$n^2=[(A^2+9b^2g^2)^{1/2}+A]/2B \ , \tag{10}$$

$$\kappa=[(A^2+9b^2g^2)^{1/2}-A]/2B \ , \tag{11}$$

where

$$A=(z+2b)(z-b)+g^2 \ ,$$

$$B=(z-b)^2+g^2 \ .$$

For future convenience in comparing observations, we shall now develop our formulas in wavelengths rather than frequencies. Let $\Delta\lambda\equiv\lambda-\lambda_{ij}$ represent the wavelength displacement, and let

$$u\equiv\frac{\lambda-\lambda_{ij}}{\lambda_{ij}}=\frac{\Delta\lambda}{\lambda_{ij}} \tag{12}$$

denote the fractional displacement. We shall always be interested in some small range about $\lambda\simeq\lambda_{ij}$, therefore we always have $u\ll1$, so u is a convenient quantity. Replacing the variable ν by u, we have $\nu=c/\lambda=c\lambda_{ij}^{-1}(1+u)^{-1}$, and $\nu_{ij}=c\lambda_{ij}^{-1}$. Making these replacements in (7)–(9) and keeping only terms of the lowest order in u, we have

$$b=\frac{c}{12\pi^2}\lambda_{ij}^2 A_{ji} g_i \left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right], \quad (13)$$

$$z=4\pi c^2\lambda_{ij}^{-2}u , \quad (14)$$

$$g=c\lambda_{ij}^{-1}\Gamma_{ij} . \quad (15)$$

Because the Čerenkov line emission is not located in the exact position $\lambda=\lambda_{ij}$, it has a small red shift, so u is not an indefinitely small quantity. In fact, practically throughout the whole effective range of u, we always have

$$g \ll z, \quad b \ll z . \quad (16)$$

For example, for Ly-α, $\lambda_{21}=1.216\times10^{-5}$ cm, $\Gamma_{21}=A_{21}=6.25\times10^{8}$ s^{-1}. Inserting these values in (14) and (15), we find that $g \ll z$ whenever $\Delta\lambda$ is greater than 2.0×10^{-4} Å. On the other hand, under the ordinary physical conditions we can safely assume $N_2 \ll N_1 \simeq N$. Then even for N_1 as high as $N_1\simeq N\simeq10^{17}$ cm^{-3}, $b \ll z$ will be true whenever $\Delta\lambda$ is greater than 1.1×10^{-3} Å. A smaller N_1 will give a still smaller lower limit of $\Delta\lambda$. We recall that, for Ly-α the Čerenkov line has an effective width of the order of 10 Å;[1] so in fact, in the whole effective-emission range, Eq. (16) always holds. Similar consideration applied to other lines leads to the same conclusion. Thus we need only retain terms of the lowest order in g/z and b/z in Eqs. (10) and (11), and we obtain

$$n^2-1=3(b/z) ,$$
$$\kappa=(3/2)(b/z)(g/z) . \quad (17)$$

Substituting (13), (14), and (15) into (17), finally, we have

$$n^2-1=\frac{1}{16\pi^3 c}\lambda_{ij}^4 A_{ji} g_j \left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right] u^{-1} , \quad (18)$$

$$\kappa=\frac{1}{128\pi^4 c^2}\lambda_{ij}^5 \Gamma_{ij} A_{ji} g_j \left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right] u^{-2} . \quad (19)$$

Equations (18) and (19) are the basic formulas for n and κ, respectively, around λ_{ij}. From (18) we see that the refractive index n varies as $(n^2-1)\propto u^{-1}\propto\Delta\lambda^{-1}$. We shall see below that the Čerenkov spectral emissivity varies approximately as $J_\lambda \propto n^2-1$, so $J_\lambda\propto\Delta\lambda^{-1}$. However, the internal absorption varies as $\kappa\propto u^{-2}\propto\Delta\lambda^{-2}$, i.e., the absorption decreases with $\Delta\lambda$ more rapidly than the emissivity. At the position $\lambda\sim\lambda_{ij}$, Čerenkov radiation can not be absorbed completely, and the Čerenkov line will be produced.

