Electron—H-atom collisions in the presence of a laser field: One-photon free-free transitions

A. Dubois and A. Maquet
Laboratoire de Chimie Physique, Université Pierre et Marie Curie, 11, Rue Pierre et Marie Curie, F-75231 Paris Cédex 05, France

S. Jetzke
Fakultät für Physik, Universität Bielefeld, D 4800 Bielefeld, Federal Republic of Germany
(Received 24 February 1986)

We report the derivation of an analytical expression of the one-photon transition amplitude for free-free transitions occurring in the course of the collision of fast electrons with a hydrogen atom, in the presence of a laser. The calculation is performed within the framework of the conventional time-dependent perturbation theory to lowest nonvanishing order. The projectile electron wave function is approximated as a plane wave and the infinite sums running over the H-atom spectrum are accounted for exactly by using a compact representation of the Coulomb Green's function. As an illustration of the usefulness of our formalism, we discuss the occurrence of dips of two distinct origins in the cross sections and compare the respective importances of absorption and stimulated emission.

I. INTRODUCTION

The process we shall consider here can be described as follows: in the course of the collision of a fast electron with a hydrogen atom, in the presence of a laser beam, the electron-atom system can absorb or emit one photon. This can be symbolically written as

$$e^-(E_i,\mathbf{k}_i)+\mathrm{H}(1s)\pm\gamma(\omega,\boldsymbol{\epsilon})\rightarrow e^-(E_f,\mathbf{k}_f)+\mathrm{H}(1s)\ . \quad (1)$$

Here E_i (E_f) and $\mathbf{k}_i$ ($\mathbf{k}_f$) are the energies and wave vectors of the incoming (outgoing) projectile electron; $\pm\gamma(\omega,\boldsymbol{\epsilon})$ represents the absorption (+) or the stimulated emission (−) of one laser photon with frequency ω and polarization $\boldsymbol{\epsilon}$. E_i and E_f are connected via the energy conservation relation

$$E_f=E_i\pm\hbar\omega\ . \quad (2)$$

Recent reviews on the subject and on related processes can be found in Refs. 1—5. At very high laser intensities the dressing of the projectile and of the target becomes important and one has to resort to nonperturbative approaches.[6—9] However, for the moderate intensities of most of the available lasers, perturbation theory still provides a sensible description of the main features of the process. Within this framework two approaches can be contemplated, depending on the chosen description of the compound system e^--H. For instance, the process can be described as a first-order radiative transition between states of the continuous spectrum of the H^- ion.[10—12] However, if the projectile is fast enough, another, somewhat simpler approach, based on the Born approximation, can be used.[13—15] In this latter case, the process has to be considered as of second order, the main difficulty encountered in the calculation arising then from the presence of the infinite sum running over the whole hydrogenic spectrum. Within the context of the problem considered here, it has been overcome in various ways, either by solving an inhomogeneous differential equation,[13] or by using a Sturmian expansion of the Coulomb Green's function.[14,15] In these works, however, the transition amplitude is expressed in terms of infinite series whose analytical properties are not easily understood. We report here the derivation of a closed-form analytical expression of the amplitude, given in terms of Appell's hypergeometric functions of two variables, obtained by using an integral representation of the Coulomb Green's function.[16] The distinctive advantage of deriving an analytical expression is that it is an easy matter to investigate its behavior in limiting cases of physical interest, such as the soft-photon or the small-momentum-transfer limits or even to get its analytical continuation towards the region of higher photon frequencies, i.e., when other expansions are inadequate.

The organization of the paper is as follows: in Sec. II we briefly present the formalism and delineate the various levels of approximation used in the calculation. The derivation of the final expression of the amplitude is outlined in Sec. III, in which the soft-photon ($\omega\rightarrow 0$) and the small-momentum-transfer ($\boldsymbol{\Delta}\rightarrow\mathbf{0}$, $\boldsymbol{\Delta}=\mathbf{k}_i-\mathbf{k}_f$) limits are also investigated. Numerical results are presented in Sec. IV, in which we discuss the relative magnitudes of the different terms contributing to the total amplitude. We also compare the transition rates of absorption and stimulated emission for various values of the parameters governing the collision dynamics. A brief discussion ends the paper in Sec. V.

II. THEORY

As already mentioned, the relevant cross section for the scattering of an electron by a hydrogen atom, with absorption (emission) of one photon from a single-mode

laser field, can be defined into two different ways, depending on the chosen description of the process. If it is described as a first-order radiative transition between states belonging to the continuous spectrum of the H^- ion, the differential cross section reads in atomic units[11]

$$d\sigma/d\Omega(\mathbf{k}_f)=(\alpha/2\pi)k_f\omega\,|\,(\mathbf{r}\cdot\boldsymbol{\epsilon})_{f,i}\,|^2\,. \tag{3}$$

This expression has the dimensions of (length)^5, i.e., is a transition probability, normalized to both the incoming electron density and photon flux. In atomic units it is given in terms of a_0^5, where a_0 is the Bohr radius. Here α is the fine-structure constant and $(\mathbf{r}\cdot\boldsymbol{\epsilon})_{f,i}$ is the dipole transition matrix element between states of the continuous spectrum of the H^- ion, with asymptotic wave numbers $\mathbf{k}_i$ and $\mathbf{k}_f$.

