Resonance effects in electron-atom scattering in a chaotic nonresonant laser field

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The presence of a chaotic laser field with a bandwidth is shown to lead to new resonancelike structures in the double-differential cross sections for electron scattering from a real atomic system, in contrast to potential scattering. On the basis of the exact results, a simpler procedure is suggested to estimate the magnitude of this effect, which explicitly invokes the laser line-shape function and demonstrates how the field correlations affect the projectile and the bound system in markedly different ways. Finally, the feasibility of experimentally detecting these new features is discussed.

Charged-particle scattering in the presence of intense, chaotic radiation fields has been receiving attention lately,[1-4] since high-power multimode lasers are expected to produce nearly chaotic fields. Explicit calculations so far have been confined to potential scattering,[1,2,4] while the general formalism of Becker *et al.*,[3] which is fully relativistic, assumes asymptotically free particles. The results of Refs. 2 and 4 show that the only effect of a nonzero bandwidth is to give a spread to the peaks in the double-differential cross sections ($d^2\sigma/d\Omega\,dE_f$) at final energies corresponding to the exchange of an integral number of photons, which are δ functions in the case of a coherent field. Here we report the results for electron scattering by a real atomic system, taken to be hydrogenic, which exhibit some novel features because of the existence of atomic energy levels. Apart from the expected peaks around the incident electron energy, these calculations indicate the existence of another series involving energy changes corresponding to the atomic transition energies, but with sidebands corresponding to multiphoton exchanges at the mean frequency of the laser. These features can be explained by recognizing that the projectile electron and the target atom respond differently to the field correlations. This is confirmed by a heuristic derivation of the contribution of these new peaks to the differential cross section $d\sigma/d\Omega$, which also stresses the role of the laser line shape in this context.

Consider the scattering of electrons by hydrogen atoms in a plane-polarized electromagnetic field, whose amplitude and phase undergo Gaussian fluctuations. In the Coulomb gauge and dipole approximation, the incident electron of average momentum $\mathbf{k}$, is represented by

$$\chi_{\mathbf{k}}(\mathbf{r},t)=\exp\left[i\mathbf{k}\cdot\mathbf{r}-i\int_{-\infty}^{t}\tfrac{1}{2}[\mathbf{k}-e\mathbf{A}(\tau)/c]^2d\tau\right], \qquad (1)$$

where $\mathbf{A}$ is the vector potential (we use atomic units with $|e|=1$). For the ground state of the hydrogen atom, first-order perturbation theory gives

$$\psi_0(\mathbf{r},t)=\left[e^{-i\omega_0 t}\phi_0(\mathbf{r})+i\sum_k M_{k0}e^{-(\gamma_k+i\omega_k)t}\phi_k(\mathbf{r})\times\int_{-\infty}^{t}e^{(\gamma_k+i\omega_{k0})t'}E(t')dt'\right]e^{ie\mathbf{A}\cdot\mathbf{r}/c}, \qquad (2)$$

where $\mathbf{E}$ is the field strength, ϕ_k is an unperturbed atomic state of energy ω_k, γ_k the level width for the transition $|k\rangle\rightarrow|0\rangle$, $\omega_{kn}=\omega_k-\omega_n$, and

$$M_{k0}=\langle k\,|\,e\mathbf{r}\cdot\hat{\mathbf{E}}_0\,|\,0\rangle\,, \qquad (3)$$

with $\hat{\mathbf{E}}_0$ denoting the direction of polarization of the field. The first Born transition probability per unit of time for direct elastic scattering in which the initial and final momenta of the electron are, respectively, $\mathbf{k}_i$ and $\mathbf{k}_f$, is then given by

$$\langle W_{if}\rangle=\left\langle\lim_{T\rightarrow\infty}\left[\frac{1}{2T}\left|-i\int_{-T}^{T}dt\langle\chi_{\mathbf{k}_f}\psi_0\,|\,V\,|\,\psi_0\chi_{\mathbf{k}_i}\rangle\right|^2\right]\right\rangle, \qquad (4)$$

