Coherent trapping in continuum-continuum transitions

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We study a quantum model in which transitions occur between structureless continua. The population is in one of the continua initially. When the continuum-continuum coupling is very strong, we show that the continua are decoupled and population trapping occurs.

It is well known that coherent population trapping can occur in bound-bound transitions.[1-7] Recent studies show that under certain conditions, population trapping can be found in bound-continuum transitions.[8-11] For example, in strong laser excitation of an auto-ionizing resonance some of the population can be trapped in discrete states under the so-called confluence condition.[8]

So far all coherent trapping phenomena are associated with the presence of at least two discrete states. Coherent population trapping in all models has a universal feature. Under certain conditions, one can construct a superposition of discrete states which decouples from the rest of the system and which traps population. The question arises, whether it is possible to find population trapping if there is no discrete state.

In this paper we show that it is possible; population trapping can exist in pure continuum-continuum (CC) transitions. In other words, when some of the population is initially in one continuum, under certain conditions some of the population will stay in this continuum forever, even though it is strongly coupled to infinitely many other continua. This is not due to discrete levels imbedded in the continua. Here we assume that the continua are completely structureless.

We consider a model, as shown in Fig. 1, which the initial population is in a certain continuum, say $|\omega_0\rangle$ and that there are other continua labels $|\omega_m\rangle$, $m=\pm 1, \pm 2, \ldots$. These continua are coupled through the transition operator D and we assume each continuum only couples to its nearest-neighbor continua, i.e., the continuum $|\omega_m\rangle$ only couples to $|\omega_{m\pm 1}\rangle$.

In the Schrödinger picture one can expand the wave function of the system as

$$|\psi(t)\rangle = \sum_m \int d\omega_m C_{\omega_m} |\omega_m\rangle \; . \qquad (1)$$

The equations of motion are then given by[12]

$$\dot{C}_{\omega_0} = -iE_{\omega_0} C_{\omega_0} - i\int d\omega_1 D_{0,1} C_{\omega_1} - i\int d\omega_{-1} D_{0,-1} C_{\omega_{-1}} \; , \qquad (2a)$$

$$\dot{C}_{\omega_m} = -iE_{\omega_m} C_{\omega_m} - i\int d\omega_{m+1} D_{m,m+1} C_{\omega_{m+1}} - i\int d\omega_{m-1} D_{m,m-1} C_{\omega_{m-1}}, \quad m=\pm 1, \pm 2, \ldots \; , \qquad (2b)$$

where we have assumed that the continua are flat, i.e., that the matrix elements $\langle \omega_{m\pm 1} | D | \omega_m \rangle$ are independent of $\omega_{m\pm 1}$ and ω_m. We denote the transition matrix elements as $D_{\omega_m,\omega_{m\pm 1}} = D_{m,m\pm 1}$.

We assume that the initial probability amplitude distribution of the continuum $|\omega_0\rangle$ is given by

$$C_{\omega_0} = \sqrt{(\gamma/\pi)} \frac{1}{\omega_0 + i\gamma} \; , \qquad (3)$$

so we have $\int d\omega_0 |C_{\omega_0}|^2 = 1$, where γ is the width of the initial amplitude distribution.

To solve Eqs. (2) we define K_m as

$$K_m = \int d\omega_m C_{\omega_m}, \quad m = 0, \pm 1, \pm 2, \ldots \; . \qquad (4)$$

We now take the Laplace transformation of Eqs. (2). After dividing both sides of the transformed equations by $(s + iE_{\omega_m})$, we integrate with respect to ω_m. We get a set of equations for $\tilde{K}_m$ (in the Laplace domain)

$$\tilde{K}_0 = \pi g(s) - i\pi D_{0,1} \tilde{K}_1 - i\pi D_{0,-1} \tilde{K}_{-1} \; , \qquad (5a)$$

$$\tilde{K}_m = -i\pi D_{m,m+1} \tilde{K}_{m+1} - i\pi D_{m,m-1} \tilde{K}_{m-1} \; , \qquad (5b)$$

where $g(s) = -2i\sqrt{\gamma/\pi}/(s+\gamma)$.

