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Diverging length scales in diffusion-limited aggregation

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Applying finite-size scaling analysis to diffusion-limited aggregation (DLA) clusters grown in finite width strips on a square lattice we find that $l_\parallel$ and $l_\perp$, the cluster lengths along and perpendicular, respectively, to the direction of growth, diverge as $N^{\nu_\parallel}$ and $N^{\nu_\perp}$, respectively, where N is the number of particles in the cluster. We find numerically that $\nu_\parallel \sim \frac{2}{3}$ and $\nu_\perp \sim \frac{1}{2}$. From the finite-size scaling analysis we derive the expression $D = 1 + (1-\nu_\parallel)/\nu_\perp$ for the fractal dimension D of DLA clusters on a square lattice. The value $D \sim \frac{5}{3}$ predicted from this relation agrees with the expected result.

Diffusion-limited aggregation (DLA) introduced by Witten and Sander[1] provides a simple model for a variety of aggregation and growth phenomena[2,3] in which diffusion is the rate-limiting step. Since the diffusion of the particles is often the dominant mechanism in many processes,[2,3] the diffusion-limited aggregation has attracted considerable interest recently.

One of the basic assumptions,[1] which has provided the impetus for a number of studies of DLA, has been that DLA clusters are self-similar fractals.[4] That is, there exists only a single fractal dimension D which describes the divergence of any cluster length (such as the radius of gyration, caliper diameter, x span, y span, etc.) with the cluster size N. On the other hand, some recent studies have indicated that DLA clusters on a square lattice have a highly anisotropic structure,[5-7] rather than a uniform circular shape found in off-lattice simulations of DLA.[8] This implies that DLA clusters grow preferentially along the four primary axes on a square lattice. Thus, there exists the possibility that DLA clusters are not self-similar and lengths measured in different directions on the cluster diverge with different powers of the cluster mass. Since anisotropy plays an important role in diffusion-limited processes associated with pattern formation[9,10] in directional solidification and dendritic growth, the existence of different diverging lengths in diffusion-limited aggregation would be of considerable importance.

In this Rapid Communication we employ a finite-size scaling method[11] to investigate the possibility that in DLA clusters, lengths along the direction of anisotropy and perpendicular to it, diverge with different exponents. We apply finite-size scaling analysis[11] to Monte-Carlo simulation data on DLA clusters in strips of finite width on a square lattice. The results indicate that, in fact, two different exponents are needed to describe DLA clusters on a square lattice, implying that DLA clusters are self-affine fractals,[4] rather than self-similar fractals.[4]

Let us first recall that if there are two diverging lengths in a cluster then the shape of a large cluster of size N can be described by two different exponnets $\nu_\parallel$ and $\nu_\perp$ (instead of one $\nu = 1/D$) giving the behavior of the mean lengths $l_\parallel$ and $l_\perp$ along and perpendicular to the direction of the anisotropy, respectively,

$$l_\parallel \sim N^{\nu_\parallel},\ l_\perp \sim N^{\nu_\perp} . \tag{1}$$

On a finite strip of width L, the average height h of a cluster of size N is expected to grow as[11]

$$h = L^{\nu_\parallel/\nu_\perp} f(N/L^{1/\nu_\perp}) , \tag{2}$$

where $f(x)$ is the scaling function. In the limit $N \rightarrow \infty$ the cluster is essentially one dimensional and h must grow linearly with N. Therefore, for $x \gg 1, f(x) \rightarrow x$, and

$$h = L^{(\nu_\parallel - 1)/\nu_\perp} N . \tag{3}$$

In previous works on DLA clusters in strip geometry[12,13] it was assumed that there exists only one exponent in the problem and the large N limit of h was written as[12,13]

$$h = L^{1-D} N . \tag{4}$$

In comparing (3) and (4), we find

$$D = 1 + (1 - \nu_\parallel)/\nu_\perp . \tag{5}$$

In order to test the possibility that two exponents are needed to describe the scaling behavior in DLA clusters we have carried out Monte Carlo simulations of DLA clusters on strips of width $L = 24-4096$ on a square lattice. In our simulations clusters can grow from any of the L initial sites on the strip according to the usual DLA rule for the growth of deposits in a strip geometry.[12,13]

We have first determined the fractal dimension D, defined in (4), from the dependence of the average density on the strip width L. We defined the average density $\bar{\rho}$ by $\bar{\rho} = 1/L(dN/dh)$, where h is the average height in the direction of the growth of the clusters. Figure 1 is a log-log plot of $\bar{\rho}$ against L for $L = 32$, 64, 128, 256, and 512. The values of $\bar{\rho}$ were measured after the deposit had grown

