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Polarization effects in elastic photon-atom scattering

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We have calculated two linear polarization effects in the elastic scattering of x rays and γ rays (in the energy range a few keV to little above 1 MeV) from targets with Z ranging from 13 to 92, using Rayleigh scattering amplitudes obtained numerically with the procedures of Kissel *et al.* We consider both polarization of an unpolarized beam in scattering and asymmetries in the scattering of an initially polarized beam. Contrary to the simple form-factor predictions these polarization effects depend on atomic number (Z) and photon energy (E). Results obtained using only K-shell Rayleigh amplitudes are not too different from those using total-atom amplitudes. In some cases polarization properties are more sensitive to Delbrück amplitudes than the differential cross sections. Theoretical predictions are compared with the limited experimental data for both types of measurements.

I. INTRODUCTION

With the availability of more exact Rayleigh scattering amplitudes,[1,2] it has been possible to investigate in greater detail the various aspects of elastic scattering of γ rays.[3–5] In this paper we study polarization effects associated with elastic scattering of x rays and γ rays from unpolarized targets, using these more exact Rayleigh amplitudes. We note a revival of experimental interest[6,7] in studying these polarization properties and we anticipate further use of synchrotron radiation in polarization studies. We also note interesting recent experimental work on magnetic scattering,[8,9] which goes beyond our present formalism. Studies of the polarization effects in elastic photon-atom scattering require improvement, both from the point of view of theory and of experiment. Polarization experiments have so far been carried out only using low-resolution scintillation detectors. Use of high-resolution semiconductor detectors[7] is expected to produce more accurate data and consequently more reliable information on the physical processes involved. On the theoretical side, the earlier predictions are based either on form-factor (FF) approximation or numerically calculated K-shell (not full-atom) amplitudes obtained by Brown and his group[10] for the single element mercury at four energies. Form-factor approximation often fails to give good predictions for the differential cross section,[1,2,4] and we shall see that the same is true for polarization properties. The FF prediction is very simple: polarization properties are independent both of the atomic number of the target element and of the energy of the photon. Brown's amplitudes, on the other hand, can only be justified when scattering from K electrons dominate, they are inaccurate in some cases,[1] and the data is too limited to permit interpolation in Z.

In view of these considerations, we present a study of the dependence of polarization properties of scattering amplitudes on scattering angles, atomic number of the target, and energy of the photon. We compute polarizations and polarization asymmetries using "exact" amplitudes obtained numerically through the multipole expansion of the second-order S-matrix element. We present predictions for photon energies in the range from 145 to 1332 keV and for a range of target elements from aluminum ($Z=13$) to uranium ($Z=92$). We describe in Sec. II the observables of elastic scattering, the scattering amplitudes, and their properties. There are two primary experimental procedures to measure polarization properties: (1) measuring the polarization of scattered photons from an unpolarized beam, (2) measuring the asymmetry in scattering by a polarized beam. We present the principles of these measurements in Sec. III. Comparisons of our predictions with available experimental results are presented in Sec. IV. We also discuss the validity of simpler form-factor predictions and the sensitivity of results to various component contributions. Some conclusions are given in Sec. V.

II. OBSERVABLES, SCATTERING AMPLITUDES, AND THEIR PROPERTIES

Elastic scattering of photons by an atom is a process in which the internal energy of the atom does not change and the incident and scattered photons have the same energy. The atom is a composite system and all its constituents (nucleons and bound electrons) contribute to the scattering of photon. To a good approximation the amplitudes for scattering from these constituents may be taken as additive, summed to form the total scattering amplitude. The bound-electron contribution is referred to as Rayleigh scattering. The contributions from the nucleus consist of nuclear Thomson scattering (classical Thomson scattering considering the nucleus as a charge distribution or, for the energies of this work, as a point charge) and nuclear resonance scattering, virtually exciting the nuclear structure. In addition there is Delbrück scattering off the vacuum fluctuations in the atomic field. A further physical assumption in this calculation is that we consider the scattering process as an isolated photon-atom interaction. We also assume no polarization properties of the target are observed.

With these assumptions, the observables of elastic scattering are simply the momentum and polarization $(\mathbf{k}_i,\epsilon_i)$ and $(\mathbf{k}_f,\epsilon_f)$, of incident and scattered photons, together with the nuclear charge Z of the atom from which scattering occurs.

For elastic scattering, $|\mathbf{k}_i|=|\mathbf{k}_f|=\omega/c$ and the general form of any of the terms contributing to the total scattering amplitude is

$$A=M(\epsilon_i\cdot\epsilon_f^*)+N(\epsilon_i\cdot\hat{\mathbf{n}}_f)(\epsilon_f^*\cdot\hat{\mathbf{n}}_i)\ . \tag{1}$$

Here $\hat{\mathbf{n}}_i$ and $\hat{\mathbf{n}}_f$ are unit vectors along $\mathbf{k}_i$ and $\mathbf{k}_f$ and the amplitudes M and N depend on the energy of the photon ω and the scattering angle θ. If A represents the total scattering amplitude summed over all contributions, then the differential elastic scattering cross section is

$$\frac{d\sigma}{d\Omega}=|A|^2\ . \tag{2}$$

The complex polarization vectors ϵ satisfy the conditions

$$\epsilon^*\cdot\epsilon=1,\quad \epsilon\cdot\mathbf{k}=0\ . \tag{3}$$

Alternatively, polarization of a photon can be described in terms of the real Stokes parameters[11]

$$\begin{aligned}&\xi_1=\epsilon_x^*\epsilon_x-\epsilon_y^*\epsilon_y,\quad \xi_2=\epsilon_x\epsilon_y^*+\epsilon_x^*\epsilon_y\ ,\\&\xi_3=i(\epsilon_x\epsilon_y^*-\epsilon_x^*\epsilon_y)\ ,\end{aligned} \tag{4}$$

