Dielectric properties of a disordered Bose condensate

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(Received 10 January 1985; revised manuscript received 24 June 1985)

The frequency-dependent and wave-number-dependent dielectric properties of a condensate of a boson gas with static disorder are evaluated. For weak disorder the condensate is superfluid and the disorder induces a mass enchancement. For strong disorder the condensate is insulating and the static dielectric function is finite. Numerical solutions for the frequency dependence of the density relaxation function and the dynamical conductivity are given. The dielectric properties of a charged Bose gas with charged impurities and a repulsive Bose gas with neutral impurities are evaluated explicitly.

I. INTRODUCTION

It is well known that strong enough disorder localizes classical particles[1] as well as fermions.[2] For a system of fermions this disorder-induced phase transition from a metal to an insulator is known as the Anderson transition.

The influence of disorder on phononlike excitations has recently attracted interest. In this field of "dirty bosons,"[3] phonon and photon localization have been studied.[4,5]

We have previously studied the influence of static disorder on interacting bosons in the condensate phase with collective modes as elementary excitations.[6] We found a disorder-induced transition from a superfluid state to an insulator state. The transition point has been characterized as a metallic state. The method which was used in our work[6] was analogous to that of Götze,[7] who calculated the propagation of density and current fluctuations self-consistently for a noninteracting electron gas. This approach gives a transition from a metallic phase to an insulating phase within one frame. Essential for the existence of a mobility edge in this theory was the idea of self-consistency.

In the work of Ref. 7 the decay channel for the current was the particle-hole spectrum of the noninteracting electron gas. The influence of disorder on a system with particle-hole excitations and collective excitations (plasmons) has been discussed for an interacting electron gas.[8] The boson condensate is interesting because here the current can decay only into the collective modes.

In this paper we give some numerical results of our equations given in Ref. 6. In particular, the frequency-dependent properties of our models are discussed. As models we use a charged Bose gas disturbed by charged impurities and a repulsive Bose gas with neutral impurities. The paper is organized as follows: In Sec. II the models used and the self-consistency equations of our theory are explained. The charged Bose gas is discussed in Sec. III, and the results for the repulsive Bose gas are given in Sec. IV. A conclusion is given in Sec. V.

II. MODELS AND SELF-CONSISTENCY EQUATIONS

In the following we report the dielectric properties of a Bose gas characterized by the Hamiltonian with kinetic energy H_0, interacting part H_I, and disorder part H_D with

$$H_0=\sum_{\mathbf{k}}\epsilon_{\mathbf{k}}a^{\dagger}_{\mathbf{k}}a_{\mathbf{k}} , \tag{1a}$$

$$H_I=\tfrac{1}{2}\sum_{\mathbf{q}}\rho^{\dagger}(\mathbf{q})V(\mathbf{q})\rho(\mathbf{q}) , \tag{1b}$$

$$H_D=\sum_{\mathbf{q}}U(\mathbf{q})\rho^{\dagger}(\mathbf{q}) . \tag{1c}$$

Here $a^{\dagger}_{\mathbf{k}}$ is the creation operator for a boson with wave vector $\mathbf{k}$, $\epsilon_{\mathbf{k}}=k^2/2m$ is the parabolic dispersion relation of bosons with density n and mass m. $V(\mathbf{q})$ is the Fourier transform of the interaction of the bosons, and the operator

$$\rho(\mathbf{q})=\sum_{\mathbf{k}}a^{\dagger}_{\mathbf{k}-\mathbf{q}/2}a_{\mathbf{k}+\mathbf{q}/2}$$

is the operator for density fluctuations. $U(\mathbf{q})$ is the Fourier transform of the random potential. For the charged Bose gas[9] with plasmons as elementary excitations[10] we use $V(\mathbf{q})=4\pi e^2/q^2$ and

$$\langle\,|\,U(\mathbf{q})\,|\,\rangle^2=n_i(4\pi e^2/q^2)^2 ,$$

and for the repulsive Bose gas[11] with phononlike excitations we use $V(\mathbf{q})=U^0$ and

$$\langle\,|\,U(\mathbf{q})\,|^2\rangle=(6\pi^2/q_0^2)U^2\Theta(q_0-q) .$$

n_i is the density of charged scatterers with charge e, U^0 is the strength of the repulsive interaction of the bosons, and q_0 and U are the range and the strength of the random potential.

In the theory of Ref. 6 the Kubo density-density relaxation function $\phi(\mathbf{q},z)$ for wave vector $\mathbf{q}$ and complex frequency z is given in terms of the thermodynamic wave-vector-dependent compressibility $g^J(\mathbf{q})$ and the frequency-dependent current relaxation kernel $M(z)$ via

$$\phi(\mathbf{q},z)=\frac{g^J(\mathbf{q})}{z-\frac{n}{m}\frac{q^2}{g^J(\mathbf{q})}\frac{1}{z+M(z)}} \tag{2}$$

and $g^J(\mathbf{q})$ was calculated in the random-phase approximation (RPA) (Refs. 12 and 13) as

$$g^J(\mathbf{q})=\frac{g^0(\mathbf{q})}{1+V(\mathbf{q})g^0(\mathbf{q})} . \tag{3a}$$