B. The Čerenkov spectral emissivity J_λ (J_u)

It is known from the theory of Čerenkov radiation that the power emitted in frequency interval $(\nu,\nu+d\nu)$ by an electron moving with velocity $\beta=v/c$ is $P_\nu d\nu =(4\pi^2e^2\beta\nu/c)(1-1/n^2\beta^2)d\nu$. Let $N(\gamma)d\gamma$ be the number density of relativistic electrons in the energy interval $(\gamma,\gamma+d\gamma)$, $\gamma\equiv1/(1-\beta^2)^{1/2}=mc^2/m_0c^2$, then the power emitted in the interval $(\nu,\nu+d\nu)$ by these electrons is $N(\gamma)d\gamma P_\nu d\nu$. For an isotropic velocity distribution of relativistic electrons, as in normal astrophysical circumstance, the Čerenkov emission will also be isotropic. Hence the spectral emissivity in unit volume and unit solid angle is

$$J_\nu d\nu=\frac{1}{4\pi}\int_{\gamma_1}^{\gamma_2}N(\gamma)d\gamma P_\nu d\nu$$
$$=\frac{\pi e^2}{c}\nu d\nu\int_{\gamma_1}^{\gamma_2}N(\gamma)d\gamma\beta\left[1-\frac{1}{n^2\beta^2}\right] .$$

For the relativistic electron of interest $\beta\simeq1$ and $\gamma\gg1$, so $\beta^{-2}\simeq1+\gamma^{-2}$, $\beta\simeq1-\frac{1}{2}\gamma^{-2}$. Also we notice that in the actual effective emission range, the refractive index of gas n is not far from unity, $n \gtrsim 1$ [see Eq. (18)]. Therefore we have

$$\int_{\gamma_1}^{\gamma_2}\left[1-\frac{1}{n^2\beta^2}\right]\beta N(\gamma)d\gamma\simeq\int_{\gamma_1}^{\gamma_2}(n^2-1-\gamma^{-2})N(\gamma)d\gamma$$
$$\simeq(n^2-1-\gamma_c^{-2})N_e ,$$

$$N_e\equiv\int_{\gamma_1}^{\gamma_2}N(\gamma)d\gamma ,$$

where n_e is the number density of relativistic electrons and γ_c is the characteristic energy of electrons in a given source. The definition of γ_c is $\int_{\gamma_1}^{\gamma_2}\gamma^{-2}N(\gamma)d\gamma\equiv\gamma_c^{-2}N_e$, $\gamma_1<\gamma_c<\gamma_2$. Hence

$$J_\nu d\nu\simeq\frac{\pi e^2}{c}N_e\nu(n^2-1-\gamma_c^{-2})d\nu . \quad (20)$$

Transforming ν to u, we have $-J_\nu d\nu=J_\lambda d_\lambda=J_u du$ and $du=\lambda_{ij}^{-1}d\lambda=\lambda_{ij}^{-1}(-c\nu^{-2}d\nu)$, so Eq. (20) becomes

$$J_u du=\pi ce^2Ne\lambda_{ij}^{-2}(n^2-1-\gamma_c^{-2})du . \quad (21)$$

The limiting width of the Čerenkov line, $\Delta\lambda_{\rm lim}$ or $u_{\rm lim}\equiv\Delta\lambda_{\rm lim}/\lambda_{ij}$, can be derived from Eq. (21). Setting $n^2-1-\gamma_c^{-2}=0$ and inserting the expression for n [Eq. (18)] into Eq. (21), we get the limiting linewidth as

$$u_{\rm lim}=C_0\gamma_c^2=\frac{1}{16\pi^3c}\lambda_{ij}^4A_{ji}g_j\left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right]\gamma_c^2 , \quad (22)$$

where

$$C_0=\frac{1}{16\pi^3c}\lambda_{ij}^4A_{ji}g_j\left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right] . \quad (23)$$

The Čerenkov radiation will be cut off at the wavelength displacement $u_{\rm lim}=\Delta\lambda_{\rm lim}/\lambda_{ij}$. Inserting Eqs. (18), (22), and (23) into Eq. (21), the spectral emissivity is following $J_u du=\pi ce^2Ne\lambda_{ij}^{-2}C_0(u^{-1}-u_{\rm lim}^{-1})du$ or

$$J_u du=NeC_1(u^{-1}-u_{\rm lim}^{-1})du , \quad (24)$$

where

$$C_1\equiv\frac{e^2}{16\pi^2}\lambda_{ij}^2A_{ji}g_j\left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right] . \quad (25)$$

For convenience in comparing observations, the wavelength will be used as a variable rather than a frequency. Therefore, Eq. (24) is the basic formula for emissivity.