where $\epsilon=\epsilon_x\hat{\mathbf{x}}+\epsilon_y\hat{\mathbf{y}}$ with $|\epsilon_x|^2+|\epsilon_y|^2=1$ and $(\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{k}})$ a complete orthonormal set of unit vectors. Note that for a given photon

$$\xi_1^2+\xi_2^2+\xi_3^2=1\ , \tag{5}$$

so that $|\xi_n|\le 1$ and only two are independent. The ξ_n do not specify the absolute phase of ϵ, which is not an observable of these processes. The Stokes parameters $\bar{\xi}_n$ characterizing a beam of photons of given momentum may be defined as an average over Stokes parameters of individual photons. In this case

$$(\bar{\xi}_1^2+\bar{\xi}_2^2+\bar{\xi}_3^2)^{1/2}\le 1 \tag{6}$$

and measures the degree of polarization of the beam.

In addition to the pair of invariant amplitudes (M,N), two other choices are often made: linear polarization amplitudes $(A_{\parallel},A_{\perp})$ and circular polarization amplitudes $(A_{\rm NSF},A_{\rm SF})$ corresponding to no spin-flip or spin-flip. Here we need only consider $(A_{\parallel},A_{\perp})$, which are obtained by resolving the photon polarization vectors into components parallel and perpendicular to the scattering plane,

$$\epsilon=\epsilon^{\parallel}\hat{\epsilon}^{\parallel}+\epsilon^{\perp}\hat{\epsilon}^{\perp}\ , \tag{7}$$

where $\hat{\epsilon}^{\parallel}$ and $\hat{\epsilon}^{\perp}$ are unit vectors parallel and perpendicular to the plane, both perpendicular to the photon propagation direction $\hat{\mathbf{k}}$. One has

$$\hat{\epsilon}_i^{\perp}\cdot\hat{\epsilon}_f^{\perp}=1,\quad \hat{\epsilon}_i^{\parallel}\cdot\hat{\epsilon}_f^{\parallel}=\cos\theta,\quad \hat{\epsilon}_i^{\perp}\cdot\hat{\epsilon}_f^{\parallel}=\hat{\epsilon}_i^{\parallel}\cdot\hat{\epsilon}_f^{\perp}=0\ . \tag{8}$$

Then, one may write

$$A=\hat{\epsilon}_i^{\parallel}\cdot\hat{\epsilon}_f^{\parallel *}A_{\parallel}+\hat{\epsilon}_i^{\perp}\cdot\hat{\epsilon}_f^{\perp *}A_{\perp}\ , \tag{9}$$

where

$$A_{\parallel}=M\cos\theta-N\sin^2\theta,\quad A_{\perp}=M\ . \tag{10}$$

Note that the form (9) indicates that if photon polarization is initially parallel or perpendicular to the scattering plane, it continues to have that property after scattering. On squaring, the general cross section may be written in terms of the Stokes parameters, taking the directions $\hat{\mathbf{x}}=\hat{\epsilon}^{\parallel}$, $\hat{\mathbf{y}}=\hat{\epsilon}^{\perp}$, $\hat{\mathbf{z}}=\hat{\mathbf{k}}$ (so that y perpendicular to the scattering plane is common to both coordinate systems):

$$\begin{aligned}\frac{d\sigma}{d\Omega}=&\tfrac{1}{4}(|A_{\parallel}|^2+|A_{\perp}|^2)+\tfrac{1}{4}\xi_{1i}\xi_{1f}(|A_{\parallel}|^2+|A_{\perp}|^2)\\&+\tfrac{1}{4}(|A_{\parallel}|^2-|A_{\perp}|^2)(\xi_{1i}+\xi_{1f})\\&+\tfrac{1}{4}(A_{\parallel}A_{\perp}^*+A_{\parallel}^*A_{\perp})(\xi_{2i}\xi_{2f}+\xi_{3i}\xi_{3f})\\&+\tfrac{1}{4}i(A_{\parallel}A_{\perp}^*-A_{\parallel}^*A_{\perp})(\xi_{3i}\xi_{2f}+\xi_{2i}\xi_{3f})\ .\end{aligned} \tag{11}$$

Note only three of these four combinations of amplitudes are independent, corresponding to not observing the overall phase of the scattering amplitude A. Explicitly,

$$\begin{aligned}(|A_{\parallel}|^2+|A_{\perp}|^2)^2=&(|A_{\parallel}|^2-|A_{\perp}|^2)^2\\&+(A_{\parallel}A_{\perp}^*+A_{\parallel}^*A_{\perp})^2\\&+[i(A_{\parallel}A_{\perp}^*-A_{\parallel}^*A_{\perp})]^2\ .\end{aligned} \tag{12}$$

The unpolarized scattering cross section, corresponding to an average over incident-photon polarization and a sum over scattered-photon polarization, is immediately obtained from Eq. (11) as

$$\frac{d\sigma^{\rm unpol}}{d\Omega}=\tfrac{1}{2}(|A_{\parallel}|^2+|A_{\perp}|^2)\ . \tag{13}$$

Three other combinations of these amplitudes may be studied if polarization properties are observed, but only one unless both initial- and final-state polarization properties are observed. If Eq. (11) is averaged over incident-photon polarizations (unpolarized beam, characterized by all Stokes parameters for the beam being zero) the resulting cross section for a specified outgoing photon state is

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{4}(|A_{\parallel}|^2+|A_{\perp}|^2)+\tfrac{1}{4}\xi_{1f}(|A_{\parallel}|^2-|A_{\perp}|^2)\ . \tag{14}$$

Thus the outgoing beam is partially linearly polarized in an amount characterized by P (degree of polarization), with

$$-P\equiv\bar{\xi}_{1f}=\frac{|A_{\parallel}|^2-|A_{\perp}|^2}{|A_{\parallel}|^2+|A_{\perp}|^2}\ ,$$
$$\bar{\xi}_{2f}=\bar{\xi}_{3f}=0\ . \tag{15}$$

