Quantum theory of the micromaser: Symmetry breaking via off-diagonal atomic injection

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We propose and analyze an experiment in which atoms in a coherent superposition of atomic states are injected into a high-Q maser cavity. We show that the symmetry of the field in the cavity is "broken" in the same way as results from a classical signal injected into a laser cavity. This broken symmetry can be detected by monitoring the atomic excitation of a probe atom.

I. INTRODUCTION

Recent experimental developments make it possible to investigate the interaction of one single atom with the electromagnetic field in a high-Q maser cavity.[1] We are here interested in the effect of atomic coherence on such a maser field. We consider therefore a beam of two-level atoms prepared in a coherent superposition of both states injected into the cavity. It is crucial, however, that all the atoms have the same phase, otherwise the coherence effects will average out. We envision a double-cavity setup. The atoms enter the first cavity in their upper state and build up a coherent radiation field with an unknown but specific phase angle. Alternatively, a coherent microwave pump field can be used in the cavity. When the atoms leave the cavity they are in a coherent superposition of states (as determined by the first cavity interaction) and are injected into the other cavity (cf. Fig. 1).

We show that the atoms will create a field with the same phase angle that determines atomic coherence. Thus the symmetry of the field with respect to phase angle is "broken." The same effect has been observed for a classical signal injected into a laser.[2,3] The coherence of the atoms (or the classical signal, respectively) is transferred to the field in the cavity. The phase angle of the coherent atomic states superposition can be varied by changing the phase of the pump field or the distance between the two cavities. We show that the probability for the atoms leaving the second cavity to be in the upper state depends on the relative phase angle between incoming atoms and field. Thus, when we measure the excitation probability of the leaving atoms, we are probing the coherence of the field as produced by the incoming (off-diagonal) atomic beam.

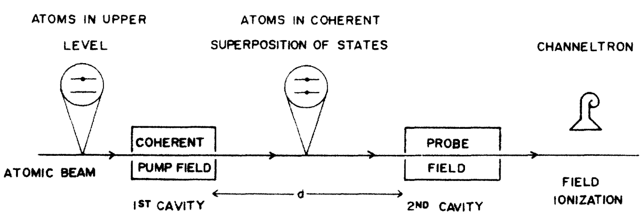

FIG. 1. Scheme of the proposed two-cavity setup. A coherent pump field is either built up by the incoming atoms in a first cavity or injected into a waveguide from an external source.

The phase and amplitude of the single-atom maser field can also be measured by the method described here. In this experiment,[1] a beam of Rydberg atoms in the upper laser level is injected into a high-Q maser cavity in order to generate a maser field. For short periods of time, probe atoms in a superposition of states are injected. This superposition is produced by an external coherent microwave field before they enter the cavity. The analysis of the excitation probability of these probe atoms (when they leave the cavity) provides the necessary information for this phase and amplitude determination.

It should be mentioned that the technique discussed here is analogous to the Ramsey two-field method invented in connection with high-resolution radio-frequency spectroscopy.[4]

The amplitude and phase measurement of the field in the single-atom maser is also interesting since sub-Poissonian statistics and squeezing can be anticipated.[5,6] The investigation of such nonclassical fields is of considerable current interest.

II. ATOMIC BEAM PREPARATION

To create a beam of two-level atoms in a phase-locked coherent superposition of states, we first inject the atoms into a high-Q cavity. If all the atoms are in their upper state when injected, they generate a coherent field in the cavity. Also, it is sufficient to let the atoms pass a wave guide with a classical microwave signal. The phase of the field will be impressed upon the atoms that leave the cavity in a superposition of upper state $|a\rangle$ and lower state $|b\rangle$. Their state vector is

$$|\psi\rangle=\alpha_0|a\rangle+\beta_0|b\rangle\ , \tag{1}$$

where the phase difference of α_0 and β_0 corresponds to the phase of the field and is the same for all atoms.