where the interaction potential is given by

$$V=\frac{1}{|\mathbf{r}_1-\mathbf{r}_2|}-\frac{1}{r_1}\,, \qquad (5)$$

$\mathbf{r}_1$ and $\mathbf{r}_2$ being the coordinates of the incident and atomic electrons, respectively. [The outer angular brackets in Eq. (4) denote ensemble averaging.] To proceed further, the field correlation function has to be specified, which we take to be[2]

$$\langle E(t)E(t')\rangle=\frac{\mathscr{E}_0^2}{2}\cos[\omega(t-t')]\exp(-\Delta\omega\,|\,t-t'\,|)\,, \qquad (6)$$

where $\mathscr{E}_0^2$ is the variance of the field strength, ω the mean frequency (assumed to be $\ll\omega_{k0}$ for all k), and $\Delta\omega$ the bandwidth. The final result of performing the ensemble averaging indicated in Eq. (4) using Eq. (6), which is too lengthy to be outlined here,[5] can be written as

$$\langle W_{if}\rangle=V_{00}^2H(0,E_{if})+V_{00}XQ+[S(\Delta\omega,\omega)-T]X^2\mathscr{E}_0^2+R\,, \qquad (7)$$

where

$$X = \mathrm{Im}\left[\sum_k \frac{V_{0k}M_{k0}}{\omega_{k0}}\right], \tag{7a}$$

$$V_{nk} = \int\int \psi_n^*(\mathbf{r}_2)e^{i\mathbf{q}\cdot\mathbf{r}_1}V(\mathbf{r}_1,\mathbf{r}_2)\psi_k(\mathbf{r}_2)d\mathbf{r}_1\,d\mathbf{r}_2, \tag{7b}$$

$$S(x,y) = H(x,E_{if}+y) + H(x,E_{if}-y), \tag{7c}$$

$$R = -\frac{\mathscr{E}_0^2\Delta\omega}{4}\sum_k \frac{V_{0k}^2 M_{k0}^2}{\gamma_k\omega_{k0}^2}S(\gamma_k,\omega_{k0}), \tag{7d}$$

$$H(x,y) = 2\exp\left[-\frac{\lambda_0^2}{2}\cos(2\phi)\right]\sum_{n=-\infty}^{\infty}\sum_{k=0}^{\infty}\frac{(\lambda_0^2/4)^{\nu+2k}}{k!(\nu+2k)!}\,\frac{\alpha(x)\cos(2n\phi)+\beta(y)\sin(2n\phi)}{\alpha^2(x)+\beta^2(y)}, \tag{7e}$$

$$\alpha(x) = (\nu+2k)\Delta\omega + (\lambda_0^2/2)(\Delta\omega\cos\phi + \omega\sin\phi) + x, \quad \nu = |n|,$$

$$\beta(y) = n\omega - y, \quad \lambda_0 = (\mathbf{q}\cdot\hat{\mathbf{E}}_0)\mathscr{E}_0/\omega^2, \quad \tan\phi = 2\,\Delta\omega/\omega, \quad \mathbf{q} = \mathbf{k}_i - \mathbf{k}_f,$$

and $E_{if} = E_i - E_f = (k_i^2 - k_f^2)/2$. The expressions for Q and T are not given here[5] since they are of no consequence in the following discussion. In Eq. (7) the first term on the right-hand side represents scattering by the static potential V_{00} and the other terms are due to the dressing of the atom by the field. As $\Delta\omega \to 0$, the term $R \to 0$, while all the others reduce to what one would obtain by averaging the corresponding result[6] for a coherent field of amplitude E_0, over a Gaussian probability distribution

$$P(E_0)dE_0 = e^{-E_0^2/\mathscr{E}_0^2}d(E_0/\mathscr{E}_0)^2, \tag{8}$$

which confirms a result already shown to hold in the cases considered in Refs. 2 and 3. In contrast, the term R is purely a product of the bandwidth, and contributes an extra term to the double-differential cross section:

$$\frac{d^2\sigma_r}{d\Omega\,dE_f} = \sum_k \left[\frac{d^2\sigma_r}{d\Omega\,dE_f}\right]_k, \tag{9}$$

where

$$\left[\frac{d^2\sigma_r}{d\Omega\,dE_f}\right]_k = \frac{k_f}{k_i}\,\frac{\mathscr{E}_0^2\Delta\omega}{32\pi^3}\,\frac{|V_{0k}M_{k0}|^2}{\omega_{k0}^2\gamma_k}S(\gamma_k,\omega_{k0}), \tag{10}$$