Another approach is to consider the H atom and the projectile electron as independent and to treat perturbatively the Coulomb as well as the laser interactions. If, in addition, the projectile is fast enough, its wave function can be approximated by a plane wave and exchange effects can be neglected. Within this approximation the process has to be described as second order, since two interactions are responsible for the transition[13]

$$H_e=(1/r_{12}-1/r_1);\;\; H_r=A_0\boldsymbol{\epsilon}(a+a^\dagger)\cdot\mathbf{p}\,. \tag{4}$$

Here H_e corresponds to the Coulomb interaction between the projectile and the atom, $\mathbf{r}_1$ being the position of the projectile and $r_{12}=|\,\mathbf{r}_1-\mathbf{r}_2\,|$ where $\mathbf{r}_2$ is the position of the atomic electron. H_r is the dipole interaction Hamiltonian where A_0 is the amplitude of the vector potential of the laser field with polarization $\boldsymbol{\epsilon}$ and a and $a^\dagger$ are the usual photon annihilation and creation operators for the considered mode.

The relevant second-order transition amplitude is of the general form

$$T_{f,i}^{(2)}=\langle f\,|\,(H_r+H_e)G_0(E_i)(H_r+H_e)\,|\,i\rangle\,, \tag{5}$$

where $|\,i\rangle$ and $|\,f\rangle$ are the initial and final eigenstates of the Hamiltonian operator H_0 of the unperturbed, uncoupled system $H_0=H_{\rm at}+H_{\rm el}+H_f$, Here $H_{\rm at}$ corresponds to the hydrogen atom, $H_{\rm el}$ is the kinetic energy operator of the projectile electron (hereafter labeled with the index 1), H_f describes the laser field and $G_0(z)=(z-H_0)^{-1}$ is the resolvent operator associated with H_0. Among the four possible combinations of the interaction Hamiltonians only two of them actually do contribute to the process defined by the energy conservation relation Eq. (2) and one gets

$$T_{f,i}^{(2)}=\langle f\,|\,H_rG_0(E_i)H_e\,|\,i\rangle+\langle f\,|\,H_eG_0(E_i)H_r\,|\,i\rangle\,. \tag{6}$$

As the interaction Hamiltonians H_r and H_e can both act into the projectile or the atomic configuration space, this leads to four amplitudes which are conveniently described with the help of the diagrams displayed in Fig. 1.[15] For the sake of future discussion, we shall refer to the contributions of the diagrams I and II as "direct" since they correspond to the interaction of the laser field with the projectile and are usually the only ones taken into account in lowest-order bremsstrahlung calculations. On the other hand, the contributions of the diagrams III and IV will be named "atomic" as they depict the interaction of the photon with the atomic electron.

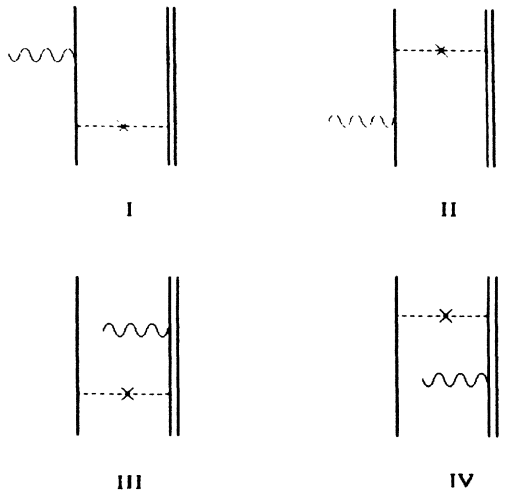

FIG. 1. Diagrams contributing to the one-photon free-free transitions occurring in the course of an e^-–H-atom collision. The diagrams denoted I and II are referred to as "direct" and those denoted III and IV are referred to as "atomic" diagrams in the text.

When specialized to the case of a single-mode laser with occupation number $N\gg1$, the contribution of the photon field reduces to a multiplicative factor common to each amplitude entering the expression, Eq. (6), of the overall transition matrix element $T_{f,i}^{(2)}$. It is then an easy matter to sum over the free-wave projectile states, using in particular the known result

$$\left\langle \mathbf{k}_f\left|\frac{1}{r_{12}}-\frac{1}{r_1}\right|\mathbf{k}_i\right\rangle=\frac{4\pi}{\Delta^2}[\exp(i\boldsymbol{\Delta}\cdot\mathbf{r}_2)-1]\,, \tag{7}$$

where $\boldsymbol{\Delta}=\mathbf{k}_i-\mathbf{k}_f$. This allows us to express $T_{f,i}^{(2)}$ in terms of atomic matrix elements. Then, by substituting into the expression of the transition probability, normalized to the incoming electron density and photon flux, one obtains the following general formula for the differential cross section:

$$\frac{d\sigma}{d\Omega(\mathbf{k}_f)}=\frac{8\pi\alpha}{\Delta^4}\frac{k_f}{\omega}\Big|i\omega[\langle 1s\,|\,e^{i\boldsymbol{\Delta}\cdot\mathbf{r}}G_C(E_{1s}-\omega)\mathbf{r}\cdot\boldsymbol{\epsilon}\,|\,1s\rangle + \langle 1s\,|\,\mathbf{r}\cdot\boldsymbol{\epsilon}G_C(E_{1s}+\omega)e^{i\boldsymbol{\Delta}\cdot\mathbf{r}}\,|\,1s\rangle] + \frac{\boldsymbol{\Delta}\cdot\boldsymbol{\epsilon}}{\omega}\langle 1s\,|\,(e^{i\boldsymbol{\Delta}\cdot\mathbf{r}}-1)\,|\,1s\rangle\Big|^2\,. \tag{8}$$

