Jackiw state and higher-order squeezing of the electromagnetic field

Robert Lynch
Physics Department, University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
(Received 18 November 1985)

Hong and Mandel have recently suggested a scheme which allows generalization of "squeezing" to higher-order (even) moments of the electromagnetic field. The new criterion for squeezing is that the expectation value of the moment be less than that for the coherent state. In this paper it is explicitly shown that both electric-field quadrature components are squeezed to fourth order in a "Jackiw state," thus proving that the coherent state does not minimize the fourth-order uncertainty product. The implications of this result for Hong and Mandel's proposal are briefly considered.

Recently Hong and Mandel have published interesting work[1] on higher-order squeezing of the electromagnetic field. Their criterion for obtaining squeezing of the quadrature components of the electric field $\hat{E}_1,\hat{E}_2$ to (even) order N is

$$\langle(\Delta\hat{E}_i)^N\rangle < \langle(\Delta\hat{E}_i)^N\rangle_{\text{coh}} \quad (i=1 \text{ or } 2)\ , \tag{1}$$

i.e., the Nth-order fluctuation in the given state is less than that which obtains in a coherent state. The basis for this generalization appears quite sound: it is known that for the coherent state, $\langle(\Delta\hat{E}_1)^2\rangle_{\text{coh}}=\langle(\Delta\hat{E}_2)^2\rangle_{\text{coh}}$ and that the uncertainty product $\langle(\Delta\hat{E}_1)^2\rangle_{\text{coh}}\langle(\Delta\hat{E}_2)^2\rangle_{\text{coh}}$ has its minimum value.[2] However, for $N>2$ it is not clear that these conditions hold. In fact, in this paper it will be explicitly demonstrated that for $N=4$ a state exists such that the product $\langle(\Delta\hat{E}_1)^4\rangle\langle(\Delta\hat{E}_2)^4\rangle$ takes a value less than that for a coherent state.

Consider a single-mode field in a Jackiw state[3]

$$|\Psi_1\rangle=\omega(|\alpha\rangle+\epsilon\exp(-|\alpha|^2/2)|0\rangle)\ , \tag{2}$$

where ω is a normalization factor, ϵ a positive small parameter, and $|\alpha|$ is large. After a straightforward calculation one finds to order ϵ, keeping terms of order $|\alpha|^4$,

$$\begin{aligned} R_1 &\equiv \langle\Psi_1|(\Delta\hat{E}_1)^4|\Psi_1\rangle/\langle\Psi_1|\Psi_1\rangle \\ &= 3|g|^4[1+\tfrac{2}{3}\epsilon\exp(-|\alpha|^2)|\alpha|^4\cos(4\lambda)] \\ &= R_2 \equiv \langle\Psi_1|(\Delta\hat{E}_2)^4|\Psi_1\rangle/\langle\Psi_1|\Psi_1\rangle\ . \end{aligned} \tag{3}$$

Here $3|g|^4$ is the expectation value $\langle(\Delta\hat{E}_1)^4\rangle_{\text{coh}}=\langle(\Delta\hat{E}_2)^4\rangle_{\text{coh}}$ in a coherent state, and $\lambda\equiv\mathbf{k}\cdot\mathbf{r}-\phi+\theta$ where $\mathbf{k}$ is the wave vector of the field, $\mathbf{r}$ the position vector, and ϕ and θ are phase angles for the electric field and coherent state $|\alpha\rangle$, respectively.

It is observed that for the choice of phases, for example, such that $\lambda=\pi/4$ both R_1 and R_2 take values less than that which obtains in a coherent state, i.e., both quadrature components exhibit "fourth-order squeezing." Actually, rather than showing squeezing this is an exhibition of the fact that the coherent state does not minimize the product of fourth-order fluctuations, which is the result promised above.

Finally, it should be remarked that whether this calculation presents a difficulty for the Hong and Mandel criterion (1) appears to be a matter of opinion. On the one hand, the criterion is of the nature of a definition of higher-order squeezing, and thus cannot be argued with on these strict terms. On the other hand, however, the criterion is being introduced in a well-established field, with an extensive literature, and should dovetail with previous work. The interest in ordinary, second-order squeezing comes from, among other things, a desire to (legitimately) "beat" the uncertainty principle, in obtaining reduced fluctuations in one quadrature component of the field, at the expense of increased fluctuations of the other component, such that the product of the two moments equals the minimum set by the uncertainty principle. The finding above that both fourth-order moments are squeezed in a Jackiw state indicates that the limits set by the uncertainty principle are not reached with the coherent state in fourth order. The search for the fourth-order minimum uncertainty state will be the subject of a future paper.[4]

[1]C. K. Hong and L. Mandel, Phys. Rev. Lett. **54**, 323 (1985); Phys. Rev. A **32**, 974 (1985).
[2]R. J. Glauber, Phys. Rev. **130**, 2529; **131**, 2766 (1963).
[3]R. Jackiw, J. Math. Phys. **9**, 339 (1968). It is interesting to note, although not strictly pertinent to the subject of this paper, that this state exhibits ordinary (second-order) squeezing.
[4]R. Lynch (unpublished).