Alternatively, for an incident photon beam characterized by a certain known degree of linear polarization $\bar{\xi}_{1i}$, the intensity of the outgoing beam (polarization of the outgoing beam not measured) will be given by

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{4}(|A_{\parallel}|^2+|A_{\perp}|^2)+\tfrac{1}{4}\bar{\xi}_{1i}(|A_{\parallel}|^2-|A_{\perp}|^2)\ ,$$
$$\bar{\xi}_{2i}=\bar{\xi}_{3i}=0\ . \tag{16}$$

Since $\bar{\xi}_{1i}$ is defined in terms of the scattering plane, a 90° rotation of the scattering plane will change the sign of ξ_{1i} and so the observed intensity at the same scattering angle. Therefore we see that, considering polarization of only one of the incident or the outgoing photon, one can get information only about the one combination of amplitudes $|A_{\parallel}|^2-|A_{\perp}|^2$. (Both types of experiments, namely, starting with an incident unpolarized photon beam and measuring the linear polarization of the outgoing photon, or starting with a linearly polarized photon beam and measuring the intensity of the outgoing photon, have been performed.)

To get information on the other combinations of amplitudes, experiments observing polarization of both the incident photon beam and the scattered photon beam are necessary. All such experiments are difficult and none have yet been performed. As an example, the cross section for a circularly polarized photon beam with degree of circular polarization $\bar{\xi}_{3i}$ will be

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{4}(|A_{\parallel}|^2+|A_{\perp}|^2)+\tfrac{1}{4}\bar{\xi}_{3i}\xi_{3f}(A_{\parallel}A_{\perp}^*+A_{\parallel}^*A_{\perp}) \tag{17}$$

and so $A_{\parallel}A_{\perp}^*+A_{\parallel}^*A_{\perp}$ can be measured if final circular polarization ξ_{3f} is observed. The same combination could be measured considering linear polarization at 45° in the incident and scattered beam:

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{4}(|A_{\parallel}|^2+|A_{\perp}|^2)+\tfrac{1}{4}\bar{\xi}_{2i}\xi_{2f}(A_{\parallel}A_{\perp}^*+A_{\parallel}^*A_{\perp})\ . \tag{18}$$

The third combination would require, for example, measuring circular polarization in one beam and a suitable linear polarization in the other.

Of the different coherent contributions constituting elastic scattering (Rayleigh, nuclear Thomson, Delbrück, and nuclear resonance), in the energy range of our interest the contribution of nuclear resonance scattering is very small and can be neglected. Delbrück scattering begins to be important from photon energies below and around 1 MeV, particularly at intermediate angles. We include the Delbrück amplitude as calculated in Born approximation by Kahane[12] and Papatzacos and Mork.[13] To obtain nuclear Thomson (NT) amplitudes at these energies we may take the nucleus as a point charge and write

$$A_{\mathrm{NT}}^{\perp}=\frac{-Z^2m}{M}r_0,\quad A_{\mathrm{NT}}^{\parallel}=A_{\mathrm{NT}}^{\perp}\cos\theta\ . \tag{19}$$

Here m/M is the ratio of the electron rest mass to the nuclear mass and θ is the angle of scattering.

Rayleigh scattering amplitudes are obtained numerically in partial waves for inner-shell electrons;[1,2] the contributions from outer-shell electrons are obtained using the prescription in Ref. 1 subsequently refined in Ref. 2. Total $A_{\parallel}$ and $A_{\perp}$ amplitudes are calculated by adding the $A_{\parallel}$ and $A_{\perp}$ amplitudes of all the coherent contribution terms. The degree of polarization P is then calculated using Eq. (15) and the asymmetry in scattering obtained from Eq. (16).

Simple approximate predictions for Rayleigh scattering amplitudes are obtained in form-factor approximation. This approach, adequate in certain circumstances, is also popular due to the relative ease of calculation of FF's and due to their availability in tabular form. FF theories are basically high-energy-limit small-momentum-transfer theories which also neglect small terms of order $(Z\alpha)^2$. Form factors are functions of momentum transfer q and Z, but not of energy or angle separately. Form factors, the Fourier transform of the atomic charge distribution, are available for nonrelativistic[14] and relativistic[15] distributions. There is also a significantly better (though more complex) relativistic theory which includes binding-energy corrections for the electrons, known as the modified form-factor approximation.[16] In the form-factor approximation (nonrelativistic, relativistic, modified), Rayleigh scattering amplitudes are given by

$$A_{\perp}=-f(q),\quad A_{\parallel}=-f(q)\cos\theta\ . \tag{20}$$

Thus polarization P calculated from Eq. (15) using any form-factor theory is independent of target atom and photon energy, and is given by

$$P=\frac{1-\cos^2\theta}{1+\cos^2\theta}\ . \tag{21}$$

Similarly, from Eq. (16), the asymmetry in the scattering of a polarized beam will be independent of target atom and photon energy in form-factor approximation.