This formula, as it stands, is valid for both emission and absorption, the difference between these processes lying only in the energy conservation relation, Eq. (2), which governs the respective magnitudes of k_i, k_f, and Δ. We note that this form of the cross section should be equivalent to the expression Eq. (3), within the limits of validity of the Born approximation for describing the pro-

jectile. Note also that in the first two terms, corresponding respectively to the diagrams III and IV, $G_C(W)=(W-H_{at})^{-1}$ is the Coulomb Green's function. The third term, which represents the contribution of the diagrams I and II, reduces here to the ground-state atomic form factor

$$\langle 1s \mid (e^{i\Delta\cdot r}-1) \mid 1s\rangle = 16(\Delta^2+4)^{-2}-1 . \tag{9}$$

We note, for the sake of future discussion, that the term -1 which appears on the left-hand side corresponds to the mere contribution of the atomic nucleus to the process. The computation of the terms containing the Coulomb Green's function is described in the following section.

III. CALCULATION

We describe here the analytical calculation of atomic matrix elements of the general form

$$M_{1s}(\Delta,\epsilon;\Omega)=\langle 1s \mid e^{i\Delta\cdot r}G_C(\Omega)\mathbf{r}\cdot\epsilon \mid 1s\rangle . \tag{10}$$

Our starting point will be the formula derived by Klarsfeld,[17] for the closely related matrix element

$$M_{\mu,\mu'}(\Delta,\Delta';\Omega)=\int d\mathbf{r}\int d\mathbf{r}'\frac{1}{r}e^{-\mu' r}e^{-i\Delta'\cdot r}G_C(\mathbf{r},\mathbf{r}';\Omega)e^{+i\Delta\cdot r'}e^{-\mu r'}\frac{1}{r'} \tag{11a}$$

$$=\frac{-16\pi x}{[(\mu+x)^2+\Delta^2][(\mu'+x)^2+\Delta'^2]}\int_0^1 du\, u^{-1/x}(1-2\beta u+\gamma^2u^2)^{-1} . \tag{11b}$$

Here $x=\sqrt{-2\Omega}$ and

$$\beta=\frac{(\mu^2-x^2+\Delta^2)(\mu'^2-x^2+\Delta'^2)+4x^2\Delta\cdot\Delta'}{[(\mu+x)^2+\Delta^2][(\mu'+x)^2+\Delta'^2]} , \tag{12a}$$

$$\gamma^2=\frac{[(\mu-x)^2+\Delta^2][(\mu'-x)^2+\Delta'^2]}{[(\mu+x)^2+\Delta^2][(\mu'+x)^2+\Delta'^2]} . \tag{12b}$$

Note that similar formulas have been derived by several authors in different contexts; see, for instance, Refs. 18 and 19.

The sought after matrix element $M_{1s}(\Delta,\epsilon;\Omega)$ can easily be obtained from $M_{\mu,\mu'}(\Delta,\Delta';\Omega)$, via the following transformation:

$$M_{1s}(\Delta,\epsilon;\Omega)=\frac{1}{\pi}\frac{\partial^2}{\partial\mu\partial\mu'}\lim_{\Delta'\to 0}\left[i\epsilon\cdot\frac{\partial}{\partial\Delta'}\right]M_{\mu,\mu'}(\Delta,\Delta';\Omega)\Big|_{\mu=\mu'=1} , \tag{13}$$

where the dipole operator $\epsilon\cdot\mathbf{r}$ has been introduced as resulting from the operation

$$\lim_{\Delta'\to 0}\left[i\epsilon\cdot\frac{\partial}{\partial\Delta'}e^{-i\Delta'\cdot r}\right] . \tag{14}$$

One then gets

$$M_{1s}(\Delta,\epsilon;\Omega)=-i2^7x^3(\Delta\cdot\epsilon)\frac{\partial^2}{\partial\mu\partial\mu'}[(\mu+x)^2+\Delta^2]^{-2}(\mu'+x)^{-4}\int_0^1 du\, u^{1-1/x}(1-2\tilde{\beta}u+\tilde{\gamma}^2u^2)^{-2}\Big|_{\mu=\mu'=1} \tag{15}$$

where $\tilde{\beta}$ and $\tilde{\gamma}^2$ are obtained from the expressions Eqs. (12a) and (12b) of β and γ^2 by replacing $\Delta'=0$. Noting that the remaining integral represents an Appell's hypergeometric function of two variables F_1,[19,20] one has

$$M_{1s}(\Delta,\epsilon;\Omega)=-\frac{i2^7x^4(\Delta\cdot\epsilon)}{2x-1}\frac{\partial^2}{\partial\mu\partial\mu'}[(\mu+x)^2+\Delta^2]^{-2}(\mu'+x)^{-4}F_1(2-1/x;2,2;3-1/x;u^+,u^-)\,|_{\mu=\mu'=1} \tag{16}$$

where u^+ and u^- are the zeros of the denominator in the integral Eq. (15):

$$u^{\pm}=\frac{(\mu'-x)(\mu-x\pm i\Delta)}{(\mu'+x)(\mu+x\pm i\Delta)} . \tag{17}$$