From Eq. (24) we see a remarkable characteristic of the Čerenkov line. In the actual effective-emission range $u \ll u_{\rm lim}$, hence in the main emission range Eq. (24) can be approximately written as

$$J_u du \simeq N_e C_1 u^{-1} du$$
$$= \frac{e^2}{16\pi^2} N_e \lambda_{ij}^2 A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j} \right] u^{-1} du \ . \quad (26)$$

From Eq. (26) we see it differs from the usual spontaneous radiation transition. The Čerenkov line emission is not determined by the population density in the upper level N_j but by the difference in population between the lower and the upper levels, $(N_i/g_i)-(N_j/g_j)$. Since in the normal condition of gas and for the lowest levels of atoms we always have $N_i \gg N_j$, we could find the remarkable result that the Čerenkov line emission is nearly proportional to the population in the lower level N_i, $J_u \propto N_i$. It implies that the Čerenkov emission line will be produced even if the gas is at a very low temperature. It provides a possibility of obtaining emission lines of unstable molecules. Also, it may well help to explain why QSO's have unusually low $I(\text{Ly-}\alpha)/I(\text{H}_\beta)$ intensity ratio, for it means that if there is an increase in the population of the first excited level N_2 then H_β will be stronger while Ly-α will be the same, leading to a lower $I(\text{Ly-}\alpha)/I(\text{H}_\beta)$ ratio, whereas, for the usual spontaneous emission, an increase in N_2 will increase Ly-α and not H_β, which causes a higher $I(\text{Ly-}\alpha)/I(\text{H}_\beta)$ ratio.

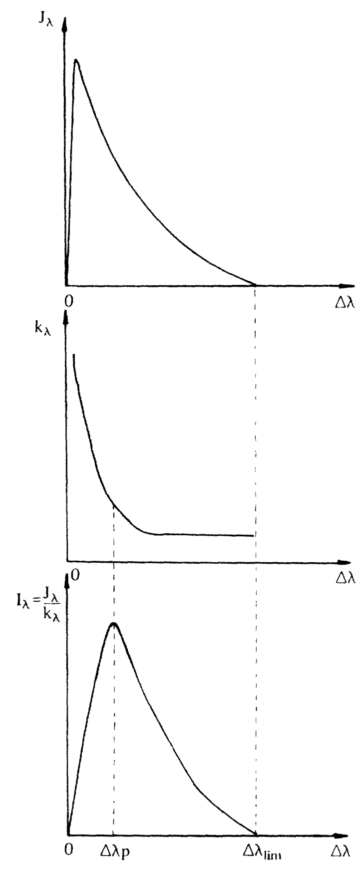

FIG. 2. The Čerenkov emissivity J_λ, absorption coefficient k_λ, and the emergent intensity I_λ of the Čerenkov line (optically thick case).

The original profile of the Cerenkov line is given by the function of $J_u \sim u$ (or $J_\lambda \sim \Delta\lambda$, see Fig. 2). It lies on the red side of λ_{ij}. The total line emissivity is given by an integration of Eq. (24), i.e., $J_c = \int_{u_{\rm min}}^{u_{\rm lim}} J_u du$. Compared to the line emissivity of spontaneous transition $J_s = (1/4\pi) N_j A_{ji} h\nu_{ij}$, J_c is very small except for a extremely high density of relativistic electrons N_e. However, in some cases it becomes comparable to the spontaneous radiation, for example, if the gas is at a very low temperature, or the gas is very dense. In that case, the emission lines from spontaneous transition would be seriously weakened due to the line absorption k_1, while the Čerenkov emission line can easily escape from the inner part of the gas cloud due to the red shift.