III. EXPERIMENTS

So far experiments have been performed examining some of the linear photon polarization properties of elastic scattering. Two types of experiments have been done, as illustrated in Fig. 1. In either case the measurable quantity is the intensity of a scattered beam. In one group of experiments (we call these type A), unpolarized photon sources have been used to provide the incident beam for the scattering experiment. The incident beam is elastically scattered from a chosen target, and the linear polarization P [equivalent to $\bar{\xi}_{1f}$ of Eq. (15)] of the scattered pho-

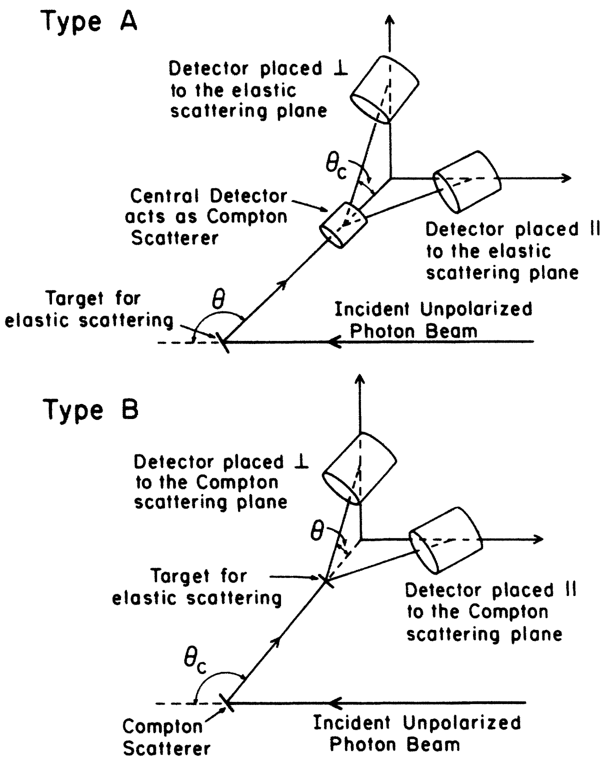

FIG. 1. Schematic diagram of two types of polarization experiments performed in the energy region of our interest.

ton beam is determined, using a polarimeter based on Compton scattering.[17–23]

In another group of experiments (we call these type B), an incident unpolarized photon beam is first Compton scattered to produce a partially polarized photon beam. The degree of polarization of the beam produced due to Compton scattering ($\bar{\xi}_c$) is given as[11]

$$\bar{\xi}_c = \frac{-\sin^2\theta_c}{1+\cos^2\theta_c+(k_0-k)(1-\cos\theta_c)} \tag{22}$$

in terms of the Compton scattering angle θ_c and the incident and scattered photon energy k_0 and k (in units of mc^2). Note this is always negative, corresponding to partial polarization perpendicular to the plane of Compton scattering. This polarized photon beam is then elastically scattered from a chosen target, and the intensity [Eq. (16)] of the elastically scattered photon beam is measured.[24–26] In practice one measures the intensity separately in two planes—parallel and perpendicular to the Compton scattered plane—corresponding to taking $\bar{\xi}_{1i}=\pm\bar{\xi}_c$, i.e., polarization perpendicular or parallel to the elastic scattering plane. Experimental results are often expressed in terms of the asymmetry ratio R of these two intensities as

$$R = \frac{(|A_\parallel|^2+|A_\perp|^2)+\bar{\xi}_c(|A_\parallel|^2-|A_\perp|^2)}{(|A_\parallel|^2+|A_\perp|^2)-\bar{\xi}_c(|A_\parallel|^2-|A_\perp|^2)} = \frac{1-\bar{\xi}_c P}{1+\bar{\xi}_c P}\,, \tag{23}$$

in terms of the combination P of amplitudes defined in Eq. (15). Thus experiments of type B lead to a value for the same quantity P that is measured in type-A experiments.

In type-A experiments, the polarimeter usually consists of three scintillation detectors: one central detector and two side detectors. The elastically scattered photon beam from a chosen target [partially linearly polarized perpendicular to the elastic scattering plane—since generally $|A_\perp| \geq |A_\parallel|$—with a negative $\bar{\xi}_{1f}$ as in Eq. (15)] is incident on the central detector, which acts as a Compton scatterer. The intensities of the Compton-scattered photon beam are measured by the two side detectors. The intensity of the photon beam after Compton scattering through an angle θ_c is (see Ref. 11)

$$\left[\frac{d\sigma}{d\Omega}\right]_c \simeq [1+\cos^2\theta_c+(k_0-k)(1-\cos\theta_c)]-\bar{\xi}_c(\sin^2\theta_c)\,, \tag{24}$$

where $\bar{\xi}_c$ is the polarization of the incident beam relative to the Compton scattering plane. In practice one measures the intensities separately by two detectors placed parallel and perpendicular to the elastic scattering plane, corresponding to taking $\bar{\xi}_c=\pm\bar{\xi}_f$, i.e., polarization generally perpendicular or parallel to the Compton scattering plane. Experimental results are often expressed in terms of the asymmetry ratio R of these two intensities,

$$R = \frac{[1+\cos^2\theta_c+(k_0-k)(1-\cos\theta_c)]-\bar{\xi}_{1f}(\sin^2\theta_c)}{[1+\cos^2\theta_c+(k_0-k)(1-\cos\theta_c)]+\bar{\xi}_{1f}(\sin^2\theta_c)}\,. \tag{25}$$

The Compton factors of Eq. (25) can all be expressed in terms of the quantity $\bar{\xi}_c$ as defined by Eq. (22), while $\bar{\xi}_{1f}\equiv P$ according to Eq. (15), so that

$$R = \frac{1+\bar{\xi}_{1f}\bar{\xi}_c}{1-\bar{\xi}_{1f}\bar{\xi}_c} = \frac{1-P\bar{\xi}_c}{1+P\bar{\xi}_c}\,. \tag{26}$$

Since $\bar{\xi}_c$ can be calculated, the measurement of R in type-A experiments indeed leads to a value for P.

Thus far all these experiments have been done using scintillation detectors, which possess low energy resolution compared to the high-resolution semiconductor detectors now available. It is to be noted that (accurate) Compton polarimeters based on Ge(Li) detectors have been developed in recent years.[27]

We should note that the dependence of photon scattering cross section on bound-electron spin has been studied,[28] using 129-keV circularly polarized γ rays from an ^{191}Os source, scattered through angles from 21° to 55° by a magnetized iron-cobalt alloy target. The formalism used in this work to compute Rayleigh scattering amplitudes would have to be extended before this experiment can be compared with our theory.