The action of the operators $\partial/\partial\mu$ and $\partial/\partial\mu'$ is obtained from the known derivation formulas of the functions F_1. Then, specializing to the case $\mu=\mu'=1$ and after some algebra, the following expression of the amplitude can be obtained:

$$M_{1s}(\Delta,\epsilon;\Omega)=i2^7(\Delta\cdot\epsilon)[A+B(+\Delta)+B(-\Delta)] , \tag{18a}$$

where

$$A=\tfrac{1}{2}(1-x^2)^{-1}(\Delta^2+4)^{-3}\times\left[2-\frac{(1+x)(\Delta^2+4)}{(2x-1)[(1+x)^2+\Delta^2]}\right] ; \tag{18b}$$

$$B(\pm\Delta)=\frac{8x^5}{(1+x)^6[(1+x)^2+\Delta^2]^3(2x-1)(3x-1)} \times\left[2-\frac{1+x\mp i\Delta}{1+x\pm i\Delta}\right] \times F_1(3-1/x;3,2;4-1/x;u^{\pm},u^{\mp}) . \quad (18c)$$

A few remarks concerning this expression may be of interest here. First, it can be shown that the preceding Appell's functions of two variables contained in Eqs. (18) can be expressed in fact as a finite sum of Gauss hypergeometric functions ${}_2F_1$. This may be readily shown from the integral representation of the F_1 functions.[17,18,20,21] However, since the actual computation of the F_1 functions as they stand does not present any special difficulty, we do not make explicit here their expression in terms of ${}_2F_1$ functions as it is too complicated to be reproduced here. Another point that deserves to be mentioned is that, given the numerous contiguity relations existing between F_1 functions whose parameters differ by an integer, the expression Eq. (18) is not unique and many variants might be derived.

We have found useful the compact form displayed here, in particular from the computational standpoint: the numerical calculation of these expressions has been performed on a 512k Macintosh microcomputer. Our numerical results have been independently checked on using a more general code based on the use of a Sturmian representation of the Coulomb Green's function.[22]

Another nice feature of this expression is that given the properties of the F_1 functions, it clearly displays the occurrence of simple poles in the atomic amplitude $M_{1s}(\Delta,\epsilon;\Omega)$. These poles are associated with the values $x=1/n$, $n=2,3,4,\ldots$ which correspond to laser frequencies matching an atomic resonance $\omega_n=\frac{1}{2}-1/2n^2$. For values of the frequency close to these resonances, one of the second-order atomic matrix elements will become very large and will dominate the other terms in the total transition amplitude.

The limiting behavior of the amplitude can be investigated in two cases of physical interest.

(i) *Soft-photon limit:* $\omega\to 0$. In this limit one has respectively: $\Omega=-\frac{1}{2}\pm\omega\to-\frac{1}{2}$ and $x\to 1$ and the lowest-order term of the expression Eqs. (18) reduces to

$$\lim_{\omega\to 0} M_{1s}(\Delta,\epsilon;\Omega)=-i(\Delta\cdot\epsilon)2^4 3(\Delta^2+12)(\Delta^2+4)^{-4} . \quad (19)$$

Since this term is preceded by the multiplicative factor $i\omega$ in the expression Eq. (8) of the total amplitude its contribution becomes vanishingly small with respect to the direct term. This point will be discussed in more details in Sec. IV.

(ii) *Small momentum transfer.* The limit of small momentum transfer is easily obtained via the approximation $e^{i\Delta\cdot\mathbf{r}}-1\sim i\Delta\cdot\mathbf{r}$, which corresponds to the Bethe-Born approximation. The atomic amplitude $M_{1s}(\Delta,\epsilon;\Omega)$ then reduces to the dipole second-order matrix element

$$\tilde{M}_{1s}(\Delta,\epsilon;\Omega)=i\langle 1s\mid\Delta\cdot\mathbf{r}G_C(\Omega)\epsilon\cdot\mathbf{r}\mid 1s\rangle . \quad (20)$$

The explicit form of this amplitude can be obtained from the general expression Eqs. (18) by investigating the limit $\Delta\to 0$. One easily gets

$$\tilde{M}_{1s}(\Delta,\epsilon;\Omega)=-i2^3(\Delta\cdot\epsilon)\left[\frac{x^2-2}{4(1-x^2)^2}+\frac{x}{(1-x^2)^2(1+x)^2(2x-1)}\,{}_2F_1(1,-1-1/x;3-1/x;\tilde{u})\right] , \quad (21)$$

where $\tilde{u}=(1-x)^2/(1+x)^2$. This result, or similar ones, had been derived by several authors in different contexts.[17,23–25]

It is interesting to note that, within the framework of the Bethe-Born approximation, the contribution of the direct term Eq. (7) associated to the diagrams I and II, Fig. 1, vanishes identically and the overall amplitude reduces to the above atomic terms Eq. (21). If, in addition, one allows the frequency ω to become small, one easily shows that

$$\lim_{\omega\to 0}\tilde{M}_{1s}(\Delta,\epsilon;\Omega)=-\tfrac{9}{4}(i\Delta\cdot\epsilon) , \quad (22)$$

which corresponds, as expected, to half the value of the static dipole polarizability of the H atom in its ground state.

These limits, as well as the general features of our results concerning the dependence of the differential cross section on the parameters governing the collision dynamics, are discussed in the next section.

IV. RESULTS AND DISCUSSION

As already noted by several authors,[13,14,25–29] the behavior of the cross section depends critically on the relative magnitudes of the direct (diagrams I and II) and atomic terms (diagrams III and IV). The contributions of those terms, in turn, depend on the parameters governing the collision dynamics, i.e., the laser polarization and frequency and the momenta of the incoming and outgoing electron.