Such a three-term recurrence relation implies that the ratio $\tilde{K}_m/\tilde{K}_{m-1}$ can be expressed as an infinite continued fraction. The initial condition establishes $\tilde{K}_0$ as special, so the continued fraction must be derived separately for $m > 0$

$$\tilde{K}_m/\tilde{K}_{m-1} = \cfrac{-i\pi D_{m,m-1}}{1 + \cfrac{\pi^2 |D_{m,m+1}|^2}{1 + \cfrac{\pi^2 |D_{m+1,m+2}|^2}{1 + \cdots}}} \qquad (6a)$$

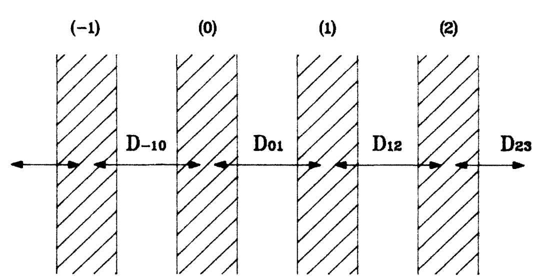

FIG. 1. A sketch of the interconnected continua.

and for $m<0$

$$\tilde{K}_m/\tilde{K}_{m+1}=\cfrac{-i\pi D_{m,m+1}}{1+\cfrac{\pi^2|D_{m,m-1}|^2}{1+\cfrac{\pi^2|D_{m-1,m-2}|^2}{1+\cdots}}} . \quad (6b)$$

In a few cases[13] a simple analytic result can be obtained. Consider the case when all D's are equal, i.e., $D_{m,m\pm1}=D$. Then we find for $m>0$

$$\tilde{K}_m/\tilde{K}_{m-1}=F/(i\pi D) , \quad (7a)$$

and for $m<0$

$$\tilde{K}_m/\tilde{K}_{m+1}=F/(i\pi D) , \quad (7b)$$

where

$$F=\cfrac{\pi^2D^2}{1+\cfrac{\pi^2D^2}{1+\cdots}}$$

$$=-\tfrac{1}{2}[1-(1+4\pi^2D^2)^{1/2}] . \quad (7c)$$

Note the limiting forms $F\to\pi^2D^2$ $(D\to0)$ and $F\to\pi D$ $(D\to\infty)$.

The required solutions are then found to be

$$\tilde{K}_0=\frac{\pi g(s)}{1+2F} \quad (8a)$$

and

$$\tilde{K}_m=[F/(i\pi D)]^{|m|}\tilde{K}_0 , \quad (8b)$$

where

$$g(s)=-\frac{2i\sqrt{\gamma/\pi}}{s+\gamma} . \quad (9)$$

From the solutions for $\tilde{K}_0$ and $\tilde{K}_m$ we obtain:

$$\tilde{C}_{\omega_0}=\frac{1}{s+iE_{\omega_0}}\left[C_{\omega_0}+i\frac{4F}{1+2F}\frac{\sqrt{\gamma/\pi}}{s+\gamma}\right], \quad (10a)$$

$$\tilde{C}_{\omega_m}=\left[\frac{1}{\pi}\frac{\tilde{K}_m}{s+iE_{\omega_m}}\right]$$

$$=\frac{1}{s+iE_{\omega_m}}\left[\frac{F}{i\pi D}\right]^{|m|}\frac{-2i}{1+2F}\frac{\sqrt{\gamma/\pi}}{s+\gamma} . \quad (10b)$$

In the long-time limit the photoelectron spectrum for the mth continuum is given by

$$W_m(\omega_m)=|(s+iE_{\omega_m})\tilde{C}_{\omega_m}|^2_{s=-iE_{\omega_m}} \quad (11a)$$

and the total population of the continuum $|\omega_m>$ is

$$P_m=\int W_m(\omega_m)d\omega_m . \quad (11b)$$

By substituting Eqs. (10) into Eqs. (11) we find

$$P_0=\left|\frac{1-2F}{1+2F}\right|^2 , \quad (12a)$$

$$P_m=\left|\frac{2}{1+2F}\left[\frac{F}{\pi D}\right]^{|m|}\right|^2 , \quad (12b)$$

Perhaps it is worth emphasizing the anti-intuitive character of these results. The population does not become equally distributed among the various continua, even as $D\to\infty$.

Inspection of Eq. (7c) shows that $Z\equiv\pi^2D^2$ is the effective coupling strength parameter. In the strong coupling limit $Z\gg1$, we have $P_0=1$ and $P_m=0$, $m=\pm1,\pm2,\ldots$. This tells us that when the CC couplings are strong enough to make $Z\gg1$, the population will be trapped in the continuum $|\omega_0\rangle$, even though all the continua in the model are strongly coupled and are structureless. We have recently shown[13] that $Z\gg1$ is also the condition for CC saturation in intense-laser ionization of atoms.