IV. RESULTS

Here we compare our results with other theoretical values and the values obtained from experiments. We also present information on the sensitivity of polarization to the contributions of outer-shell electrons and component coherent amplitudes.

We present, in Figs. 2—4, the variation of linear polarization with scattering angle for three different elements (aluminum, molybdenum, and uranium) at different photon energies. Also shown in the figures are the form-factor result, represented by a dashed curve (independent of energy and element). It is evident from these figures that polarizations do change with energy, in contrast to form-factor predictions, which are most successful at low energies. The change is appreciable beyond low-Z elements (thus also demonstrating Z dependence). The form factor predicts that maximum polarization (i.e., all photons with polarization vectors perpendicular to the plane of scattering) always occurs at a 90° angle of scattering. We observe that except for low-Z elements the maximum is not at 90° and shifts with energy. In form-factor approximation, a zero is predicted at 90° for scattering of photons of linear polarization parallel to the plane of scattering (i.e., $A_{\parallel}=0$). Our amplitudes strongly violate this prediction at all but the lowest energies. Instead of $A_{\parallel}$ going through zero at 90° as in the form-factor predictions, we find that the real part of $A_{\parallel}$ has a sign change usually well before 90° (as early as 60° in the case of lead, 50° in the case of molybdenum at 1332 keV) and the imaginary part of $A_{\parallel}$ has a sign change well below 90° (as early as 45°) for energies below the pair-production threshold and well after 90° (as late as 120° in the case of lead, 127.5° in the case of molybdenum at 1332 keV) for higher energies. We observe that form-factor-predicted values of polarization agree well with our values for low-atomic-number elements, at lower energies (which are, however, still well above threshold energies), and when the momentum transfer is not too large. We have observed essentially form-factor-like behavior for lead at 37 and at 8 keV.

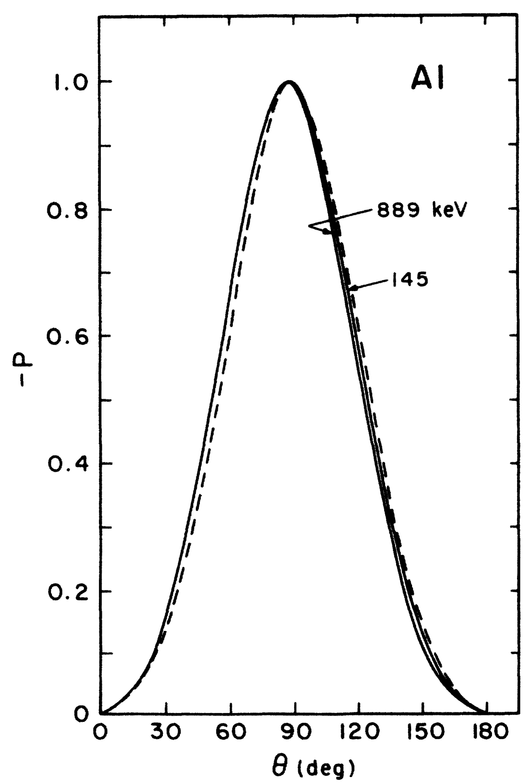

FIG. 2. Variation of polarization with scattering angle for photon energies 145 and 889 keV for aluminum ($Z=13$), according to our calculations (solid curves). The dashed curve shows the energy-independent FF values.

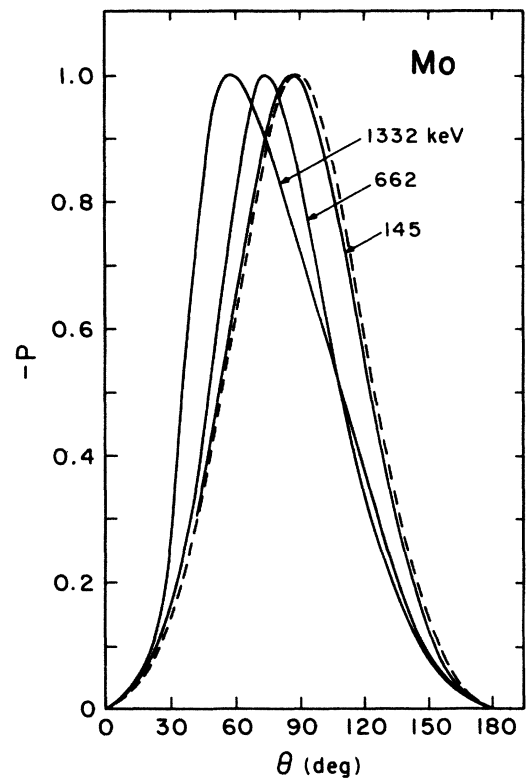

FIG. 3. Variation of polarization with scattering angle for molybdenum ($Z=42$) at different photon energies, according to our calculations. The dashed curve again shows the energy-independent FF values.

The variation of polarization with target atomic number Z for different photon energies at three different scattering angles is presented in Fig. 5. We observe that polarization is almost independent of Z at low photon momentum transfers (for 145 keV at all angles except back angles), which one expects from form-factor predictions.