The time of flight between the two cavities shall be denoted by t_0. The atomic density operator in the interaction picture will be after the time t_0,

$$\begin{aligned}\rho_A(t_0)&=e^{iHt_0/\hbar}\rho_A(0)e^{-iHt_0/\hbar}\\&=|\alpha_0|^2|a\rangle\langle a|+|\beta_0|^2|b\rangle\langle b|\\&\quad+\alpha_0\beta_0^*e^{i\omega t_0}|a\rangle\langle b|+\alpha_0^*\beta_0e^{-i\omega t_0}|b\rangle\langle a|\ ,\end{aligned} \tag{2}$$

where ω is the frequency between the atomic levels. The velocity spread of the atoms in the beam has to be small enough to ensure $\omega\Delta t \ll 1$, i.e.,

$$\Delta v/v \ll 1/(\omega t_0)\ . \tag{3}$$

If this condition is fulfilled, the velocity spread does not destroy the atomic coherence. This limit results in an upper bound for t_0, but as long we stay below this bound, t_0 can be varied to obtain a different phase angle.

III. INFLUENCE OF THE ATOMIC COHERENCE ON THE FIELD

When we assume that the energy of the atomic transition matches with the frequency of the cavity mode under consideration, the Hamiltonian is[7]

$$H=\tfrac{1}{2}\hbar\omega\sigma_z+\hbar\omega a^\dagger a+\hbar g(\sigma_+a+\sigma_-a^\dagger) \tag{4}$$

with the usual definition of the Pauli spin matrices σ_z and $\sigma_\pm\equiv\sigma_x\pm i\sigma_y$, and the coupling constant g.

If r is the rate of atomic injection, we can calculate the change of the field in the cavity due to the atoms in the well-known way.[8] If the field density matrix is denoted by $\rho_F=\sum_{n,m}\rho_{n,m}|n\rangle\langle m|$ in the number representation, we have

$$\begin{aligned}\left[\frac{\partial\rho_{n,m}}{\partial t}\right]_{\text{atoms}}&=r(\rho_{n,m}\{|\alpha_0|^2[\cos(g\tau\sqrt{n+1})\cos(g\tau\sqrt{m+1})-1]+|\beta_0|^2[\cos(g\tau\sqrt{n})\cos(g\tau\sqrt{m})-1]\}\\&\quad+\rho_{n-1,m-1}|\alpha_0|^2\sin(g\tau\sqrt{n})\sin(g\tau\sqrt{m})+\rho_{n+1,m+1}|\beta_0|^2\sin(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m+1})\\&\quad+i\alpha_0\beta_0^*e^{i\omega t_0}[\rho_{n,m+1}\cos(g\tau\sqrt{n+1})\sin(g\tau\sqrt{m+1})-\rho_{n-1,m}\sin(g\tau\sqrt{n})\cos(g\tau\sqrt{m})]\\&\quad+i\alpha_0^*\beta_0e^{-i\omega t_0}[\rho_{n,m-1}\cos(g\tau\sqrt{n})\sin(g\tau\sqrt{m})-\rho_{n+1,m}\sin(g\tau\sqrt{n+1})\cos(g\tau\sqrt{m+1})])\ ,\end{aligned} \tag{5}$$

where τ is the time an atom spends in the cavity.

We now assume that $|\beta_0|$ is small compared to $|\alpha_0|$ and expand the sin and cos functions to the second order in the argument. We then obtain

$$\begin{aligned}\left[\frac{\partial\rho_{n,m}}{\partial t}\right]_{\text{atoms}}&=-\tfrac{1}{2}r|\alpha_0|^2g^2\tau^2(n+1+m+1)\rho_{n,m}+r|\alpha_0|^2g^2\tau^2\sqrt{nm}\,\rho_{n-1,m-1}\\&\quad-\tfrac{1}{2}r|\beta_0|^2g^2\tau^2(n+m)\rho_{n,m}+r|\beta_0|^2g^2\tau^2\sqrt{(n+1)(m+1)}\rho_{n+1,m+1}\\&\quad+\tfrac{1}{24}r|\alpha_0|^2g^4\tau^4[(m+1)^2+(n+1)^2+6(n+1)(m+1)]\rho_{n,m}-\tfrac{1}{6}r|\alpha_0|^2g^4\tau^4[(n+m)\sqrt{nm}\,]\rho_{n-1,m-1}\\&\quad+i\alpha_0\beta_0^*e^{i\omega t_0}rg\tau(\sqrt{m+1}\rho_{n,m+1}-\sqrt{n}\,\rho_{n-1,m})+i\alpha_0^*\beta_0e^{-i\omega t_0}rg\tau(\sqrt{m}\,\rho_{n,m-1}-\sqrt{n+1}\rho_{n+1,m})\ .\end{aligned} \tag{6}$$