$g^0(\mathbf{q})$ is the compressibility of the free Bose gas given as

$$g^0(\mathbf{q})=2n/\epsilon_{\mathbf{q}} . \tag{3b}$$

The current relaxation kernel determines the dynamical conductivity $\sigma(z)$ via[14]

$$\sigma(z)=\frac{ne^2}{m}\frac{i}{z+M(z)} \tag{4a}$$

and the dynamical polarizability $X(z)$

$$X(z)=i\sigma(z)/z . \tag{4b}$$

In a mode-coupling approximation $M(z)$ is expressed as[7]

$$M(z)=\frac{1}{3nm}\sum_{\mathbf{q}} q^2\langle |U(\mathbf{q})|^2\rangle\phi(\mathbf{q},z) . \tag{5}$$

Equations (2) and (5) imply a self-consistency problem;[7] the asymptotic solutions of the problem have been discussed before.[6] It has been shown that in the ideal-conductor phase $M(z\to 0)\sim z$, at the transition point $M(z\to 0)\sim i\alpha$ $(\alpha>0)$, and in the insulator phase $M(z\to 0)\sim -1/z$.

III. DIELECTRIC PROPERTIES OF A CHARGED BOSE GAS

The interacting Bose gas is characterized by the energy scale $\omega_p=(4\pi ne^2/m)^{1/2}$, which is the $q\to 0$ plasmon energy of the interacting boson gas without disorder. The relevant length scale is $q_0^{-1}=(2m\omega_p)^{-1/2}$ and the connection with the dimensionless RPA parameters r_s (Ref. 12) is $(q_0a_B)^4=(12/r_s^3)$. $a_B=\hbar^2/me^2$ is the Bohr radius.

It was found in Ref. 6 that the parameter A, given by

$$A=0.0885\frac{n_i}{n}\left[\frac{1}{n^{1/3}a_B}\right]^{3/4} , \tag{6}$$

determines various phases. For $A<1$ the system of bosons is superfluid and for $A>1$ the bosons are localized. $A=1$ determines the phase-transition point, and here the system is in a metallic phase. For $n_i=n$ the phase transition condition $A=1$ can be rewritten analogous to the form of the so-called Mott criterion[15] for the metal-insulator transition of fermions (n_c is the critical density, where the transition takes place)

$$n_c^{1/3}a_B=0.040 . \tag{7}$$

So for $n<n_c$ the system is in a localized phase; for $n>n_c$ the boson system is superfluid. The dependence of A on n_i and n is crucial. By increasing n (for $n_i=n$) the disorder in the system is increased. But also the screening properties are improved. Equation (7) tells us that in our approximation screening wins over disorder. For the electron systems realized in semiconductors, for example, phosphorus-doped silicon, this behavior is seen in experiment.[16] For an interacting electron gas a relation of the form $n_c^{1/3}a_B=f$ has been given within our theoretical frame.[8]

The homogeneous dielectric function $\epsilon(\omega)$ is given by the polarizability $X(\omega)$ of the boson gas

$$\epsilon(\mathbf{q}=0,\omega)=\epsilon(\omega)=1+4\pi X(\omega) .$$

In the insulator phase the static polarizability $X(0)$ is given by

$$\frac{1}{4\pi X(0)}=4A\left[1-\left[\frac{4\pi X(0)}{1+4\pi X(0)}\right]^{1/4}\right] \tag{8}$$

with the asymptotic solutions

$$4\pi X(0)=\begin{cases}\frac{1}{4A}, & A\to\infty\\ \frac{5}{8}\frac{A}{A-1}, & A\to 1^+ .\end{cases} \tag{9}$$

In the insulator phase we have a finite static polarizability and in Fig. 1 we have shown the dependence of the inverse polarizability on n/n_c according to Eq. (8). The asymptotic solutions (9) are shown as dotted ($A\to\infty$) and dashed curves ($A\to 1^+$). The scaling law (dashed curve) is in good agreement with the solution of (8) for $0.9<n/n_c<1.0$. The inset compares the $A\to\infty$ solution of $4\pi X$ with the self-consistent solution. The factor $A/(A-1)$ may be interpreted as a Clausius-Mosotti formula:

$$\epsilon(0,0)=1+\frac{5}{8}\frac{1}{1-1/A} \quad (A\to 1^+) .$$

The frequency-dependent dielectric function is expressed as

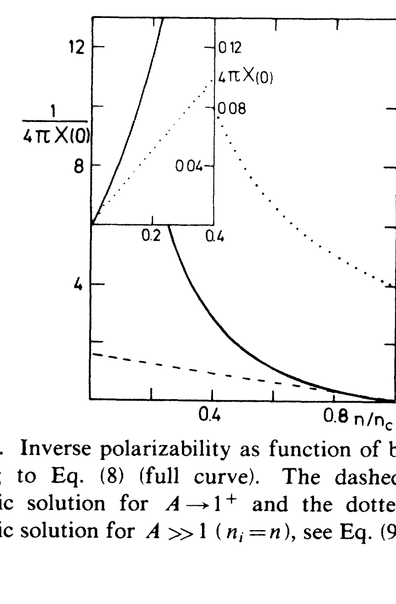

FIG. 1. Inverse polarizability as function of boson density n according to Eq. (8) (full curve). The dashed curve is the asymptotic solution for $A\to 1^+$ and the dotted curve is the asymptotic solution for $A\gg 1$ ($n_i=n$), see Eq. (9).