In Fig. 2 we have plotted P_0 and P_m given by Eqs. (12) with three different Z values. When $Z\ll1(Z=0.01)$ the CC transition channels are not "open" and most of the population will stay where it was at $t=0$. When the interaction strength is greater, so that $Z=1$, from Fig. 2 we can see that most of the population has left the continuum $|\omega_0\rangle$ where the population was initially located. At the critical value $Z=\frac{3}{4}$ one finds $P_0=0$. However, when the interaction strength is increased further and $Z\gg1$ $(Z=100)$, almost all the population is trapped in the initial continuum $|\omega_0\rangle$. There is a very small amount of population that leaves the continuum $|\omega_0\rangle$, and it is approximately equally distributed in all other continua.

The coherent trapping in a bound-bound or bound-continuum transition model is due to the fact that there are two transition channels to a given state in the model. The coherent cancellation of these two channels means that there is a superposition state which is decoupled com-

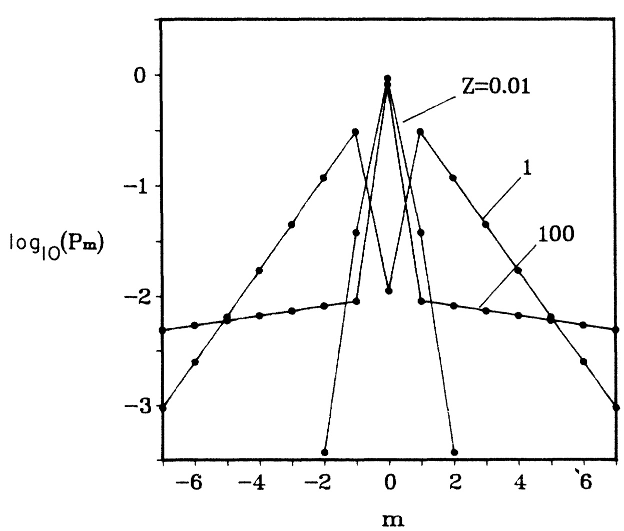

FIG. 2. The total population of different continua with three different Z values where the label m means the mth continuum.

pletely from the rest of the system. For example, consider a Lambda system with the interactions $|1\rangle - |2\rangle$ and $|2\rangle - |3\rangle$ and a population loss from $|2\rangle$ due to the coupling to a continuum. The two transition channels to $|2\rangle$, i.e., $|1\rangle - |2\rangle$ and $|3\rangle - |2\rangle$, can cancel with each other and a superposition state can be constructed which is decoupled from $|2\rangle$, resulting in the population trapping. This is also the reason that we can find the coherent trapping in CC transitions. In our model each continuum, say $|\omega_m\rangle$, is coupled to its nearest-neighbor continua $|\omega_{m\pm1}\rangle$ and these two channels are represented by the two terms in Eq. (5b). In the case $Z \gg 1$, we have $\tilde{K}_{m+1} = -\tilde{K}_{m-1}$ for all m except $m=0$ and these two terms are exactly canceled up to terms of order $1/Z^{1/2}$. Therefore the coherent trapping in CC transitions is also due to the coherent interference of the two transition channels to a given continuum, making this continuum decouple from the others.

As we know,[13] if $Z \ll 1$ the CC transitions are not open and roughly speaking the continua are decoupled. When $Z \gg 1$, due to the coherent cancellation of pairs of CC transition channels, the continua again are decoupled. Therefore if the initial population is distributed in several continua in our model, when $Z \gg 1$ they will keep their original distribution. In the discussion above we simply consider a special case where the initial population is in one continuum.

If the CC transition matrix elements $D_{m,m\pm1}$ in an atomic system represent the dipole coupling term $e\mathbf{r}\cdot\mathbf{E}$ arising from laser irradiation, Z will be proportional to the laser intensity. In most cases in typical atoms, and at optical wavelengths, the laser intensity is required to be above 10^{14} W/cm^2 to make $Z \gg 1$. Also we know that far from the ionization threshold the transition matrix elements $D_{m,m\pm1}$ are approximately independent of m. Thus at sufficiently high but achievable laser intensities CC transitions in an atomic system may exhibit population trapping.

For example, consider our predictions in the case of a fast electron with kinetic energy E_0 interacting with an atom and with a laser field of frequency ω_L. In the low-intensity regime, the laser field does not play an essential role and only the elastic atomic scattering is significant. When the laser intensity is increased, the fully elastic peak will decrease as scattered peaks with energy $E_0 \pm n\hbar\omega_L$ appear, which are due to absorbing or emitting n photons by the electron. When the laser field is increased further to make $Z \gg 1$, our model suggests that the scattered peaks with energy $E_0 \pm n\hbar\omega_L$ will decrease and the elastic peak around E_0 will (almost) recover its initial intensity, which is due to the population trapping in CC transitions. It is conceivable that the effect can play a role in electron-atom scattering.

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