The laser polarization plays a purely geometrical role which, as we shall see, can give rise to a strong asymmetry between absorption and emission. On the other hand, the energetics of the process depends prominently on the laser frequency and on the magnitudes of the projectile momenta. Note that, as the validity of our approximate treatment is restricted to relatively large values of the momenta (or energies) of the electron, one is naturally led to mainly discuss the influence of the laser frequency ω on the process. Within this context, one can distinguish two

distinct regimes according to whether ω is small (soft-photon approximation) or is of the same order of magnitude than characteristic atomic frequencies. The results of the preceding section will provide us the clue for discussing the variations of the cross sections in respectively the soft-photon and the vacuum-ultraviolet (vuv) or optical frequency ranges. We shall discuss afterward the limit of small momentum transfer $\Delta \to 0$ (Bethe-Born approximation) and extend the discussion to the general case in which none of these simplifying assumptions holds. In this regime, the angular variations as well as the dispersion curves for the cross sections display several interesting features such as deep minima, resulting either from purely geometrical considerations or even from destructive interferences between the different amplitudes.

(i) *Soft-photon limit:* $\omega \to 0$. When inserting the corresponding limiting form Eq. (19) of the atomic terms in the expression of the overall transition amplitude Eq. (8), one observes that they become negligibly small with respect to the direct term, associated to the diagrams III and IV, which is dominant given its dependence in ω^{-1}. This allows us to check that, in the soft-photon limit, the radiative cross section diverges and is proportional to the elastic scattering one:

$$\lim_{\omega \to 0} \frac{d\sigma}{d\Omega} \sim 8\pi\alpha k_f \omega^{-3} (\Delta \cdot \epsilon)^2 [16(\Delta^2+4)^{-2} - 1]^2 \Delta^{-4} . \quad (23)$$

This result is a form of the Low theorem,[30] which precisely states that the bremsstrahlung cross section diverges at low frequencies and remains proportional to the elastic scattering cross section.

The behavior of the differential cross section for one-photon absorption in this range of frequencies is illustrated in Fig. 2, in which we display its angular dependence for $E_i = 5.0$ and $\omega = 0.01$ a.u. One observes that, excepted at very small scattering angle $\theta < 3°$, corresponding to small momentum transfer, the direct term is overwhelmingly dominant. In this range, the value of the cross section is given, with great accuracy, by the expression Eq. (23). The occurrence of the minima, which appear here at very small scattering angle, is an almost general feature of the absorption cross sections and will be commented on below. Note also that, at wider angles, the cross sections for stimulated emission and for absorption display the same behavior, both processes being equiprobable in the low-frequency domain.

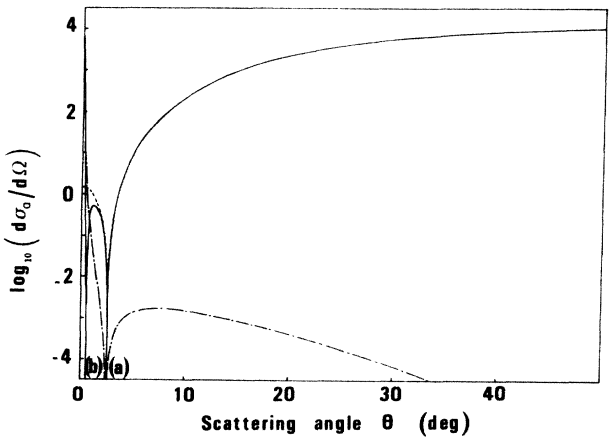

FIG. 2. Variations of $\log_{10}(d\sigma_a/d\Omega)$ with the scattering angle θ for one-photon absorption. $\epsilon \| \mathbf{k}_i$, $E_i = 5.0$ a.u. and $\omega = 0.01$ a.u. Dashed line: contribution of the direct terms; dot-dashed line: contribution of the atomic terms; solid line: overall cross section. The occurrence of the minima denoted (a) and (b) is discussed in the text.

(ii) *Optical and vuv frequencies.* When the laser frequency increases the dominance of the direct term becomes less marked, except at large scattering angles. This behavior is exemplified in Figs. 3 and 4, in which we present the angular variations of the absorption cross section for $E_i = 5.0$ and frequencies $\omega = 0.1$, and $\omega = 0.35$ a.u., respectively. We have considered here the particular geometry $\epsilon \| \mathbf{k}_i$ which simplifies to some extent the angular dependence without altering the generality of the discussion. In Fig. 4, for instance, we observe that the atomic term is dominant at angles $\theta < 25°$ and remains important until $\theta \cong 40°$.

In those figures the differential cross sections display two minima noted (a) and (b). Both kinds of minima correspond to values of the scattering angle for which the cross section is actually zero, nevertheless the origin of these zeroes is different in each case.

One observes that the minima denoted (a) appear in fact at angles such that the scalar product $\Delta \cdot \epsilon = 0$. Since this product is common to the direct and atomic terms, they both cancel as well as the overall amplitude. Note that this feature is peculiar to the case of transitions between rotationally symmetric s states. For the special geometry chosen here ($\epsilon \| \mathbf{k}_i \| \hat{\mathbf{z}}$), the values of these angles are simply given by the relation $\theta = \cos^{-1}(k_i/k_f)$ (note that this condition should be modified in the case of more general geometries corresponding to different orientations of the laser polarization[22]). This result allows us to predict the occurrence of deep minima in the absorption cross section, situation in sharp contrast with the case of emission in which the condition $k_i/k_f < 1$ cannot be met. This dissymmetry between absorption and emission will be discussed in more detail following.