We observe (Table I) that in these situations linear polarizations calculated using Rayleigh amplitudes for scattering off electrons of the innermost shell (K shell) or from a few inner shells (e.g., K and L shells) do not differ appreciably from those obtained from Rayleigh amplitudes for all the electrons of the atom. This happens even when the photon energy is not too much above the K threshold (for example, in the case of 145 keV for lead). Either polarization is form-factor-like and independent of shell (low energy) or it is K-shell dominated (high ener-

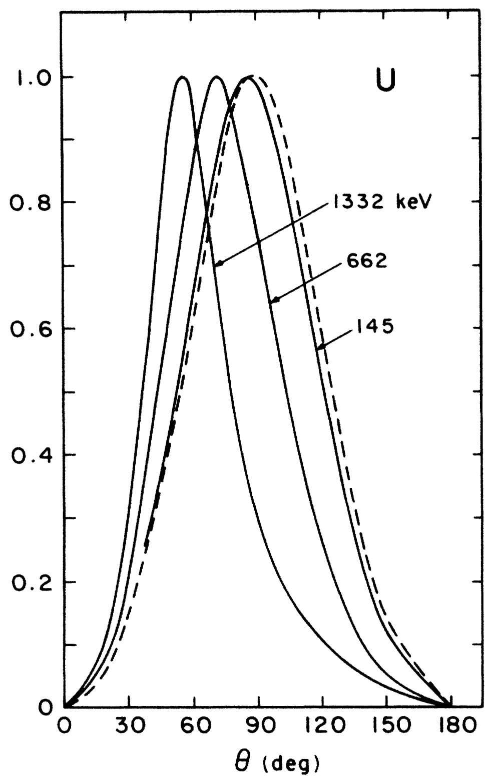

FIG. 4. Variation of polarization with scattering angle for uranium ($Z=92$) at different photon energies, according to our calculations. The dashed curve shows the energy-independent FF values.

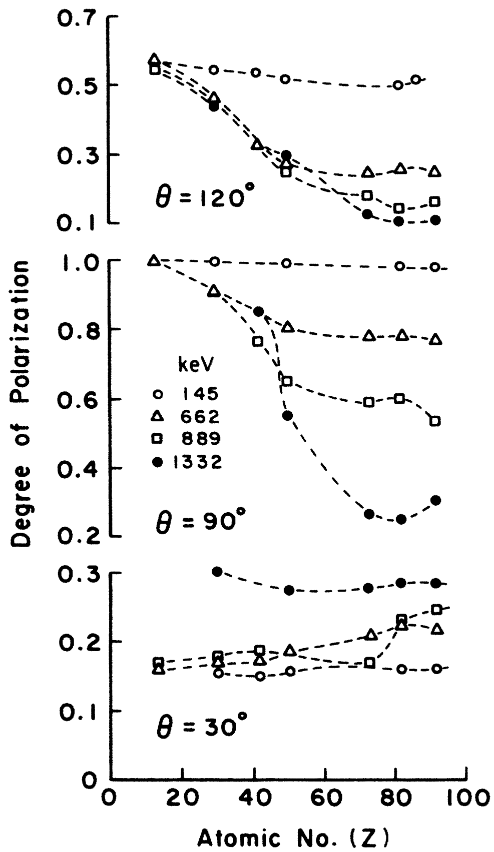

FIG. 5. Variation of polarization with atomic number (Z) for different photon energies at three scattering angles.

gies). This is quite different from what we observe for the scattering cross sections, since the amplitudes are never shell independent. In the past, using polarization predictions from Brown-Mayers K-shell Rayleigh amplitudes, it had been suggested that the difference from experiment could be either due to L-shell scattering or Delbrück scattering. We now see that outer-shell electrons were not responsible. Of course, if Delbrück amplitudes do matter, outer-shell Rayleigh amplitudes must be included in order to obtain the correct relative weight of Rayleigh and Delbrück amplitudes.

This leads us to examine more carefully the sensitivity of polarization properties to Delbrück amplitudes, presuming they are roughly of the order of the Born-approximation Mork-Papatzacos result. The ideal situation to observe the real part of the Delbrück amplitude should be in the region of photon energy and scattering angle, and for elements, for which the imaginary part of the Delbrück amplitude is negligible compared to its real part, and the real part of Delbrück amplitude is bigger or at least comparable to the sum of Rayleigh and nuclear

TABLE I. Polarizations calculated for three energies and various angles of scattering from lead, using whole-atom Rayleigh amplitudes together with nuclear Thomson amplitudes (designated "whole atom"), compared with polarizations calculated using only K-shell Rayleigh amplitudes.

θ	145 keV		662 keV		1332 keV	
(deg)	K shell	Whole atom	K shell	Whole atom	K shell	Whole atom
0	0	0	0	0	0	0
10	0.0164	0.0140	0.0199	0.0177	0.0212	0.0245
30	0.153	0.153	0.195	0.211	0.239	0.241
60	0.637	0.6549	0.835	0.851	0.993	0.991
90	0.984	0.984	0.785	0.774	0.571	0.546
120	0.541	0.512	0.256	0.247	0.246	0.225
150	0.125	0.115	0.0481	0.048	0.0631	0.0584
180	0	0	0	0	0	0

Thomson amplitudes. Within the energy range of interest of this paper, scattering at large angles ($\gtrsim 90°$) from high-Z targets at energies above and around 1 MeV is appropriate to observe the real part of the Delbrück amplitude. One commonly used approach is to measure the total elastic scattering cross section and compare it with the theoretical cross sections with and without including a given individual amplitude contribution (Rayleigh, nuclear Thomson, or Delbrück). In a similar way we may study the sensitivity of polarization properties in elastic scattering of γ rays to the Delbrück amplitude. To compare the sensitivity of cross section *vis à vis* polarization to Delbrück scattering we present in Table II polarization and cross-section values for lead at large scattering angles for three different photon energies. The subscript R + NT means only Rayleigh and nuclear Thomson amplitudes are considered in computing the corresponding quantities, whereas tot means Delbrück amplitudes are also included with Rayleigh and nuclear Thomson amplitudes. We see from Table II that the polarization is appreciably more sensitive to Delbrück amplitudes than cross sections. This is because in this range of photon energy the parallel component of the Delbrück amplitude $A_D^{\parallel}$ is becoming more dominant compared to $A_D^{\perp}$ with increasing angles and thereby P [Eq. (15)] changes greatly without making a large change in $|A_D^{\parallel}|^2 + |A_D^{\perp}|^2$. Of course it is also true that a cross-section measurement is easier than a polarization measurement. For instance, in an optimum case (Pb at 1332 keV at 120°) one would need to compare the merits of distinguishing 10% polarization from 20% polarization versus a 25% difference in cross section.