where we have used the fact that $V_{0k}M_{k0}$ is pure imaginary. Equation (10) constitutes the central result of this communication. This can be further simplified on identifying $2\gamma_k$ with the spontaneous transition probability per unit time from $|k\rangle$ to $|0\rangle$,[7] which gives

$$\left[\frac{d^2\sigma_r}{d\Omega\,dE_f}\right]_k = \frac{k_f}{k_i}\,\frac{3\mathscr{E}_0^2\Delta\omega\, c^3}{64\pi^3}\,\frac{|V_{0k}|^2}{\omega_{k0}^5}S(\gamma_k,\omega_{k0}). \tag{11}$$

Now, it can be verified from the definition of $H(x,y)$ that this function consists of peaks around $y = n\omega$, so that $(d^2\sigma_r/d\Omega\,dE_f)_k$ has two central peaks at $E_f = E_i \pm \omega_{k0}$, with sidebands at intervals of ω. For hydrogen, the dominant peaks are evidently around an energy difference corresponding to the $2p$ level, viz., $\frac{3}{8}$ a.u. Figure 1 illustrates the situation at a scattering angle of 10° for $E_i = 100$ eV and a field of strength 10^8 V/cm, $\mathbf{E}\|\mathbf{q}$, $\hbar\omega = 1.17$ eV and $\Delta\omega = 10^{-4}\omega$. The peaks at $E_f = E_i + \frac{3}{8}$ and $E_f = E_i + \frac{3}{8} - \omega$ are representative of the new genre and are seen to be of the same strength as the ones at $E_f = E_i$ and $E_f = E_i + \omega$.

To estimate the contribution $(d\sigma_r/d\Omega)_k$ of Eq. (11) to the differential cross section $d\sigma/d\Omega$, we note that

$$H(0,y) \to 2\pi \sum_{n=-\infty}^{\infty} I_n(\lambda_0^2/2)e^{-\lambda_0^2/2}\delta(y - n\omega) \quad \text{as } \Delta\omega \to 0, \tag{12}$$

where I_n is a Bessel function of imaginary argument. Therefore, assuming $\Delta\omega$ and γ_k to be quite small, we have from Eqs. (10) and (12),

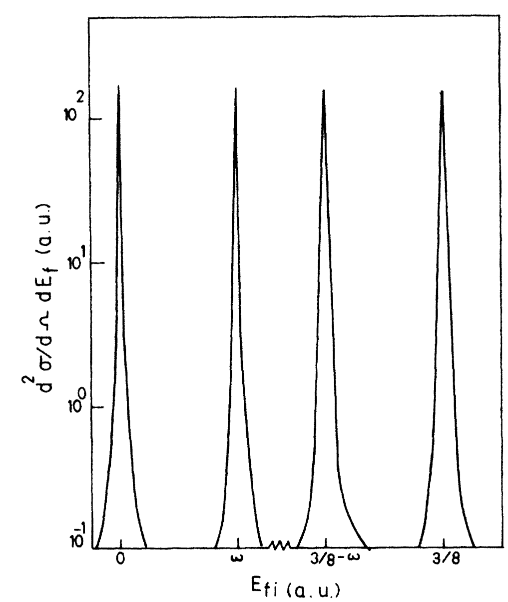

FIG. 1. Double-differential cross section for the scattering of electrons from hydrogen at an angle of 10° in a laser field polarized parallel to the change in momentum. E_{fi} is the energy gained by the projectile. The other parameters are $\hbar\omega = 1.17$ eV, $\Delta\omega = 10^{-4}\omega$, $\mathscr{E}_0 = 10^8$ V/cm, and $E_i = 100$ eV.

$$\left[\frac{d\sigma_r}{d\Omega}\right]_k = \int \left[\frac{d^2\sigma_r}{d\Omega\, dE_f}\right]_k dE_f = \frac{\mathscr{E}_0^2\Delta\omega}{16\pi^2}\sum_n \frac{k_f}{k_i}\frac{|V_{0k}M_{k0}|^2}{\gamma_k\omega_{k0}^2}e^{-\lambda_0^2/2}I_n(\lambda_0^2/2), \tag{13}$$

where $k_f^2=2(E_i\pm\omega_{k0}-n\omega)$.