C. Absorption coefficient

For an optically thick dense gas, to find the intensity of the emergent flux we must consider both absorption and the process of radiative transfer. In the simple dust-free case, there are two absorptions that are relevant to the Čerenkov line emission. One is the line absorption k_1 in the vicinities of atomic lines, directly related to the extinction coefficient κ given in Eq. (19). The other is the photoelectric absorption k_2. For the optical wavelength range which we shall be mainly interested in, free-free absorption is small and can be neglected. Thus, for us, the total absorption is

$$k = k_1 + k_2 \ . \quad (27)$$

The relation between k_1 and κ is $k_1 = 4\pi\kappa/\lambda = (4\pi/\lambda_{ij})(1-u)\kappa \simeq (4\pi/\lambda_{ij})\kappa$. Inserting the expression (19) and keeping the terms of lowest order of u, we have

$$k_1 = \frac{1}{32\pi^3 c^2} \lambda_{ij}^4 \Gamma_{ij} A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j} \right] u^{-2} = C_2 u^{-2} \ ,$$

where

$$C_2 = \frac{1}{32\pi^3 c^2} \lambda_{ij}^4 \Gamma_{ij} A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j} \right] . \quad (28)$$

Obviously, k_1 can also be obtained from the well-known formula

$$k_\nu = \frac{c^2 N_i g_j}{8\pi\nu^2 g_i} \left[1 - \frac{g_i N_j}{g_j N_i} \right] A_{ji} \varphi_{ji}(\nu) \ ,$$

where $\varphi_{ji}(\nu)$ is the Lorentz profile factor

$$\varphi_{ji}(\nu) = \frac{\Gamma_{ij}/4\pi^2}{(\nu-\nu_{ij})^2 + (\Gamma_{ij}/4\pi)^2} \simeq \frac{\Gamma_{ij}}{4\pi^2(\nu-\nu_{ij})^2} \ ,$$

and the photoelectric absorption coefficient is $k_2 = \sum_l N_l \sigma_l$, where the summation extends over all levels for which the photoionization potential is less than the photon energy $h\nu$. For hydrogen atoms, the photoionization cross section for l is $\sigma_l = 2.8 \times 10^{29} \nu^{-3} l^{-5}$ or

$$\sigma_l = 1.04\times10^{-2} l^{-5} \lambda_{ij}^3 (1+u)^3 \simeq 1.04\times10^{-2} l^{-5}\lambda_{ij}^3 ,$$

thus

$$k_2 = 1.04\times10^{-2}\lambda_{ij}^3 \sum_l N_l l^{-5} . \quad (29)$$

The line absorption $k_1 \propto u^{-2}$ decreases rapidly with increasing u, and so is effective only in a very narrow range next to λ_{ij}, while the photoelectric absorption k_2 is nearly independent of wavelength shift. Over the whole width of the Čerenkov line, k_2 is actually the main absorption that determines the integrated intensity. The main effect of k_1 is to shift the line towards the red side past λ_{ij}. There is a critical wavelength shift $\Delta\lambda_p$ or u_p, at which $k_1 = k_2$. For $\Delta\lambda < \Delta\lambda_p$, k_1 is the dominant component of absorption, $k_1 \gg k_2$; for $\Delta\lambda > \Delta\lambda_p$, k_2 is the dominant one, $k_1 \ll k_2$. It will be shown that this critical value is just the peak position of emergent spectral intensity, i.e., $\Delta\lambda_p$ is the Čerenkov red shift (Fig. 2).

D. The emergent spectral intensity $I_\lambda(I_u)$

Using the J_u and k given above, the emergent intensity can be calculated. The equation of radiative transfer is

$$\frac{d}{d\tau_\lambda}\left[\frac{I_\lambda(x)}{n^2}\right] = S_\lambda - \frac{I_\lambda(x)}{n^2} , \quad (30)$$

where $\tau_\lambda = kL$ is optical depth, $S_\lambda \equiv J_\lambda / kn^2$ is source function. For a uniform plane-parallel layer of emitting gas with thickness L, the solution of Eq. (30) is

$$I_\lambda = n^2 S_\lambda (1-e^{-\tau_\lambda}) = \frac{J_\lambda}{k}(1-e^{-kL}) . \quad (31)$$