$$\epsilon(\omega\to 0)=\begin{cases}1-(1-A)\dfrac{\omega_p^2}{\omega^2}, & A<1\\ 1-\dfrac{\omega_p^2}{\omega[\omega+i\omega_p(\frac{8}{5})^{1/2}]}, & A=1\\ 1+\dfrac{5}{8}\dfrac{A}{A-1}, & A\to 1^+\\ 1+\dfrac{1}{4A}, & A\to\infty\ .\end{cases} \tag{10}$$

In the ideal-conductor phase the influence of the impurities can be interpreted with $\epsilon(\omega)=1-\bar{\omega}_p^2/\omega^2$ and $\bar{\omega}_p^2=4\pi ne^2/m^*$ as a mass renormalization (mass enhancement) with

$$\frac{m^*}{m}=\frac{1}{1-A},\quad A<1\ . \tag{11}$$

The state characterized by $A<1$ is free from dissipation, and impurities induce a mass enhancement. The transition point $A=1$ is specified by metallic properties.

In ergodic systems $g^{\rm iso}(\mathbf{q})=g^J(\mathbf{q})$ holds, but in nonergodic systems the Kubo relaxation function [$g^{\rm iso}(\mathbf{q})$ is the isolated density-density susceptibility] exhibits a nonergodicity peak $\phi(\mathbf{q},z)=-f(\mathbf{q})/z$ and one gets[17,18]

$$g^{\rm iso}(\mathbf{q})=g^J(\mathbf{q})-f(\mathbf{q})\ .$$

For our model $f(\mathbf{q})$ is expressed as

$$f(\mathbf{q})=g^J(\mathbf{q})\Big/\left[1+4\pi X(0)\frac{n}{m}q^2\Big/g^J(\mathbf{q})\right]\ .$$

So we conclude that the state with $A>1$ is a nonergodic state with insulating properties. The condensate is pinned by a random array of impurities.

The isolated density-density susceptibility $g^{\rm iso}(\mathbf{q})$ determines via

$$\epsilon(\mathbf{q},\omega=0)^{-1}=\epsilon(\mathbf{q})^{-1}=1-V(\mathbf{q})g^{\rm iso}(\mathbf{q})$$

the static wave-vector-dependent dielectric function $\epsilon(\mathbf{q})$. We find

$$\epsilon(\mathbf{q})=\begin{cases}1+\dfrac{1}{(q/q_0)^4}, & A\le 1\\ 1+\dfrac{1}{1/4\pi X(0)+(q/q_0)^4}, & A>1\ .\end{cases} \tag{12}$$

The screened interaction of the bosons is given[19] by $V_c(\mathbf{q})=V(\mathbf{q})/\epsilon(\mathbf{q})$. With (12) we give the Fourier transform as

$$V_c(r)=\begin{cases}\dfrac{e^2}{r}e^{-rq_0/2^{1/2}}\cos(rq_0/2^{1/2}), & A\le 1\\ \dfrac{e^2}{r}\dfrac{1}{1+4\pi X(0)}[1+4\pi X(0)e^{-r\bar{q}_0}\cos(r\bar{q}_0)], & A>1\end{cases} \tag{13}$$

and $\bar{q}_0=q_0[1+1/4\pi X(0)]^{1/4}/2^{1/2}$. From (13) it is clear that for $A\le 1$ the interaction is screened at $r\sim 2^{1/2}/q_0$. But in the insulator phase the screened potential is a Coulomb potential with an effective charge Z^*

$$Z^*=\frac{e}{1+4\pi X(0)}<e\ . \tag{14}$$

Next we investigate the induced charge distribution $\rho_{\rm ind}$ when a static impurity $\varphi_{\rm ex}(r)=Ze/r$ has been introduced into the system. With

$$\varphi_{\rm ex}(\mathbf{q},\omega)=8\pi^2 Ze\delta(\omega)/q^2$$

the induced charge is

$$\rho_{\rm ind}(\mathbf{r},t)=-\frac{Z}{e}\int\frac{d^3\mathbf{q}}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}g^{\rm iso}(\mathbf{q})\frac{4\pi e^2}{q^2}\ . \tag{15}$$

In the dissipationless phase we get the known result[20]

$$\rho_{\rm ind}(\mathbf{r})=-\frac{Ze}{4\pi}\frac{q_0^2}{r}e^{-rq_0/2^{1/2}}\sin(rq_0/2^{1/2}) \tag{16}$$

but in the insulator phase we get

$$\rho_{\rm ind}(\mathbf{r})=-\frac{Ze}{4\pi}\frac{q_0^2}{r}\left[\frac{4\pi X(0)}{1+4\pi X(0)}\right]^{1/2}e^{-r\bar{q}_0}\sin(r\bar{q}_0) \tag{17}$$

and the induced charge is reduced in comparison to (16). The total induced charge $Q=\int d^3\mathbf{r}\,\rho_{\rm ind}(\mathbf{r})$ is then

$$Q=-Ze\begin{cases}1, & A\le 1\\ \dfrac{1}{1+1/4\pi X(0)}, & A>1\end{cases} \tag{18}$$

and the screening in the insulator phase is no longer perfect.