Note that in addition to the usual diagonal coupling between the matrix elements of ρ_F (Ref. 8) we have a coupling parallel to rows and columns (cf. Fig. 2).

Equation (6) can be rewritten using creation and annihilation operators for the field. In the interaction picture, we obtain the following result:

$$\left[\frac{\partial\rho_F}{\partial t}\right]_{\text{atoms}} = -\tfrac{1}{2}r\,|\alpha_0|^2 g^2\tau^2(aa^\dagger\rho_F+\rho_F aa^\dagger-2a^\dagger\rho_F a)-\tfrac{1}{2}r\,|\beta_0|^2 g^2\tau^2(a^\dagger a\rho_F+\rho_F a^\dagger a-2a\rho_F a^\dagger)$$
$$+\tfrac{1}{24}g^4\tau^4 r\,|\alpha_0|^2(aa^\dagger aa^\dagger\rho_F+\rho_F aa^\dagger aa^\dagger+6aa^\dagger\rho_F aa^\dagger-4a^\dagger aa^\dagger\rho_F a-4a^\dagger\rho_F aa^\dagger a)$$
$$-i\alpha_0\beta_0^* e^{i\omega(t_0-\tau)} rg\tau[a^\dagger,\rho_F]+i\alpha_0^*\beta_0 e^{-i\omega(t_0-\tau)} rg\tau[a,\rho_F]\ . \tag{7}$$

The interaction with the heat bath is described by the following master equation:[9]

$$\left[\frac{\partial\rho_F}{\partial t}\right]_{\text{bath}} = -\tfrac{1}{2}\gamma n_b(aa^\dagger\rho_F+\rho_F aa^\dagger-2a^\dagger\rho_F a)-\tfrac{1}{2}\gamma(n_b+1)(a^\dagger a\rho_F+\rho_F a^\dagger a-2a\rho_F a^\dagger)\ , \tag{7'}$$

where n_b is the number of thermal photons in the resonator and γ the coupling constant. Therefore the total time derivative of the field density matrix is

$$\dot\rho_F=\left[\frac{\partial\rho_F}{\partial t}\right]_{\text{atoms}}+\left[\frac{\partial\rho_F}{\partial t}\right]_{\text{bath}}$$
$$=\tfrac{1}{2}A(aa^\dagger\rho_F+\rho_F aa^\dagger-2a^\dagger\rho_F a)-\tfrac{1}{2}C(a^\dagger a\rho_F+\rho_F a^\dagger a-2a\rho_F a^\dagger)$$
$$+\tfrac{1}{8}B(aa^\dagger aa^\dagger\rho_F+\rho_F aa^\dagger aa^\dagger+6aa^\dagger\rho_F aa^\dagger-4a^\dagger aa^\dagger\rho_F a-4a^\dagger\rho_F aa^\dagger a)-i[sa^\dagger+s^*a,\rho_F] \tag{8}$$

with

$$A=r\,|\alpha_0|^2 g^2\tau^2+\gamma n_b\ ,$$
$$B=\tfrac{1}{3}r\,|\alpha_0|^2 g^4\tau^4\ ,$$
$$C=r\,|\beta_0|^2 g^2\tau^2+\gamma(n_b+1)\ ,$$

and

$$s=\alpha_0\beta_0^* e^{i\omega(t_0-\tau)} g\tau r\ .$$

This result is the well-known maser equation, where A is the gain coefficient, B the saturation coefficient, and C the loss coefficient, to which we have added the commutator due to the coherence of the incoming atoms. The same commutator term is obtained when an injected classical signal is considered. The symmetry-breaking nature of this term is seen best in the α representation. In this representation, Eq. (8) reads[3]