We next compare our results with the polarization values obtained from experiments (Table III). As we said before, in one type of experiment the polarization of the outgoing photon beam after elastic scattering at a chosen scattering angle is measured (type A). With this type of experiment, measurements of polarization have been primarily performed for a single element, lead, for γ rays obtained from radioactive ^{60}Co (γ-ray energies 1.332 and 1.177 MeV, averaged as 1.25 MeV) in the range of scattering angles 40°–105° (Refs. 17, 19, 22, and 23) (also for mercury at 70° angle of scattering[21]); for 662-keV photons for 64°, 90°, and 120°;[17–20] for 412-keV photons at a 90° angle of scattering;[17] and for 280-keV photons at 93.5° and 120° angles of scattering.[19]

In the other type of experiment, in which a partially polarized photon beam is incident on a chosen target and the intensities of the beam after elastic scattering are measured (type B), the asymmetry ratio R [Eq. (23)] was measured, for lead for photon energies corresponding to $0.56mc^2$ and $0.78mc^2$ (where the rest energy of the electron mc^2 is 511 keV) in the range of scattering angles 45° to 105°,[24] and for mercury at scattering angles 65°, 90°, and 110° for energies $1.28mc^2$ (Ref. 26) and at 90° for $0.64mc^2$.[25]

We present in Table III the experimental values of polarization together with the theoretical values obtained from the present calculation. As two different types of experiments are involved, we present the results in Tables III(a) and III(b). (a) includes values obtained from experi-

TABLE II. Effect of the Delbrück contribution on the elastic scattering cross sections ($d\sigma/d\Omega$) and the polarization in elastic scattering (P), for Pb at three different energies. Subscripts R + NT indicate only Rayleigh and nuclear Thomson amplitudes are included in the calculation and tot represents total amplitudes including Rayleigh, nuclear Thomson, and Delbrück amplitudes. Numbers in square brackets denote powers of ten, i.e., $2.555[-3] = 2.555\times10^{-3}$.

θ	662 keV				889 keV				1332 keV			
(deg)	P_{tot}	P_{R+NT}	$\left(\frac{d\sigma}{d\Omega}\right)_{tot}$ (b/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{R+NT}$ (b/sr)	P_{tot}	P_{R+NT}	$\left(\frac{d\sigma}{d\Omega}\right)_{tot}$ (b/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{R+NT}$ (b/sr)	P_{tot}	P_{R+NT}	$\left(\frac{d\sigma}{d\Omega}\right)_{tot}$ (b/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{R+NT}$ (b/sr)
90	0.7782	0.7740	2.555[−3]	2.547[−3]	0.8071	0.6387	6.202[−4]	6.412[−4]	0.2555	0.2614	1.047[−4]	1.055[−4]
120	0.2616	0.2477	1.8[−3]	1.805[−3]	0.1531	0.1782	4.63[−4]	4.699[−4]	0.1103	0.2247	9.957[−5]	7.515[−5]
150	0.0510	0.0475	1.587[−3]	1.598[−3]	0.0298	0.0359	4.219[−4]	4.254[−4]	0.0282	0.0584	9.968[−5]	7.794[−5]
165	0.0103	0.00944	1.544[−3]	1.556[−3]	0.0086	0.0099	4.149[−4]	4.176[−4]	0.0063	0.0155	1.039[−4]	7.956[−5]

TABLE III. Comparison of experimental polarization (P) and asymmetry ratio (R) values with the theoretical values. (a) presents experiments where polarization of elastically scattered photons has been measured (type A in the text) and (b) presents experiments where elastic scattering cross sections of polarized photons have been measured. R is shown when reported experimentally, and in these cases we have calculated the corresponding experimental P.