On the other hand, the minima denoted (b) occur at angles for which neither the atomic nor the direct ampli-

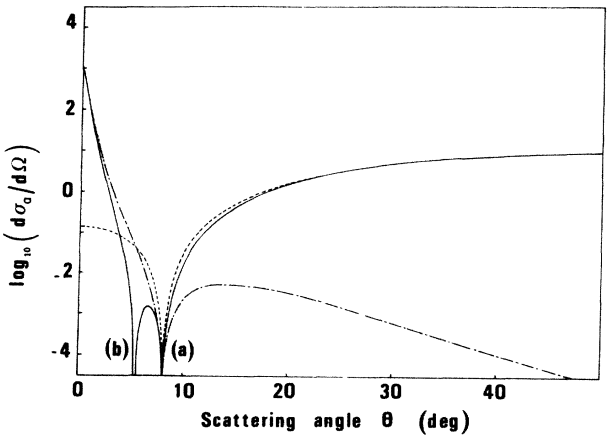

FIG. 3. Same as for Fig. 2, but with $\omega = 0.1$ a.u.

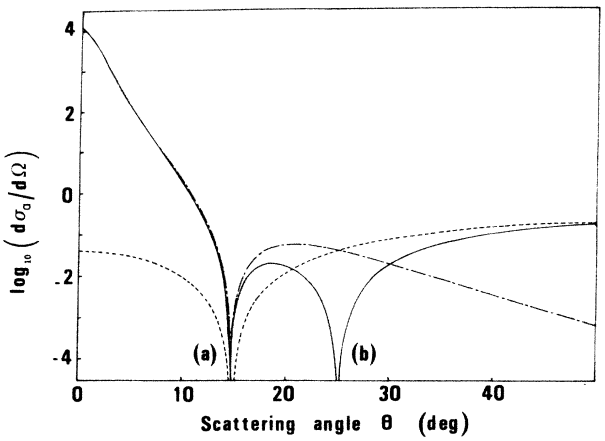

FIG. 4. Same as for Fig. 3, but with $\omega=0.35$ a.u.

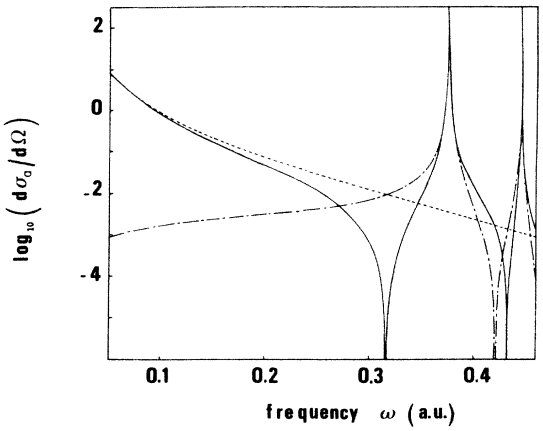

FIG. 6. Same as for Fig. 5, but with $E_i=20.0$ a.u.

tudes vanish. One can show in fact that they cancel since, for the set of parameters chosen here, they are of opposite signs and that their magnitudes are varying in opposite directions when the scattering angle, i.e., the momentum transfer, increases. This can easily be understood, when observing that at small values of the momentum transfer Δ the direct term is very small itself, see Eq. (9), and that the situation is inversed for large values of Δ.

These minima are also observed in the dispersion curves for the variations of the absorption cross section in terms of the laser frequency. In addition, such curves exhibit also the resonances occurring when the laser frequency matches an atomic transition frequency. This behavior is illustrated in Figs. 5 and 6, in which we present the variations of the absorption cross section in terms of the laser frequency, for a fixed scattering angle $\theta=10°$ and at energies of the incoming electron $E_i=5.0$ and $E_i=20.0$ a.u., respectively. The geometry is the same as in the preceding discussion. As the frequency increases, so does the momentum transfer, and one observes that the direct term decreases steadily, goes to zero when the condition $\Delta\cdot\epsilon=0$ is fulfilled and then increases again toward an almost constant value. This contrasts with the behavior of one of the atomic terms which exhibits a resonant structure when the laser frequency comes close to an atomic transition frequency: this makes the atomic contribution become dominant in a quite large domain in the neighborhood of the resonance. We note also the presence of minima between two resonances: this behavior results of the fact that the resonant atomic amplitude changes of sign in this range and can compensate the direct term contribution. We note also, in the Fig. 6, that at higher incoming electron energies, $E_i=20.0$ a.u., the condition $\Delta\cdot\epsilon=0$ cannot be met in the frequency range displayed here (the minimum would occur at $\omega=0.63$ a.u.). Accordingly, the observed minima belong to the type (b), i.e., correspond to destructive interference effects between the atomic and the direct amplitudes.