The Fourier transform of the form factor of the nonergodicity peak of the density fluctuation is given as

$$f(r)=\frac{nm}{\pi}\frac{1}{1+4\pi X(0)}\frac{1}{r}e^{-r/r_0}\cos(r/r_0) \tag{19}$$

with the decay length r_0

$$r_0 q_0=[8-4/\epsilon(\omega=0)]^{1/4}\ . \tag{20}$$

Density fluctuations do not propagate in time and are localized to a finite volume given by r_0^3. So for $A\to 1^+$ the decay length r_0 shows no divergence. The dependence of the corresponding inverse decay length on n/n_c is shown in Fig. 2 as the full line. The asymptotic solutions with (10) are shown as dotted ($n\ll n_c$) and dashed lines ($n\to n_c^-$). For $n>n_c$ the inverse decay length is zero. So the r_0 jumps at the transition point and the prefactor of $f(r)$ goes to zero at the transition point. In the following part of this section we discuss the numerical solution of Eqs. (2)–(5) for $n_i=n$.

For a good ideal conductor with $n/n_c=12$ ($A=\frac{1}{12}$) we compare in Fig. 3 the self-consistent solutions for $M'(\omega)$, $M''(\omega)$, and $\sigma'(\omega)$ (full curves) with the zeroth-order results[6,21] (linear in n_i) $M^{0'}(\omega)$, $M^{0''}(\omega)$, and $\sigma^{0'}(\omega)$ (dotted curves). The zeroth-order result is expressed as

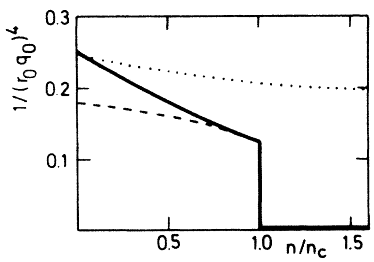

FIG. 2. Inverse decay length $1/r_0$ as function of n according to Eq. (20) (full curve). The dashed and dotted curves are the asymptotic results with $X(0)$ from (9) ($n_i=n$).

$$M^{0'}(\omega)=4A\frac{\omega_p^2}{\omega}\begin{cases}\dfrac{1}{(1-\omega^2/\omega_p^2)^{1/4}}-1, & \omega<\omega_p\\[2mm] \dfrac{1/2^{1/2}}{(\omega^2/\omega_p^2-1)^{1/4}}-1, & \omega>\omega_p\end{cases} \tag{21a}$$

$$M^{0''}(\omega)=4A\frac{\omega_p^2}{\omega}\begin{cases}0, & \omega<\omega_p\\[2mm] \dfrac{1/2^{1/2}}{(\omega^2/\omega_p^2-1)^{1/4}}, & \omega>\omega_p\ .\end{cases} \tag{21b}$$

The $(\omega-\omega_p)^{-1/4}$ singularity in the zeroth-order result is fully renormalized in the self-consistent theory. The increase of $M^{0''}(\omega)$ in the zeroth order at $\omega=\omega_p$ is explained by the decay of current into plasmons. Only for $\omega\geq\omega_p$ can the current decay into plasmons and the system is free of dissipation for $\omega<\omega_p$, see Eq. (2), for $M(z)=0$. So one has also a singularity for $\sigma^{0'}(\omega\to\omega_p)$

$$\sigma^{0'}(\omega)=\begin{cases}0, & \omega<\omega_p\\[2mm] \dfrac{1}{16\pi 2^{1/4}}\dfrac{\omega_p}{A}(\omega/\omega_p-1)^{1/4}, & \omega\to\omega_p^+\ .\end{cases} \tag{22}$$

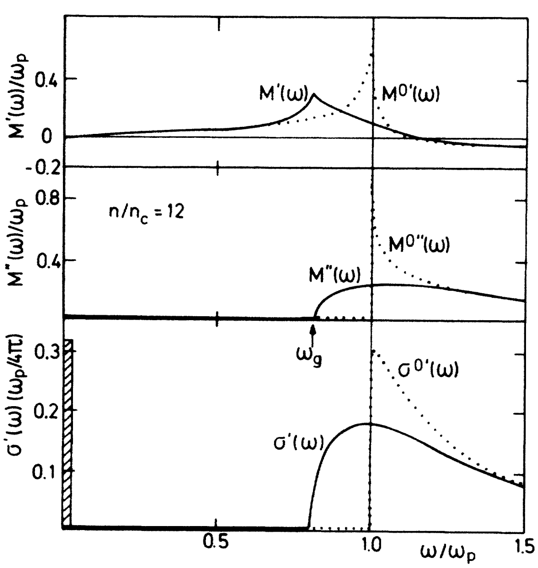

FIG. 3. Comparison of zeroth-order result (dotted line) and the solution of the self-consistency equations for the current relaxation kernel $M'(\omega)$, $M''(\omega)$, and $\sigma'(\omega)$ for $n/n_c=12$ ($n_i=n$). The dashed area is the $\delta(\omega)$ contribution to $\sigma'(\omega)$.

In Eq. (22) we have neglected this $\delta(\omega)$ contribution for $\sigma^{0'}(\omega)$, which signals the dissipationless state. As can be seen in Fig. 3, the gap frequency ω_g, where $M''(\omega)=0$ for $0<\omega<\omega_g$, is reduced in the self-consistent solution of $M''(\omega)$, while in the zeroth-order result $\omega_g=\omega_p$.