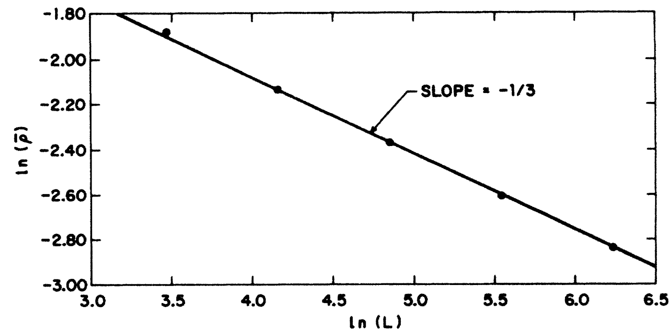

FIG. 1. Log-log plot of the average density $\bar{\rho}$ against the strip width L. The values of $\bar{\rho}$ were measured when $h > 20L$. The slope of the line drawn through the data points is $-\frac{1}{3}$, implying that $D=\frac{5}{3}$.

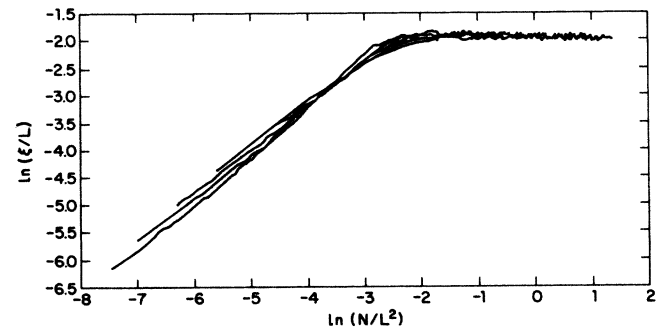

FIG. 3. The data for ξ are shown to collapse to a single scaling function according to Eq. (8) when ξ/L is plotted against N/L^2. This implies that $\nu_\perp=\frac{1}{2}$.

to a mean height of $20L$ or greater. By least-squares fitting a straight line to the results we find

$$\bar{\rho}\sim L^{D-2} \text{ with } D=1.660\pm0.0019 . \quad (6)$$

The numerical value of D is consistent with the simulation results for DLA clusters on a square lattice[8] and with the prediction $D=5/3=1.666\ldots$ of Turkevich and Scher[14] and Ball, Brady, Rossi, and Thompson[15] for DLA clusters on a square lattice.

Another quantity which can be used to determine the exponents in (2) is the width of the active zone ξ.[16,17] Since recent studies[17] have shown that ξ scales linearly with the cluster radius in DLA, here we expect ξ to scale linearly with L for $h \gg L$. We have numerically tested this assumption by measuring ξ for different strip widths L in the limit where the average deposition height h is $20L$ or greater. Figure 2 shows the dependence of ξ on L. From least-squares fitting a straight line to the data we find

$$\xi\sim L^{\nu'} \text{ with } \nu'=1.042\pm0.0016 , \quad (7)$$

in agreement with the expected behavior[17] that ξ grows linearly with the system size L. For finite N, we assume a finite-size scaling behavior of the form

$$\xi=Lg(N/L^{1/\nu_\perp}) , \quad (8)$$

where $g(x)$ is a scaling function that goes to a constant for $x\gg1$ and decays faster than any power of x for $x\ll1$.

We have fitted our data for ξ for various L to the scaling form (8) in order to determine the exponent $\nu_\perp$. Our best scaling plot, shown in Fig. 3, indicates that

$$\nu_\perp=\tfrac{1}{2} . \quad (9)$$

Assuming $\nu_\perp=\frac{1}{2}$ and $D=\frac{5}{3}$,[14,15] from relation (5) we find

$$\nu_\parallel=\tfrac{2}{3} . \quad (10)$$

To make an independent test of the predictions (9) and (10), we have used the scaling form (2) and have made scaling plots of $hL^{-\nu_\parallel/\nu_\perp}$ against $N/L^{1/\nu_\perp}$ with various values of $\nu_\parallel$ and $\nu_\perp$. We found that the best fit, shown in Fig. 4, is obtained with $\nu_\parallel=\frac{2}{3}$ and $\nu_\perp=\frac{1}{2}$, in agreement with our independent calculations based on the decay of the density $\bar{\rho}$ with L and the scaling of the screening length ξ. The large x behavior of $f(x)$ is consistent with $h\sim N$ and gives $\nu_\parallel=\frac{2}{3}$ and $\nu_\perp=\frac{1}{2}$. The small x behavior of the scaling function $f(x)$ in Fig. 4 is related to the growth of h with N for diffusion-limited deposition, as discussed in Refs. 13 and 18.