$$\dot P(\alpha,\alpha^*;t)=-\tfrac{1}{2}(A-C)\frac{\partial}{\partial\alpha}(\alpha P)+\tfrac{1}{2}A\frac{\partial^2}{\partial\alpha\partial\alpha^*}P$$
$$+\tfrac{1}{2}B\frac{\partial}{\partial\alpha}(\alpha\alpha^*\alpha P)+is\frac{\partial P}{\partial\alpha}+\text{c.c.}\ , \tag{9}$$

where $\rho_F=\int P(\alpha)\,|\alpha\rangle\langle\alpha|\,d^2\alpha$.

The steady-state solution of this Fokker-Planck equation has been found to be[3]

$$P(\alpha,\alpha^*;t)=N\exp\left[\frac{4}{A}\left[\tfrac{1}{4}(A-C)\,|\alpha|^2-\tfrac{1}{8}B\,|\alpha|^4+\frac{i}{2}(s^*\alpha-s\alpha^*)\right]\right] \tag{10}$$

with a normalization constant N.

In the steady state, $P(\alpha)$ does no longer depend on $|\alpha|$ alone but also on the angle θ, i.e., the symmetry with respect to the angle is broken. The effect can be seen from Fig. 3 where the exponent of $P(\alpha)$ is plotted. Broken symmetry is synonymous with field coherence, and

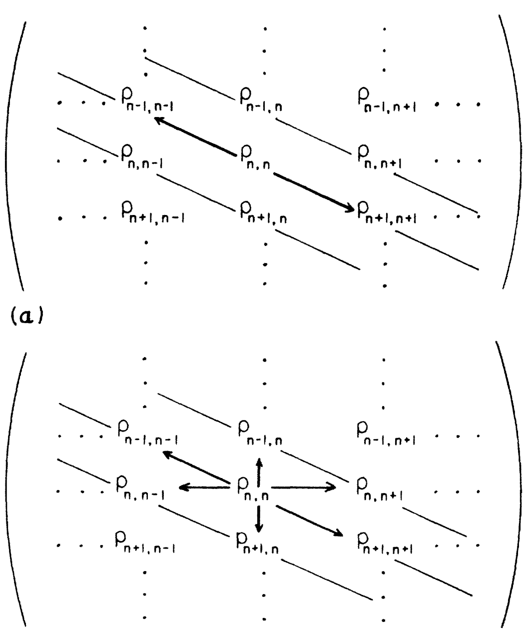

FIG. 2. (a) Density matrix of laser with coupling of the matrix elements parallel to the main diagonal. (b) Density matrix of laser when coherent atoms are injected. The coherence induces additional coupling parallel to rows and columns.

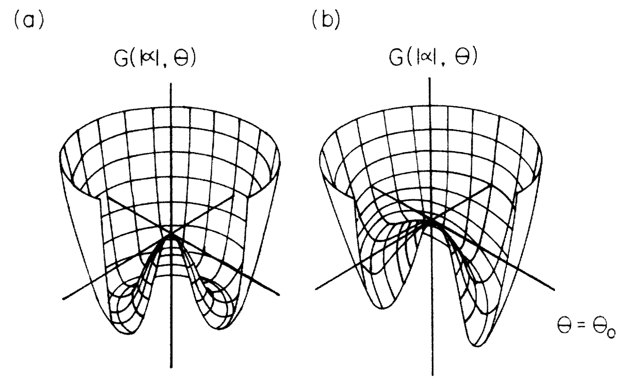

FIG. 3. Negative exponent $G(|\alpha|,\theta)$ of the field distribution function $P(\alpha)$ without injection of coherent atoms (a) and when coherent atoms with phase θ_0 are injected (b).

we may say that the coherence of the field in the first cavity is transferred to the second cavity via the atomic beam.