For the transition regime $A\sim 1$ the scaling equation[6] implies a frequency range, $0<\omega<\omega_g$, where $M''(\omega)=0$ and $\omega_g^2=|A-1|^2/4AC$ with $C=\frac{5}{8}A$. For the Coulomb gas we get for $A\to 1^+$ or $A\to 1^-$

$$\omega_g=0.63\omega_p\left|1-\frac{n}{n_c}\right|\ . \tag{23}$$

The reduction of ω_g is seen more drastically in Fig. 4, where the dynamical conductivity has been shown for $n/n_c=2$, 4, and 8. For decreasing n the gap frequency ω_g is reduced. In the theory the f sum rule[12] is fulfilled: $\int_{-\infty}^{\infty}\sigma'(\omega)d\omega=(\pi/4)\omega_p^2$. In the ideal conductor phase the $\delta(\omega)$ contribution to $\sigma'(\omega)$ (shaded area in Fig. 4) contributes to the f sum rule, and so we receive for $\omega^*\to 0$

$$\int_{\omega^*}^{\infty}d\omega\,\sigma'(\omega)=\frac{\pi}{8}\omega_p^2\begin{cases}A, & A\leq 1\\ 1, & A>1\ .\end{cases} \tag{24}$$

This is the reason why the $\sigma'(\omega)$ in Fig. 4 for $n/n_c=2$ is greater than for $n/n_c=4$ and 8.

The reduction of the gap frequency is also seen in Fig. 5, where the density relaxation function normalized to the compressibility is shown as function of frequency for $q=10^{-5}q_0$ and $n/n_c=2$ and 12. ω_m in the figure marks the maximum in the excitation spectrum and indicates the softening of the plasmon energy by disorder. The origin of this collective-mode softening will be discussed more explicitly for the repulsive Bose gas in Sec. IV. The line broadening induced by disorder becomes greater when n gets smaller. Because of the compressibility sum rule[12]

$$\int_{-\infty}^{\infty}d\omega\,\phi''(\mathbf{q},\omega)=\pi g^J(\mathbf{q})$$

(see Fig. 5) there follows because of (5) a sum rule for the imaginary part of the current relaxation kernel

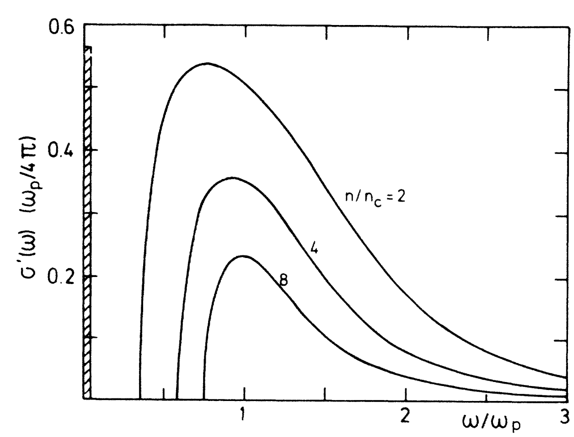

FIG. 4. Dynamical conductivity for $n/n_c=2$, 4, and 8 as function of frequency. The shaded area indicates the $\delta(\omega)$ contribution to the conductivity in the ideal-conductor phase ($n_i=n$).

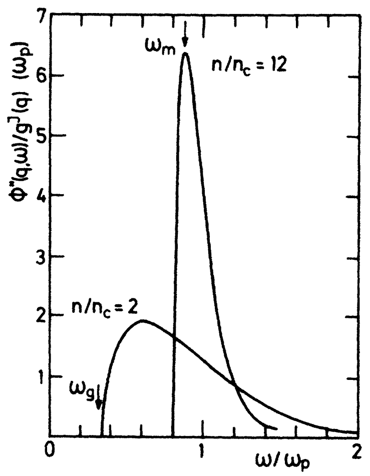

FIG. 5. Density relaxation function $\phi''(\mathbf{q}\to 0,\omega)$ as function of frequency for $n/n_c=12$ and $n/n_c=2$. The arrows indicate the gap frequency ω_g and the energy of maximal excitation ω_m ($n_i=n$, $q=10^{-5}q_0$, $n_i=n$).

$$\int_{-\infty}^{\infty} d\omega\, M''(\omega)=\pi B\omega_p^2 , \tag{25}$$

where B is the strong-insulator result for the inverse polarizability.[6] This sum rule holds also for fermions.[8] Here $B=4A$ and because in the insulator phase there is a $\delta(\omega)$ contribution to the integral in (25), we get for $\omega^*\to 0$

$$\int_{\omega^*}^{\infty} d\omega\, M''(\omega)=\pi\omega_p^2 \begin{cases} 2A, & A\le 1 \\ 2A-\frac{4}{5}(1-1/A), & A\to 1^+ \\ 0, & A\to\infty\ . \end{cases} \tag{26}$$

This sum rule can be seen in Fig. 3 by comparing the frequency integral of the zeroth-order result and of the self-consistent result, which must be equal according to Eq. (26).