In the small x limit, h is expected to grow as[18]

$$h\sim(N/L)^{\epsilon} . \quad (11)$$

This implies that $f(x)\sim x^{\epsilon}$, as $x\to0$. Using this scaling

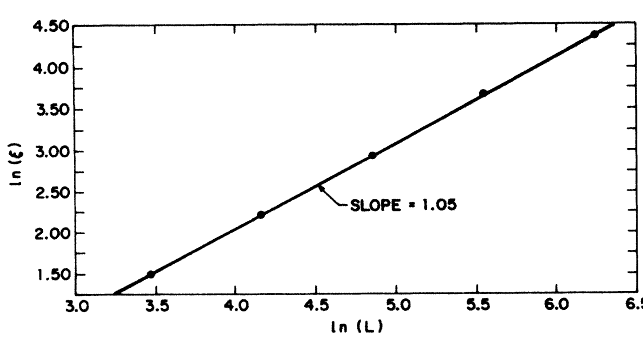

FIG. 2. Dependence of the width of the active zone ξ on the strip width L. The data indicate that ξ varies linearly with L.

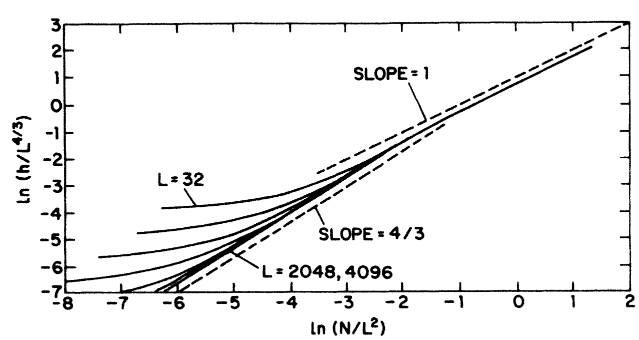

FIG. 4. The data for the average deposition height h are shown to collapse to a single scaling function according to Eq. (2) when $h/L^{4/3}$ is plotted against N/L^2. This result further supports the predicted values $\nu_\parallel=\frac{2}{3}$ and $\nu_\perp=\frac{1}{2}$.

form for $f(x)$ in (2), we find

$$\varepsilon = \nu_{\parallel}/(\nu_{\perp} - 1) = \tfrac{4}{3} \ . \qquad (12)$$

The estimate $\varepsilon = \frac{4}{3}$ is in excellent agreement with the value of the slope of the scaling plot in Fig. 4 for small x. General scaling arguments[18] give $\varepsilon = 1/[D(2-\tau)]$, where τ is the exponent describing the power-law decay of the cluster size distribution in diffusion-limited deposition. Using the best estimates, $D = \frac{5}{3}$ and $\tau = 1.55 \pm 0.05$,[13] we find $\varepsilon = 1.33 \pm 0.17$, in excellent agreement with the scaling prediction (12). In the earlier simulations[13] it was found that $\varepsilon = 1.36 \pm 0.06$ and 1.36 ± 0.05 when h was fitted to a function of the form $AN^{\varepsilon} + B$. A corrections-to-scaling fit of the form $h = AN^{\varepsilon}(1 + BN^{-y})$ gave[13] $\varepsilon = 1.33$ for earlier simulations and 1.55 ± 0.1 in simulations with better statistics. In the light of the agreement between the scaling result (12) and other estimates of ε we can only surmise that the previous larger estimate of ε is due to an inappropriate corrections-to-scaling form for h.

Further evidence for the breakdown of the single length picture is provided by the fact that the Rácz-Vicsek[18] prediction $\varepsilon = 1/(D - d + 1)$, based on the additional assumption of a single diverging length scale, gives $\varepsilon = \frac{3}{2}$ in disagreement with (12) and the simulations data. Meakin[13] has also studied the scaling relation $h_{max} \sim N^{\phi}$, where h_{max} is the average height of the "upper surface" in the deposits. According to the two length scaling picture, $h_{max} \sim l_{\parallel} \sim N^{\nu_{\parallel}}$ in a *single* DLA of size N. Using general scaling arguments, similar to those used to determine ε,[18] we find $\phi = \nu_{\parallel}/(2-\tau) = 1.48 \pm 0.15$ for the growth exponent of the upper surface. This estimate of ϕ is in excellent agreement with the simulations,[13] which gave $\phi = 1.45 \pm 0.05$, and lends further support to the two scaling length picture.

The above results strongly support the existence of two different diverging lengths in DLA clusters grown in a strip geometry on a square lattice. The question of whether the two length scales arise from the anisotropy induced by the underlying lattice or the boundaries of the strip geometry is a subtle one. Since strong anisotropy has been observed in ordinary DLA's grown on infinite lattices,[8] it seems more plausible to ascribe the emergence of the two diverging lengths to the geometry of the finite system which enhances the anistropy induced by the underlying lattice. Further studies of this phenomenon can be carried out by studying the effects of reorienting the lattice with respect to the preferred growth direction in the strip geometry.

In conclusion, our finite-size scaling analysis of DLA clusters on a strip geometry on a square lattice are consistent with a two exponent scaling of the form (1). This implies that lengths along and perpendicular to the direction of anisotropy diverge with different exponents.

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