Energy (keV)	θ (deg)	Element	Asymmetry ratio R Expt.	Asymmetry ratio R Theor.	Polarization P (%) Expt.	Polarization P (%) Theor.
			(a)			
280	93.5	Lead			96 ±6 (Ref. 19)	90
	120	Lead			42.5±5 (Ref. 19)	42.9
412	90	Lead			74 ±10 (Ref. 17)	89.6
662	45	Lead	1.41±0.04 (Ref. 20)	1.80	29.4±0.5	50.4
	60	Lead	2.12±0.04 (Ref. 20)	2.84	62.1±1.2	85.1
	64	Lead			96 ±8 (Ref. 18)	93.7
					94 ±9 (Ref. 19)	
	75	Lead	2.75±0.05 (Ref. 20)	3.66	80.8±1.4	98.9
	90	Lead	1.96±0.04 (Ref. 20)	2.63	56.2±1.1	
					78 ±8 (Ref. 18)	77.4
					90 ±14 (Ref. 19)	
					83 ±3 (Ref. 17)	
	105	Lead	1.31±0.12 (Ref. 20)	1.78	23.2±2.1	46.9
	120	Lead			28.4±6.5 (Ref. 18)	24.77
					33 ±7 (Ref. 19)	
1250[a]	53	Mercury	2.3 ±0.1 (Ref. 21)	2.44	94.1±4.1	99.7
	40	Lead			52.4±12.6 (Ref. 23)	58.7
	45	Lead			32.4±8.5 (Ref. 22)	77.2
	50	Lead			78.1±13.0 (Ref. 23)	93.1
	60	Lead			87.9±9.1 (Ref. 23)	94.4
					100.1±5.8 (Ref. 22)	
	64	Lead			100 ±15 (Ref. 19)	
	70	Lead			56.2±7.0 (Ref. 23)	65.8
	80	Lead			25.7±12.0 (Ref. 23)	40.2
	90	Lead			27 ±4.6 (Ref. 22)	25.5
					15.7±6.4 (Ref. 19)	
					6 ±2.5 (Ref. 17)	
	105	Lead			1.36±6.3 (Ref. 22)	15.5
			(b)			
286[b]	45		1.50 ±0.02 (Ref. 24)	1.60	34.5±0.5	39.8
($0.56mc^2$)	60	Lead	2.17 ±0.04 (Ref. 24)	2.45	63.6±1.2	72.5
	75		2.99 ±0.07 (Ref. 24)	3.53	85.9±2.0	97.9
	90		3.36 ±0.11 (Ref. 24)	3.33	93.3±3.0	92.8
	105		2.15 ±0.21 (Ref. 24)	2.33	62.9±6.1	69.1
327[c]	90	Mercury	1.711±0.064 (Ref. 25)	1.815	85.4±3.2	94.3
($0.64mc^2$)						
398[d]	45	Lead	1.41 ±0.02 (Ref. 24)	1.58	34.0±0.5	45.3
($0.78mc^2$)	60		1.91 ±0.03 (Ref. 24)	2.28	62.5±1.0	78.3
	75		2.48 ±0.06 (Ref. 24)	2.92	85.0±2.0	98.1
	90		2.63 ±0.09 (Ref. 24)	2.62	89.8±3.0	89.6
	105		1.76 ±0.18 (Ref. 24)	1.92	55.1±5.6	63.2
654[e]	65	Mercury	1.868±0.082 (Ref. 26)	1.865	94.0±4.1	93.7
($1.28mc^2$)	90		1.423±0.104 (Ref. 26)	1.664	54.2±3.9	77.4
	110		1.295±0.205 (Ref. 26)	1.246	39.9±6.3	34.0

[a]Calculations are for a photon energy of 1332 keV.
[b]Calculations are for 279 keV.
[c]Calculations are for 320 keV.
[d]Calculations are for 411.8 keV.
[e]Calculations are for 661.6 keV.

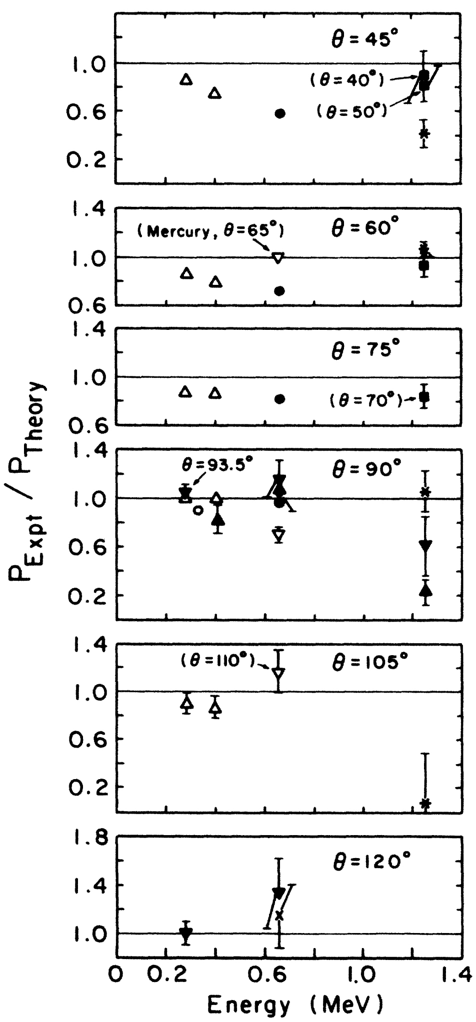

FIG. 6. Comparison of experimental polarization values with theoretical values, shown as the ratio of experimental to theoretical values ($P_{\rm expt}/P_{\rm theory}$) for lead at different photon energies and different angles of scattering. Open symbols represent experiments of type B (see text for details). ▲, Ref. 17; ×, Ref. 18; ▼ Ref. 19; ●, Ref. 20; *, Ref. 22; ■, Ref. 23; △, Ref. 24; ○, Ref. 25; ▽, Ref. 26.

ments of type A and (b) includes values obtained from experiments of type B. The comparison between theory and experiment is also represented in Fig. 6, showing the ratio of experimental polarization to the theoretical values ($P_{\rm expt}/P_{\rm theory}$) for different photon energies and scattering angles.

As seen from Fig. 6, experimental values in general (except for the largest angle $\theta = 120°$ case) tend to be lower than the theoretical values. The agreement between theory and experiment is mixed; in some cases there is agreement within experimental uncertainty and in some cases there is not. Quoted experimental uncertainty in type-B experiments, particularly for relatively small angles of scattering, seems rather small, considering the types of detectors and experimental arrangements.

V. CONCLUSION

The present extent of experimental knowledge is inadequate to characterize the general photon-atom scattering matrix element. It will be necessary to specify polarization of both initial beam and scattered photons, including at least one measurement of circular polarization.

The linear polarization of elastically scattered γ rays varies with atomic number of the target and incident photon energy, contrary to form-factor predictions.

Polarization properties of Rayleigh amplitudes are not very sensitive to outer-electron scattering amplitudes in the regimes considered here. Inner-electron amplitudes alone are sufficient to produce reasonably accurate polarization values at these energies as long as Rayleigh scattering dominates the total amplitude.

For photon energies around and little above 1 MeV and at larger scattering angles (the conditions appropriate to obtain a significant real part for the Delbrück amplitude) measurement of polarization may be a more sensitive tool than the measurement of scattering cross sections.

The general trend of the data, that experimental values are lower than theory, as well as the scatter between experimental values obtained by different group of workers, suggests a need for precision measurements of polarizations properties using high-resolution Ge(Li) detectors.

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