Equation (31) gives the $I_\lambda \sim \lambda$ relation, i.e., the calculated profile of the emergent Čerenkov line. Replacing by $u = \Delta\lambda/\lambda_{ij}$, Eq. (31) becomes

$$I_u = \frac{J_u}{k}(1-e^{-kL}) . \quad (32)$$

For the optically thin case, $kL \ll 1$, so $I_u \simeq J_u L$, i.e.,

$$I_\lambda \simeq J_\lambda L . \quad (33)$$

If it is optically thick, $kL \gg 1$, then $I_u \simeq J_u/k$, or

$$I_\lambda \simeq J_\lambda / k . \quad (34)$$

We shall be particularly interested in the optically thick case, because our main interest is in the high-energy astronomical objects, such as QSO's, supernova, solar flares, etc., and we know that these objects all have compact structures in which the gas density is high. According to Eq. (34), the emergent intensity in the optically thick case is independent of the geometric thickness L of the cloud and, as a consequence, several results can be obtained in a particularly simple form. Inserting Eqs. (24), (27), and (28) in (34) and writing C_3 for k_2, gives

$$I_u = \frac{J_u}{k_1+k_2} = \frac{N_e C_1 (u^{-1} - u_{\lim}^{-1})}{C_2 u^{-2} + C_3} , \quad (35)$$

where C_1, C_2, and C_3 are coefficients which are given by Eqs. (25), (28), and (29), respectively. Figure 2 shows the profile of I_λ in the optically thick case. For convenience, we collect here the formulas for C_0, C_1, C_2, and C_3,

$$\begin{aligned}
C_0 &= \frac{1}{16\pi^3 c}\lambda_{ij}^4 A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j}\right] \\
&= 6.72\times10^{-14}\lambda_{ij}^4 A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j}\right] , \\
C_1 &= \frac{e^2}{16\pi^2}\lambda_{ij}^2 A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j}\right] \\
&= 1.46\times10^{-21}\lambda_{ij}^2 A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j}\right] , \\
C_2 &= \frac{1}{32\pi^3 c^2}\lambda_{ij}^4 A_{ji}\Gamma_{ij} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j}\right] \\
&= 1.12\times10^{-24}\lambda_{ij}^4 \Gamma_{ij} A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j}\right] , \\
C_3 &= 1.04\times10^{-2}\lambda_{ij}^3 \sum_l N_l l^{-5} .
\end{aligned} \quad (36)$$

In the general case, the emergent spectral intensity depends on the thickness L of the emitting gas,

$$I_u = \frac{N_e C_1 (u^{-1} - u_{\lim}^{-1})}{C_2 u^{-2} + C_3}\{1 - \exp[-(C_2 u^{-2} + C_3)L]\} . \quad (37)$$

In the optically thin case we have

$$I_u = N_e C_1 (u^{-1} - u_{\lim}^{-1}) L . \quad (38)$$

Note, however, that Eq. (38) is not valid when $u \to 0$, because it was derived using condition (16), which holds only for u greater than some finite small value. On the other hand, Eqs. (35) and (37) can be safely extended to $u \to 0$ without any difficulty. To transform I_u into I_λ, which can be directly compare with observations, using $I_u du = I_\lambda d\lambda$, we have

$$I_\lambda = \frac{1}{\lambda_{ij}} I_u . \quad (39)$$

E. The total intensity of the Čerenkov line, I

The total intensity of the Čerenkov line is

$$I = \int_0^{u_{\lim}} I_u du , \quad (40)$$

where $u_{\lim}$ is given by Eqs. (22) and (23), i.e.,

$$u_{\lim} = \frac{\Delta\lambda_{\lim}}{\lambda_{ij}} = C_0 \gamma_c^2 = \frac{1}{16\pi^3 c}\lambda_{ij}^4 A_{ji} g_j \left[\frac{N_i}{g_i} - \frac{N_j}{g_j}\right]\gamma_c^2 .$$

Inserting Eqs. (35), (37), or (38) in (40) and integrating Eq. (40), we obtain the total intensity of the Čerenkov line in the corresponding case. In particular, for the optically thick case which we are interested in, the total intensity is

$$I=Y\{\ln(1+X^2)-2[1-\arctan(X)/X]\} , \quad (41)$$

where $Y=(N_e/2)C_1/C_3$, $X=(C_3/C_2)^{1/2}u_{\lim}$, and C_1, C_2, and C_3 are given by Eq. (36).