In Fig. 6 the frequency dependence of the current relaxation kernel $M(\omega+i0)=M'(\omega)+iM''(\omega)$, and the corresponding dynamical conductivity, is shown for $n/n_c=0.8$ (insulator phase), $n/n_c=1.0$ (transition point), and $n/n_c=1.2$ (ideal-conductor phase). In the insulating phase $M'(\omega\to 0)\sim -1/\omega$ and $M''(\omega)$ exhibits, according to the Kramer-Kronig relation, a $\delta(\omega)$ peak (shaded area). The dc conductivity is zero. The transition point is characterized by $M''(\omega\to 0)=\text{const}$ and $M'(\omega\to 0)\sim\omega$ with a finite dc conductivity. The state with no static dissipation shows $M''(\omega\to 0)=0$ and $M'(\omega\to 0)\sim\omega$, the conductivity has a $\delta(\omega)$ contribution (shaded area).

The corresponding density relaxation function as function of frequency for $q=10^{-5}q_0$ is shown in Fig. 7. The insulating phase is characterized by the nonergodic $\delta(\omega)$ contribution (shaded area), and in the ideal-conductor phase near the transition point the plasmon modes are strongly softened and are very broad. Note that the area under the curves are equal according to the compressibility sum rule.

The $M'(\omega)$, $M''(\omega)$, and $\sigma'(\omega)$ behavior for the insulating phase looks very like the corresponding functions of localized fermions[8,18] and localized classical particles.[22] It seems that the statistics of the quantum particles is not so

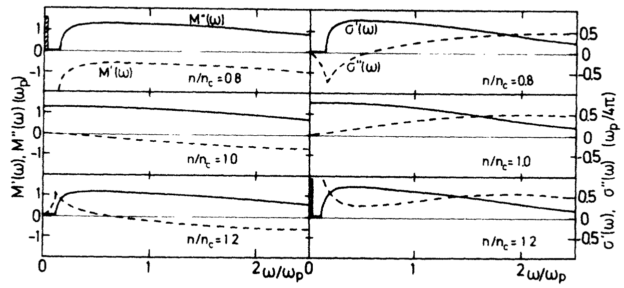

FIG. 6. Frequency dependence of the current relaxation kernel $M'(\omega)$ and $M''(\omega)$ and the dynamical conductivity $\sigma'(\omega)$ and $\sigma''(\omega)$ for $n/n_c=0.8$, 1.0, and 1.2 ($n_i=n$).

important when the particles are localized. This seems plausible when electrons and classical particles are compared. In the case of a localized Bose condensate one has to remember that the whole boson system acts as one particle, which is localized.

The gap behavior of $M''(\omega)$ and $\sigma'(\omega)$ in the insulating phase is probably a characteristic behavior of the Bose condensate. For classical properties one gets in the insulating phase $\sigma'(\omega\to 0)\sim\omega^2$ (Ref. 22) and for fermions logarithmic corrections[15] are expected: $\sigma'(\omega\to 0)\sim\omega^2(\ln\omega)^4$. For a trapped macroscopic Bose system a finite frequency gap for finite conduction could be due to pinning. We mention here that for a one-dimensional pinned charge-density wave a Mott-Berezinsky law, $\sigma'\sim\omega^2(\ln\omega)^2$, has also been found[23] for ω greater than some characteristic frequency ω_1. For a frequency $\omega<\omega_1$ an excitation gap exists also for a charge-density wave. Within our framework a gap also exists for a disordered electron system.[18,8] But there the gap in the insulator is a consequence of the approximation and can be improved by a $\sigma'(\omega\to 0)\sim\omega^2$ behavior, when a generalized hydrodynamic description of the density propagation is used.[24,25] In our case here the density propagator [Eq. (2)] is already hydrodynamic without any further approximation, and the gap seems to be a real effect.

The behavior of ω_g and ω_m, where the density relaxation is maximal, see Fig. 5, as function of n/n_c is shown in Fig. 8. For $n/n_c>1$, ω_m may be interpreted as the disorder-induced softening of the plasmon mode and indi-

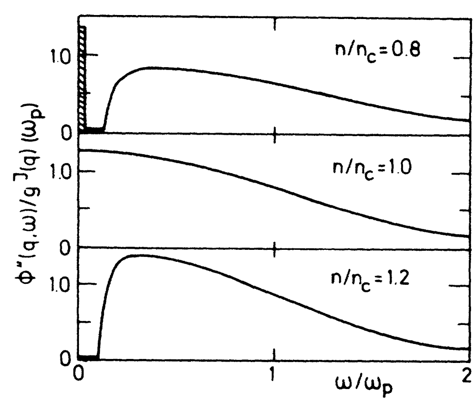

FIG. 7. Frequency dependence of the density relaxation function for the parameters as in Fig. 6 ($n_i=n$, $q=10^{-5}q_0$, $n_i=n$).

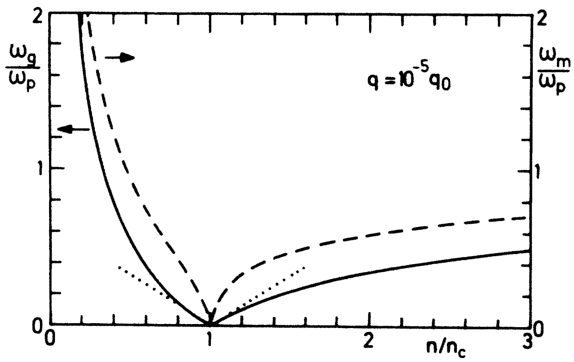

FIG. 8. ω_g (full curve) and ω_m (dashed curve) for $q=10^{-5}q_0$ as a function of density. The dotted curve is the scaling law [Eq. (23)] ($n_i=n$).

cates this softening quantitatively as a function of disorder. Because of the broadening of this mode $\omega_g<\omega_m$. The phase transition from a superfluid phase to an insulating phase occurs when ω_m has been reduced to zero.