(iii) *Stimulated emission versus absorption.* The difference between emission and absorption may become important, principally in the vicinity of the minima discussed in the preceding paragraph. This is illustrated in Fig. 7, in

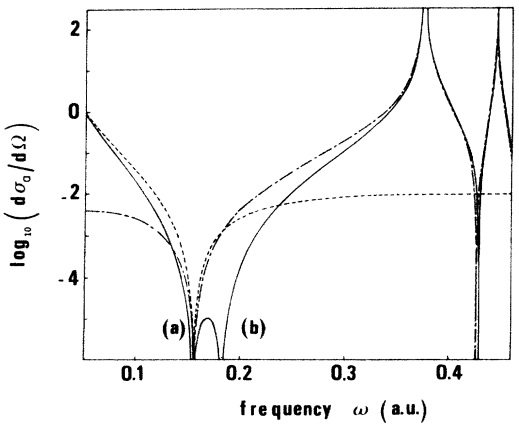

FIG. 5. Variations of $\log_{10}(d\sigma_a/d\Omega)$ in terms of the laser frequency ω for one-photon absorption. $\epsilon\|\mathbf{k}_i$, $\theta=10°$, $E_i=5.0$ a.u. Dashed line: contribution of the direct terms; dot-dashed line: contribution of the atomic terms; solid line: overall cross section.

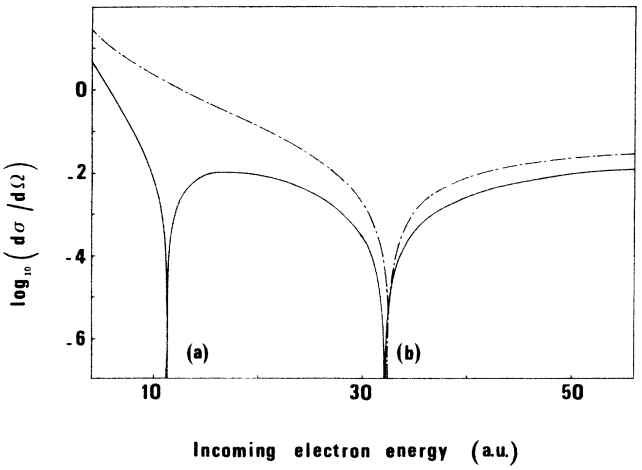

FIG. 7. Variations of $\log_{10}(d\sigma/d\Omega)$ in terms of the incoming electron energy E_i for one-photon absorption (solid line) and stimulated emission (dot-dashed line); $\epsilon\|\mathbf{k}_i$, $\omega=0.35$ a.u., and $\theta=10°$.

which we present the dependence of the cross sections for both processes in terms of the energy of the incoming electron, at fixed frequency $\omega=0.35$ a.u. and scattering angle $\theta=10°$. The orientation of the laser polarization is the same as before. The main difference observed between emission and absorption cross sections comes from the presence or the absence of the two kinds of minima discussed above. In particular, as already mentioned, the condition $\Delta\cdot\epsilon=0$ cannot be fulfilled in the emission case, for purely kinematical reasons. In the vicinity of the corresponding energy ($E_i=11.257$ a.u.) stimulated emission dominates absorption by several orders of magnitude.

The situation is a little bit more intricate in the vicinity of the other minima, which are caused by a destructive interference between the atomic and direct amplitudes. As a matter of fact, the cancelations occur in both cases at energies which, although close together, are distinct: this leads to steep variations of the ratio of the cross sections. This behavior is displayed in Fig. 8, in which we present the variations of the ratio of the cross sections for emission and absorption in terms of the laser frequency, at a given scattering angle and for two typical incoming electron energies. In the case $E_i=5$ a.u., one observes a "kinematical" maximum, denoted (a), corresponding to the vanishing of the absorption cross section, and a feature, denoted (b), associated to the destructive interferences in the amplitudes. These latter features are the only ones to survive at higher energies: this is already verified for $E_i=20.0$ a.u.

(iv) *Small momentum transfer, Bethe-Born approximation.* Our results clearly display the inadequacy of the Bethe-Born approximation to describe the process considered here, since it amounts to neglect the direct term which is often dominant, in particular at lower frequencies. In addition to the small-angle–small-momentum-transfer condition, the range of validity of this approximation is in fact reduced to the somewhat higher-frequency regime in which the atomic contribution can become resonant.

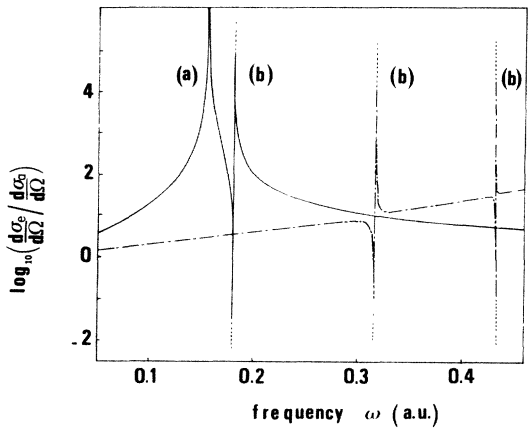

FIG. 8. Variations of the ratio of the cross sections for stimulated emission and absorption (logarithmic scale) with the laser frequency ω. $\epsilon\|\mathbf{k}_i$, $\theta=10°$. Solid line: $E_i=5.0$ a.u.; dot-dashed line: $E_i=20.0$ a.u.

V. CONCLUSION

In this paper, we have presented the results of analytical and numerical calculations of the amplitudes and cross sections for the free-free transitions occurring in the course of the collisions of fast electrons with a hydrogen atom, in the presence of a laser. The calculation has been performed within the framework of the conventional perturbation theory to lowest nonvanishing order. Analytical expressions of the transition amplitudes have been obtained by using a compact representation of the Coulomb Green's function, which allowed us to conveniently study several limits of physical interest.