F. An important simplification

From Eq. (41) we see that, in addition to the number density of relativistic electrons N_e, the intensity is also dependent on the parameters C_0, C_1, C_2, and C_3; they are functions of atomic constants $(A_{ji},\Gamma_{ij},\lambda_{ij})$ and the populations N_i and N_j. Under ordinary circumstances in a gas, at least for the lowest levels, we always have $N_1 \gg N_2 \gg N_3 \gg \cdots$, or $N_i \gg N_j$, i.e., the population in the lower level is much higher than the upper one. So $(N_i/g_i - N_j/g_j) \simeq N_i/g_i$ and $\sum_l N_l l^{-5} \simeq N_l l^{-5}$ (i.e., we only retain the first term in the summation). Thus Eqs. (36) are simplified into

$$\begin{aligned}
C_0 &\simeq 6.72\times 10^{-14}\lambda_{ij}^4 A_{ji}\frac{g_j}{g_i}NR_i , \\
C_1 &\simeq 1.46\times 10^{-21}\lambda_{ij}^2 A_{ji}\frac{g_j}{g_i}NR_i , \\
C_2 &\simeq 1.12\times 10^{-24}\lambda_{ij}^4 A_{ji}\frac{g_j}{g_i}NR_i\Gamma_{ij} , \\
C_3 &\simeq 1.04\times 10^{-2}\lambda_{ij}^3 NR_l l^{-5} ,
\end{aligned} \quad (42)$$

where $R_i=N_i/N$, N is the number density of neutral atoms, and $R_1\simeq 1$ (i.e., $N_1\simeq N$, most of the neutral atoms are in the ground level), and $R_1 \gg R_2 \gg R_3 \gg \cdots$.

G. Čerenkov red shift u_p, limiting linewidth $\Delta\lambda_{\lim}$, and the effective linewidth $\Delta\lambda_{\text{eff}}$

An important feature of the present theory is the phenomenon of Čerenkov red shift, that is, the fact that the Čerenkov line is not centered on the wavelength of the corresponding atomic line λ_{ij}, but is displaced to the red (Fig. 2). As stated in Sec. II C the Čerenkov red shift can be approximately derived from $k_1=k_2$, i.e., $C_2u_p^{-2}=C_3$. From this we find the Čerenkov red shift to be

$$u_p=\Delta\lambda_p/\lambda_{ij}=\left[\frac{C_2}{C_3}\right]^{1/2} ,$$

or (43)

$$u_p=1.04\times 10^{-11}\left[\lambda_{ij}\Gamma_{ij}A_{ji}\frac{g_j}{g_i}R_iR_l^{-1}l^5\right]^{1/2} .$$

Equation (43) can also be approximately derived from Eq. (35) by letting $dI_u/du=0$. We see that the Čerenkov red shift is determined by the population distribution and the atomic constants. Therefore, we expect different red shifts for different lines, even for lines of the same atom. Thus we have here the possibility of an explanation for the observed phenomenon that different emission lines in QSO's seem to have slightly different red shifts.[10]

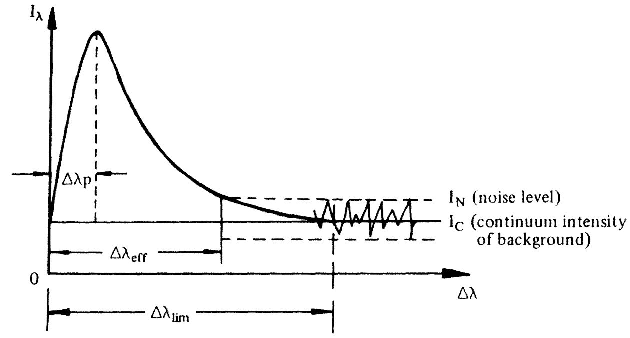

FIG. 3. Čerenkov red shift $\Delta\lambda_p$, effective width $\Delta\lambda_{\text{eff}}$, and limiting linewidth $\Delta\lambda_{\lim}$.