IV. DIELECTRIC PROPERTIES OF THE REPULSIVE BOSE GAS

For the repulsive Bose gas[11] disturbed by neutral impurities two energy scales $E_0=q_0^2/2m$ and $\omega_p=(4\pi e^2 n/m)^{1/2}$ together with the parameter $\eta^2=2U^0 n/E_0$ have been introduced in Ref. 6. The discussion of the dielectric properties of the repulsive Bose gas goes along the line shown in the last section for the charged Bose gas. So we give here only the most important results.

It was found in Ref. 6 that the parameter A, depending on U, E_0, and η, determines the state of the system. U_c is the critical strength of the random potential, where the phase transition happens. For $A<1$ ($U<U_c$) the system is superfluid, for $A>1$ ($U>U_c$) the system has the properties of an insulator, and $A=1$ ($U=U_c$) is the transition point. This behavior is summarized in the low-frequency behavior of the homogeneous dielectric constant

$$\epsilon(\omega\to 0)=\begin{cases}1-(1-A)\omega_p^2/\omega^2, & A<1\\ 1-\omega_p^2/[\omega(\omega+i(E_0^2/C_c)^{1/2}], & A=1\\ 1+\omega_p^2 C_c/[E_0^2(A-1)], & A\to 1^+ .\end{cases} \tag{27}$$

C_c parametrizes the insulator phase near $A=1$.

Analogous to the charged Fermi gas the static wave-vector-dependent dielectric constant, because of the nonergodicity of density fluctuations in the insulator phase,[17,18] is expressed as

$$\epsilon(q)=1+\frac{\eta^2 q_0^2 q^2}{q^4+q_0^4\omega_p^2/\{E_0^2[\epsilon(\omega=0)-1]\}} . \tag{28}$$

Because $\epsilon(\omega\to 0)$ diverges for $A\le 1$, we have $\epsilon(q)=1+\eta^2 q_0^2/q^2$ for $A\le 1$.

The most important difference between a repulsive Bose condensate and a charged Bose condensate is the q dependence of the collective modes. In the case of the charged Bose gas an excitation gap for the collective modes exists. For a repulsive Bose gas the collective modes are phonon-like and no excitation gap exists. In this sense a charged Bose gas is like a three-dimensional electron gas and a repulsive Bose gas is more like a two-dimensional electron gas when the particle-hole excitations for the interacting electron system are neglected.

For the density relaxation function, see Eq. (2), one gets in the superfluid phase for $M(z\to 0)=Az$ and $q\to 0$ a very sharp peak structure

$$\phi''(\mathbf{q}\to 0,\omega\to 0)=\frac{\pi}{2U^0}[\delta(\omega-\tilde{\omega})+\delta(\omega+\tilde{\omega})] \tag{29}$$

located at $\tilde{\omega}=qs(1-A)^{1/2}$ and $s=(nU^0/m)^{1/2}$. This is shown in Fig. 9 and $\tilde{\omega}$ is shown by an arrow. The shaded area is the $\delta(\omega-qs)$ peak for the free repulsive Bose gas. The finite linewidth in Fig. 9 comes from the finite $M''(\omega)$. The disorder induces a collective mode softening via reactive effects $[M'(\omega)]$ and we get a correction of the Landau criterion[26] for superfluidity due to disorder. This softening is linear in the impurity concentration. Normally dissipative effects also induce a mode softening; however, this is quadratic in the impurity concentration. A very analogous effect for a disorder-induced softening of the plasmon modes in a two-dimensional electron gas has been reported recently.[8] Whether the mode softens or hardens depends on the signature of $M'(\omega)$, see Eq. (17) of Ref. 6. For $M'(\omega)>0$ the mode becomes softer; however, for $M'(\omega)<0$ the mode hardens.

In the case of the repulsive Bose gas the zeroth-order result of the relaxation kernel is given by

$$M^{0'}(\omega)=4U^2/\omega[-\eta^3\arctan(1/\eta)+(y_2-x_2)/(x+y)] , \tag{30a}$$

$$M^{0''}(\omega)=2\pi U^2/\omega[x^{5/2}/(x+y)]\Theta(1+\eta^2-\omega^2/E_0^2) , \tag{30b}$$

and

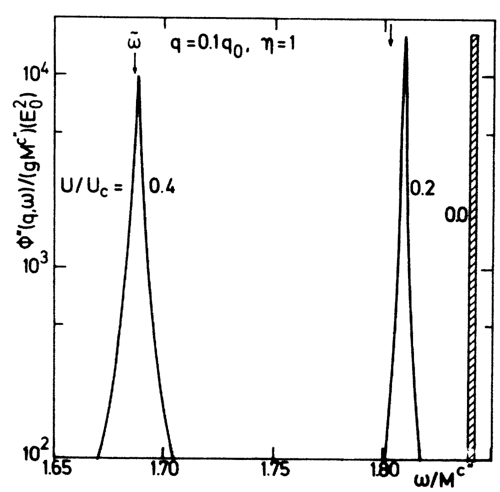

FIG. 9. $\phi''(\mathbf{q},\omega)$ normalized to compressibility as function of frequency for $U/U_c=0.2$ and 0.4. The arrows indicate $\tilde{\omega}$ (see text).