IV. PROBING THE PHASE OF MASER II

We next consider probing the phase of the field in the second cavity via the excitation probability of the outgoing atoms. The density operator of the combined system of an atom entering the cavity and the field is given by

$$\rho=\int P(\alpha)\Big(|\alpha_0|^2|a,\alpha\rangle\langle a,\alpha| + |\beta_0|^2|b,\alpha\rangle\langle b,\alpha| + \alpha_0\beta_0^* e^{i\omega t_0}|a,\alpha\rangle\langle b,\alpha| + \alpha_0^*\beta_0 e^{-i\omega t_0}|b,\alpha\rangle\langle a,\alpha|\Big)d^2\alpha\ . \tag{11}$$

The time development for the combined states $|a,\alpha\rangle$ and $|b,\alpha\rangle$ can be expressed in the following way, using the interaction picture again, where $U(\tau)$ is the time-development operator,

$$U(\tau)|a,\alpha\rangle = \sum_{m=0}^{\infty}(g\tau)^{2m}\frac{(-1)^m}{(2m!)}(aa^\dagger)^m|a,\alpha\rangle - ie^{-i\omega\tau}\sum_{m=0}^{\infty}(g\tau)^{2m+1}\frac{(-1)^m}{(2m+1)!}(a^\dagger a)^m a^\dagger|b,\alpha\rangle \tag{12a}$$

and

$$U(\tau)|b,\alpha\rangle = \sum_{m=0}^{\infty}(g\tau)^{2m}\frac{(-1)^m}{(2m)!}(a^\dagger a)^m|b,\alpha\rangle - ie^{i\omega\tau}\sum_{m=0}^{\infty}(g\tau)^{2m+1}\frac{(-1)^m}{(2m+1)!}(aa^\dagger)^m a\,|a,\alpha\rangle\ . \tag{12b}$$

Inserting these expressions into the equation for $\rho(\tau)=U(\tau)\rho U^\dagger(\rho)$ and tracing over the field states we obtain

$$\rho_A(\tau)=\int P(\alpha)\langle\alpha|U(\tau)\rho U^\dagger(\tau)|\alpha\rangle d^2\alpha\ . \tag{13}$$

The probability w_a for the atom to be in the excited state can be extracted from this equation,

$$\begin{aligned} w_a=\int P(\alpha)\Bigg[&|\alpha_0|^2\sum_{l=0}^{\infty}(g\tau)^{2l}\frac{(-1)^l}{(2l)!}\sum_{m=0}^{\infty}(g\tau)^{2m}\frac{(-1)^m}{(2m)!}\langle\alpha|(aa^\dagger)^{l+m}|\alpha\rangle \\ &+|\beta_0|^2\sum_{l=0}^{\infty}(g\tau)^{2l+1}\frac{(-1)^l}{(2l+1)!}\sum_{m=0}^{\infty}(g\tau)^{2m+1}\frac{(-1)^m}{(2m+1)!}\langle\alpha|(a^\dagger a)^{l+m+1}|\alpha\rangle \\ &+i\alpha_0\beta_0^* e^{i\omega(t_0-\tau)}\sum_{l=0}^{\infty}(g\tau)^{2l+1}\frac{(-1)^l}{(2l+1)!}\sum_{m=0}^{\infty}g(\tau)^{2m}\frac{(-1)^m}{(2m)!}\langle\alpha|(aa^\dagger)^{l+m}|\alpha\rangle\alpha^* \\ &-i\alpha_0^*\beta_0 e^{-i\omega(t_0-\tau)}\sum_{l=0}^{\infty}(g\tau)^{2l}\frac{(-1)^l}{(2l)!}\sum_{m=0}^{\infty}(g\tau)^{2m+1}\frac{(-1)^m}{(2m+1)!}\langle\alpha|(aa^\dagger)^{l+m}|\alpha\rangle\alpha\Bigg]d^2\alpha\ . \end{aligned} \tag{14}$$