Another important feature of the Čerenkov lines is that different lines should have different limiting widths. The linewidth is also determined by the atomic parameters and the population in different levels. From Eqs. (22) and (23) the limiting linewidth is

$$u_{\lim}=C_0\gamma_c^2=\frac{1}{16\pi^3c}\lambda_{ij}^4A_{ji}g_j\left[\frac{N_i}{g_i}-\frac{N_j}{g_j}\right]\gamma_c^2$$

$$\simeq 6.72\times 10^{-14}\gamma_c^2\lambda_{ij}^4A_{ji}\frac{g_j}{g_i}NR_i \quad (44)$$

or

$$\Delta\lambda_{\lim}=\lambda_{ij}u_{\lim}=6.72\times 10^{-14}\gamma_c^2\lambda_{ij}^5A_{ji}\frac{g_j}{g_i}NR_i . \quad (45)$$

There is thus a possibility of explaining the different widths of the QSO lines. It should be pointed out that the emission in the "tail part" at the long-wavelength side of the Čerenkov line (i.e., $\Delta\lambda \lesssim \Delta\lambda_{\lim}$) is very weak. In fact, the profile of the tail part is nearly horizontal and cannot be distinguished from the background continuum, and so may be submerged in a noisy background (Fig. 3). Therefore, the effective observed width of the Čerenkov line $\Delta\lambda_{\text{eff}}$ will be much smaller than the limiting width.

III. DISCUSSION

According to Eqs. (24) or (26), the emissivity $J\propto(N_i/g_i-N_j/g_j)$. If there is a nonthermal population distribution, particularly if there is a population inversion, i.e., $(N_i/g_i)<(N_j/g_j)$, then there is a Čerenkov *blue shift* rather than red shift, i.e., the peak position of the line is located at the blue side of the intrinsic wavelength λ_{ij}. At the same time, the asymmetry of the line profile is also changed, being steep on the long-wavelength side and flat on the short-wavelength side.

Although the formulas given here are developed with special reference to the allowed hydrogen transitions, they can also be used for the dipole-allowed lines of other atoms and ions, after an appropriate modification is made in Eq. (29) for the photoelectric absorption k_2.

[1]J. H. You and F. H. Cheng, Acta Phys. Sinica **29**, 927 (1980); [Chin. Phys. **2**, 16 (1982)].
[2]J. H. You, T. Kiang, F. H. Cheng, and F. Z. Cheng, Mon. Not. R. Astron. Soc. **211**, 667 (1984).
[3]J. V. Jelley, *Čerenkov Radiation and Its Applications* (Pergamon, London, 1959).
[4]K. Z. Xu, B. X. Yang, Y. U. Xi, and Z. P. Zhang, Phys. Lett. **86A**, 24 (1981).
[5]K. Davidson and H. Netzer, Rev. Mod. Phys. **51**, 715 (1979).
[6]J. A. Baldwin, Mon. Not. R. Astron. Soc. **178**, 67 (1977).
[7]D. W. Weedman, Annu. Rev. Astron. Astrophys. **15**, 69 (1977).
[8]P. S. Osmer and M. G. Smith, Astrophys. J. **210**, 267 (1976); **213**, 607 (1977).
213, 607 (1977).
[9]D. E. Osterbrock, Astrophys. J. **215**, 773 (1977).
[10]J. A. Baldwin and H. Netzer, Astrophys. J. **226**, 1 (1978).
[11]B. J. Wilkes and R. F. Caswell, Mon. Not. R. Astron. Soc. **201**, 645 (1982).
[12]M. G. Smith and R. F. Caswell, Mon. Not. R. Astron. Soc. **195**, 437 (1981).
[13]B. J. Wilkes, Mon. Not. R. Astron. Soc. **207**, 73 (1984).
[14]T. Kiang, J. H. You, F. Z. Cheng, and F. H. Cheng, Phys. Lett. **109A**, 408 (1985).
[15]*Handbook of Physics,* edited by Condon and Odishow (McGraw-Hill, New York, 1958), Chap. 6, p. 112.