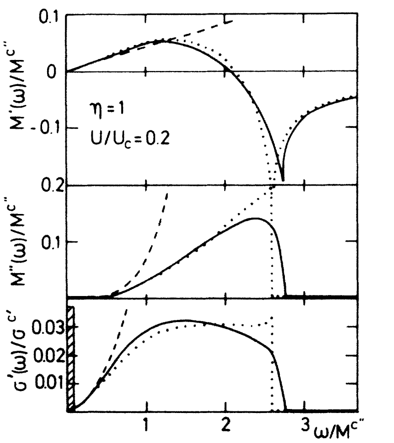

FIG. 10. Comparison of zeroth-order result (dotted curves), low-frequency expansion of zeroth-order result (dashed curves), and self-consistent theory (full curves) for $M'(\omega)$, $M''(\omega)$, and $\sigma'(\omega)$ for $A=0.04$.

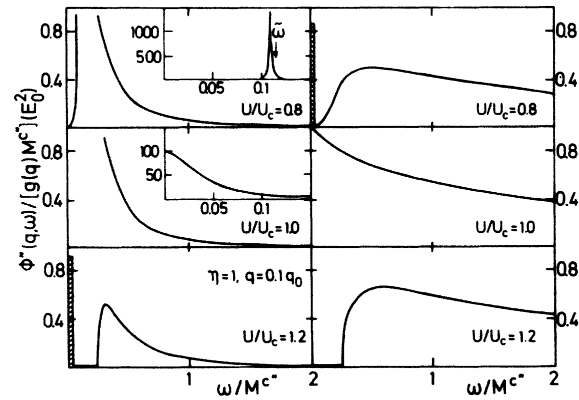

FIG. 11. Density relaxation function and dynamical conductivity as function of frequency for $A=0.64$, 1, and 1.44.

$$y=x+\eta^2 , \tag{31a}$$

$$x=-\eta^2/2+[(\omega/E_0)^2+\eta^4/4]^{1/2} , \tag{31b}$$

$$y_2=y^{5/2}\arctan(1/y^{1/2}) , \tag{31c}$$

$$x_2=x^{5/2}/2\ln|(1+x^{1/2})/(1-x^{1/2})| . \tag{31d}$$

The low-frequency expansion gives $M'\sim\omega$, $M''\sim\omega^4$, and $\sigma'(\omega)\sim\delta(\omega)+\omega^2$. The increase in $M''(\omega)$ with increasing frequency is due to the decay of current into the collective modes.

The behavior of M', M'', and $\sigma'(\omega)$ is shown in Fig. 10 for $A=0.04$ ($U/U_c=0.2$) and $\eta=1$. The dotted lines are the zeroth-order result, the dashed lines are the low-frequency expansion of the zeroth-order result, and the full lines are the solutions of the self-consistent theory. The shaded area in $\sigma'(\omega)$ indicates the $\delta(\omega)$ contribution in the superfluid phase. The finite range q_0 implies a finite frequency range where the decay of the current into the phononlike collective modes is possible and the singularity in $M'(\omega)$ of the zeroth-order results reflects this unphysically sharp cutoff. $M^{c''}(\omega=0)$ and $\sigma^{c''}(\omega=0)$ are the values for $A=1$. For low frequency $M'(\omega)>0$ and a mode softening of the collective modes, as shown in Fig. 9, occurs.

In Fig. 11 the behavior of the density relaxation function normalized to the compressibility as a function of frequency for $q=0.1q_0$ and the dynamical conductivity is shown near $A=1$ for a superfluid phase ($A=0.64$, $U/U_c=0.8$), the conductor phase ($A=1$, $U=U_c$), and an insulator phase ($A=1.44$, $U/U_c=1.2$). The dashed area in $\phi''(\mathbf{q},\omega)$ indicates the nonergodicity contribution in the insulator phase. Note that in Figs. 10 and 11 the dynamical conductivity in the superfluid phase is finite for $\omega>0$ and no gap like for the charged Bose gas exists. The origin for this behavior is the $\omega\sim q$ dependence of the elementary excitation of the system.

V. CONCLUSION

Analytical and numerical results have been presented for the influence of disorder on a Bose gas in the condensate state at temperature zero. The frequency dependence and the wave-vector dependence of the dielectric function indicate the transition from a superfluid phase to an insulator phase. We want to mention that our approximations assume no singularities in the static properties of the system, for example, the compressibility of the system. Only under these circumstances, which cannot be proved within our frame, should our theory content some of the essential physics.

The discussed models are classical models in statistical mechanics. So to study the influence of disorder on these models is interesting from an academical point of view. Moreover they have some overlap with the disordered interacting electron gas, when the particle-hole excitations are neglected.

Recently a collective excitation gap was discussed in connection with the fractional quantum Hall effect.[27] In this theory disorder was neglected. We believe that our results are relevant in this context, but further theoretical work is necessary to discuss the details.

ACKNOWLEDGMENT

This work was supported in part by the Ernst von Siemens Stipendium of the Siemens AG.

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