Note that additional factors α^* and α in the cross term, respectively, resulting from unpaired operators $a^\dagger$ and a, give an explicit dependence on the phase angle. The matrix elements $\langle\alpha|(aa^\dagger)^k|\alpha\rangle$ and $\langle\alpha|(a^\dagger a)^k|\alpha\rangle$ can be calculated by the recursion relations

$$\langle\alpha|(a^\dagger a)^k|\alpha\rangle=|\alpha|^2\langle\alpha|(aa^\dagger)^{k-1}|\alpha\rangle$$

and

$$\langle\alpha|(aa^\dagger)^k|\alpha\rangle=1+|\alpha|^2\sum_{i=1}^{k}\begin{bmatrix}k\\ i\end{bmatrix}\langle\alpha|(aa^\dagger)^{i-1}|\alpha\rangle\ .$$

If $|\alpha|\gg 1$, however, we can neglect the effects of spontaneous emission, and we get $\langle\alpha|(aa^\dagger)^k|\alpha\rangle \simeq\langle\alpha|(a^\dagger a)^k|\alpha\rangle\simeq|\alpha|^{2k}$, thus obtaining the classical expression

$$w_a=\int_0^\infty \int_0^{2\pi} P(\alpha)\{\,|\alpha_0|^2\cos^2(g\tau|\alpha|)+|\beta_0|^2\sin^2(g\tau|\alpha|)+2|\alpha_0||\beta_0|\sin(g\tau|\alpha|)\cos(g\tau|\alpha|)\sin[\theta-\phi+\omega(\tau-t_0)]\}d\theta\,|\alpha|\,d\,|\alpha|\ , \tag{15}$$

where we have defined $\theta\equiv\arg\alpha$ and $\phi\equiv\arg(\alpha_0\beta_0^*)$. $P(\alpha)$ depends on the angle θ in the same way. Therefore the integral over the cross term does not vanish. Because of

$$\frac{i}{2}(s^*\alpha-s\alpha^*)=\mathrm{Im}(g\tau r\,|\alpha_0||\beta_0||\alpha|\times\exp\{i[\omega(t_0-\tau)-\theta+\phi]\})=-|\alpha_0||\beta_0|g\tau r|\alpha|\times\sin[\theta-\phi+\omega(\tau-t_0)]\ , \tag{16}$$

we obtain an integral of the form

$$\int_0^{2\pi}\sin(\theta-\theta_0)e^{x\sin(\theta-\theta_0)}d\theta=\int_0^{2\pi}\sin\theta e^{x\sin\theta}d\theta\equiv f(x)\ . \tag{17a}$$

If the phase of the pump field or the distance between the two cavities is changed and the phase of the atoms is now θ_0', then the cross term yields

$$\int_0^{2\pi}\sin(\theta-\theta_0')e^{x\sin(\theta-\theta_0)}d\theta=\cos(\theta_0-\theta_0')f(x)\ . \tag{17b}$$

Thus we have obtained a dependence on the phase of the atoms relative to the field which can be detected by the measurement of the atomic excitation probability.

V. FLUCTUATIONS OF THE ANGLE VARIABLE

When we carry out the measurement of the excitation probability according to Sec. IV, we do not only probe the coherence of the field in the second cavity. In addition, we obtain information about the fluctuation of the phase angle. From Eqs. (15) and (17b) we expect the excitation probability of the outgoing atoms to be an oscillatory function of the phase difference $\theta_0-\theta_0'$ as defined in (17). When we assume that the ensemble average for $|\alpha_0|$, $|\beta_0|$, and τ can be carried out independently from the average over the field variables, we obtain

$$w_a=|\alpha_0|^2\langle\cos^2(g\tau|\alpha|)\rangle+|\beta_0|^2\langle\sin^2(g\tau|\alpha|)\rangle+2|\alpha_0||\beta_0|\langle\sin(g\tau|\alpha|)\cos(g\tau|\alpha|)\sin\theta\rangle\times\cos(\theta_0-\theta_0')\equiv\zeta+\xi\cos(\theta_0-\theta_0')\ . \tag{18}$$

The amplitude of the oscillation ξ in (18) can be measured by varying the phase angle for the incoming atoms as well as the constant contribution ζ. The latter can be rewritten

$$\zeta=|\alpha_0|^2-(|\alpha_0|^2-|\beta_0|^2)\langle\sin^2(g\tau|\alpha|)\rangle\ , \tag{18a}$$

and the expectation value $\langle\sin^2(g\tau|\alpha|)\rangle$ can be extracted. Since we have assumed $g\tau|\alpha|<1$, we also can calculate from this an approximate value for $\langle|\alpha|\rangle$ and $\bar n=\langle|\alpha|^2\rangle$ as well.