We may now proceed to relate these resonancelike structures to the laser line shape, which, in the present case, is Lorentzian [cf. Eq. (6)]. (The implications of a steeper wing are discussed later.) The important point to note in this context is that, if the whole system were interacting with the photons from the wings, one should expect multiphoton exchanges at these frequencies, which are absent in Eq. (13), where the sidebands always occur with a spacing of ω. The obvious conclusion is that only the atom is affected by the presence of these photons, because of the strong resonances. Let us therefore consider a single-mode laser field $\mathbf{E}=\mathbf{E}_0\sin(\omega t)$, which has a distribution of frequencies only in the interaction with the target atom. The appropriate wave functions in this case are

$$\chi_{\mathbf{k}}(\mathbf{r}_1)=\exp\left[i\mathbf{k}\cdot\mathbf{r}_i+i\lambda\sin(\omega t)-\frac{k^2}{2}t\right],\quad \lambda=\frac{e\mathbf{E}_0\cdot\mathbf{k}}{\omega^2}, \tag{14}$$

$$\psi_0(\mathbf{r}_2)=(\phi_0-\phi_0')e^{-i\omega_0 t+ie\mathbf{A}\cdot\mathbf{r}_2/c}, \tag{15}$$

$$\phi_0'(\mathbf{r}_2)=i\sum_k\frac{M_{k0}e^{-i\omega' t}E_0\phi_k}{2[i(\omega_{k0}-\omega')+\gamma_k]}. \tag{16}$$

The prime over ω in Eq. (16) is to distinguish it from ω, when the average over the spectrum is taken later on. The terms of order E_0^2 in the scattering cross section evidently arise from the matrix elements $\langle\phi_0|V|\phi_0'\rangle$+c.c., whose contribution to the S matrix is readily calculated to be

$$S_{fi}(E_0)=-\pi E_0\sum_k\sum_n J_n(\lambda_0')\left[\frac{M_{0k}V_{k0}}{i(\omega'-\omega_{k0})+\gamma_k}\delta(E_{if}-\omega'-n\omega)-\frac{M_{k0}V_{0k}}{i(\omega_{k0}-\omega')+\gamma_k}\delta(E_{if}+\omega'-n\omega)\right], \tag{17}$$

where λ_0' is defined similarly to λ_0. Clearly, the resonances of interest here occur for $\omega'\sim\omega_{k0}$. For $k_f^2=2(E_i+\omega_{k0}-n\omega)$, the scattering amplitude is obtained from Eq. (17) as

$$f_k^{(n)}=-\frac{i}{4\pi}\frac{M_{k0}V_{0k}E_0}{i(\omega_{k0}-\omega')+\gamma_k}J_n(\lambda_0'). \tag{18}$$

The corresponding cross section averaged over a normalized Lorentzian with a bandwidth $\Delta\omega$ is[8]

$$\frac{k_f}{k_i}\int|f_k^{(n)}|^2\frac{\Delta\omega\, d\omega'}{\pi[(\omega'-\omega)^2+\Delta\omega^2]} \simeq\frac{k_f}{k_i}\frac{E_0^2\Delta\omega}{16\pi^2}\frac{|M_{0k}V_{k0}|^2}{\gamma_k\omega_{k0}^2}J_n^2(\lambda_0'). \tag{19}$$

On taking the ensemble average over E_0 by means of Eq. (8), we finally recover Eq. (13) (to order $\mathscr{E}_0^2$).

The above derivation shows that the magnitude of the resonance effect has a strong dependence on the laser line shape. Now, it is well known that in practice, the Lorentzian approximation is reasonable near the center, where it has observable effects,[9] but not farther off, where the spectrum falls off much faster. It is therefore very unlikely that in the case of electron-hydrogen collisions in a laser with $\hbar\omega\simeq 1$ eV any such structure would be observed. In the experiments of Weingartshofer *et al.*[10] also these peaks would have been absent, as they used argon as the target. However, these structures may be expected to show up if the laser is detuned several natural linewidths away from the dominant dipole-favored transition frequency of the target atom, but within a few bandwidths of the laser. Such an experiment would seem feasible at present with suitable alkali atoms.

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