Among other results we predict the presence of deep minima in the absorption cross sections. Some of these minima are of purely kinematical origin and could be observed under a rather large class of scattering geometries and laser polarization orientations, for transitions between s states. As they have no equivalent in the stimulated emission case, this latter can be strongly favored with respect to absorption. Another kind of minima, associated to destructive interferences between the so-called atomic and direct amplitudes, can also occur in both the emission and absorption cases. Since they result from the cancelation of two amplitudes which are evaluated at a lowest-order approximation, their location cannot be assessed with certainty. It is even possible that a higher-order calculation would, at least partly, wash out these minima. However we wish to stress here that our calculation is consistent within the order of approximation retained and that any refinement would present considerable difficulties since it would require the computation of second-order Born amplitudes.

More general calculations, including the excitation of the atom and conducted along the same lines, except that they rely on another, Sturmian, representation of the Coulomb Green's function, will be published elsewhere.[22]

ACKNOWLEDGMENTS

We would like to acknowledge very helpful discussions with Professor M. Gavrila. The Laboratoire de Chimie Physique de l'Université Pierre et Marie Curie is a "Unité Associée au CNRS, UA No. 176." The support of the Deutsche Forschungsgemeinschaft in Sonderforschungsbereich 216 is gratefully acknowledged.

[1]M. Gavrila and M. Van der Wiel, Comments At. Mol. Phys. 8, 1 (1978).

[2]M. H. Mittleman, *Theory of Laser-Atom Interactions* (Plenum, New York, 1982).

[3]L. Rosenberg, in *Advances in Atomic and Molecular Physics*, edited by D. Bates and B. Bederson (Academic, New York, 1982), Vol. 18, p. 1.

[4]A. Lami and N. K. Rahman, in *Collisions and Half Collisions*

with Lasers, edited by N. K. Rahman and C. Guidotti (Harwood, London, 1984), p. 373.
[5]F. H. M. Faisal, Comments At. Mol. Phys. **15**, 119 (1984); J. Phys. (Paris), Colloq. **46**, 223 (1985).
[6]B. A. Zon, Zh. Eksp. Teor. Fiz. **73**, 128 (1977) [Sov. Phys.—JETP **46**, 65 (1977)].
[7]K. Unnikrishnan and M. A. Prasad, Phys. Rev. A **29**, 3423 (1984).
[8]F. W. Byron Jr. and C. J. Joachain, J. Phys. B **17**, L295 (1984).
[9]C. J. Joachain, in *Fundamentals of Laser Interactions,* edited by F. Ehlotzky (Springer, Berlin, 1985), p. 37.
[10]S. Geltman, Astrophysical J. **141**, 376 (1965).
[11]S. Geltman, J. Quant. Spectrosc. Radiat. Transfer **13**, 601 (1973).
[12]M. J. Conneely and S. Geltman, J. Phys. B **14**, 4847 (1981).
[13]V. M. Buimistrov and L. I. Trakhtenberg, Zh. Eksp. Teor. Fiz. **69**, 108 (1975) [Sov. Phys.—JETP **42**, 54 (1976)]; see also, V. M. Buimistrov, Phys. Lett. **30A**, 136 (1969).
[14]N. K. Rahman and F. H. M. Faisal, J. Phys. B **9**, L275 (1976); Phys. Lett. **57A**, 426 (1976); J. Phys. B **11**, 2003 (1978).
[15]A. Lami and N. K. Rahman, J. Phys. B **16**, L201 (1983).
[16]L. Hostler, J. Math. Phys. **5**, 591 (1964).
[17]S. Klarsfeld, Lett. Nuovo Cimento **1**, 682 (1969).
[18]M. Gavrila and A. Costescu, Phys. Rev. A **2**, 1752 (1970).
[19]E. J. Kelsey and J. Macek, J. Math. Phys. **17**, 1182 (1976), and references therein.
[20]P. Appell and M. J. Kampé de Feriet, *Fonctions Hypergéométriques et Hypersphériques. Polynômes d'Hermite* (Gauthier-Villars, Paris, 1926).
[21]A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Bateman Manuscript Project: Higher Transcendental Functions* (McGraw-Hill, New York, 1953), Vol. 1, Chap. V.
[22]S. Jetzke, J. Broad, and A. Maquet (unpublished).
[23]M. Gavrila, Phys. Rev. **163**, 147 (1967).
[24]L. P. Rapoport and B. A. Zon, Phys. Lett. **26A**, 564 (1968).
[25]A. Maquet, Phys. Rev. A **15**, 1088 (1977).
[26]M. Ya. Amusia, A. S. Baltenkov, and A. A. Paiziev, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 366 (1976) [JETP Lett. **24**, 332 (1976)].
[27]V. P. Zhdanov, Zh. Eksp. Teor. Fiz. **73**, 112 (1977) [Sov. Phys.—JETP **46**, 57 (1977)].
[28]B. A. Zon, Zh. Eksp. Teor. Fiz. **77**, 44 (1979) [Sov. Phys.—JETP **50**, 21 (1979)]; P. A. Golovinskii and B. A. Zon, Zh. Tekh. Fiz. **50**, 1847 (1980) [Sov. Phys. Tech. Phys. **25**, 1076 (1980)].
[29]M. Ya. Amusia, Comments At. Mol. Phys. **11**, 123 (1982); see also M. Ya. Amusia, N. B. Avdonina, L. V. Chernysheva, and M. Yu. Kuchiev, J. Phys. B **18**, L791 (1985).
[30]F. E. Low, Phys. Rev. **110**, 974 (1958).