If $\bar n$ is sufficiently large, the field distribution $P(\alpha)$ will be strongly peaked around $\langle|\alpha|\rangle\simeq(\bar n)^{1/2}$, and the averages over $|\alpha|$ and the angle θ can be carried out separately. We then get for the amplitude of the oscillation

$$\xi\simeq2|\alpha_0||\beta_0|\langle\sin(g\tau|\alpha|)\cos(g\tau|\alpha|)\rangle\langle\sin\theta\rangle\ . \tag{18b}$$

When we use the approximation

$$\langle\sin(g\tau|\alpha|)\cos(g\tau|\alpha|)\rangle\simeq\{\langle\sin^2(g\tau|\alpha|)\rangle[1-\langle\sin^2(g\tau|\alpha|)\rangle]\}^{1/2}\ , \tag{18c}$$

we get $\langle\sin\theta\rangle$ from (18a) and (18b). The angular distribution which we get from (10) is peaked at $\theta=3\pi/2$ (or at $\theta=\pi/2$, since the sign cannot be determined by measuring the amplitude). Therefore we have

$$-\langle\sin\theta\rangle=\langle\cos(\theta-3\pi/2)\rangle\simeq1-\tfrac{1}{2}\langle(\theta-3\pi/2)^2\rangle\ , \tag{19a}$$

if the fluctuations around the maximum value are not too large. When we insert $\cos\theta$ instead of $\sin\theta$ in (17a), we immediately obtain

$$\langle\cos\theta\rangle=\langle\sin(\theta-3\pi/2)\rangle\simeq\langle\theta-3\pi/2\rangle=0\ , \tag{19b}$$

i.e., the standard deviation for the phase angle can be obtained from (19a),

$$\Delta\theta=[\langle(\theta-3\pi/2)^2\rangle]^{1/2}\ . \tag{20}$$

Model calculations using the distribution (10) show that (18b) is generally a good approximation. In Eq. (18c), however, the right-hand side is larger than the left-hand side which results in an artificial broadening of the measured angular distribution. Nevertheless, this error can be tolerated when $\Delta\theta$ is not too small.

It is obvious from Eq. (17) that the width of the angular distribution strongly depends on the value of $|s|$. If $|s|=0$, the maximum of the $|\alpha|$ distribution is at $(\bar n)^{1/2}=[(A-C)/B]^{1/2}$ and there is no symmetry breaking. For $|s|>0$, the maximum of the $|\alpha|$ distribution is shifted to higher values of $|\alpha|$. This effect is considerable for large values of $|s|$, and one should be able to detect it via $\langle\sin^2(g\tau|\alpha|)\rangle$ from Eq. (18a). The angular distribution has a width which varies like $(|s|\langle|\alpha|\rangle)^{-1/2}$. When the magnitude of $|s|$ is estimated the velocity spread of the atomic beam has to be taken into account. It can reduce $\langle\exp[i\omega(t_0-\tau)]\rangle$ to a factor smaller than unity, so that the width $\Delta\theta$ is large enough to be detected.

VI. CONCLUSION

It is the purpose of this work to show that coherence can be transferred to a field in a cavity via atoms in a coherent superposition of states. If the excitation probability for the outgoing atoms is measured for different phase angles of the atoms relative to the field, the coherence of the field can be detected.

Phase locking of the incoming atoms is achieved by a cavity with a coherent field which the atoms traverse first. The phase angle of the atoms depends on the phase of the pump field and the distance between the two cavities. Both phases can be altered, where the external pump field has an easier experimental access. Thus all the tools for a direct measurement of the transfer of coherence via an atomic beam are provided.

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