Improvement in laser Doppler velocimetry by the use of time-interval photon statistics

F. Moreno
Departamento de Optica y Estructura de la Materia, Facultad de Ciencias, Universidad de Santander, 39005 Santander, Spain

M. A. Rebolledo
Departamento de Optica, Facultad de Ciencias, Universidad de Zaragoza, 50009 Zaragoza, Spain

R. J. López
Departamento de Optica y Estructura de la Materia, Facultad de Ciencias, Universidad de Santander, 39005 Santander, Spain

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In this paper it is shown experimentally that with the use of a laser Doppler velocimeter, the fluid velocity can be determined from the measurement of $Q_C(S)$ [the squared cosine transform of the time-interval probability, $W(\theta)$, between two consecutive photopulses] or from the measurement of $Q_R(X)$ [the square-wave transform of $W(\theta)$]. By using a computer simulation method, the accuracy of these techniques is compared with the one of the usual intensity correlation techniques. It is found that the measurement of $Q_C(S)$ or $Q_R(X)$ provides more accurate results than the intensity correlation technique. It is expected that the advantage of these techniques increases as the intensity of the detected light beam decreases.

INTRODUCTION

During the last 20 years photon statistics has been successfully used in experiments where a light beam with a fluctuating intensity is obtained.[1-5] In some experiments[1,3,4] (e.g., scattering experiments) the intensity fluctuates at random. There are other experiments where intensity fluctuations are deterministic (e.g., optical communications[3] and experiments in spectroscopy where a periodic intensity is obtained[6,7]) or quasideterministic (e.g., laser Doppler velocimetry experiments,[2,5] where periodic signals are obtained at random instants). Usually the number of photocounts, $n(t,T)$, in a time interval $(t,t+T)$ is measured for a variety of values of t and the normalized intensity correlation function $g^{(2)}(\tau)$ is computed for several values of the delay time τ. As the intensity of the analyzed light beam decreases, shot noise increases and a larger error is involved in obtaining information about the light beam from $g^{(2)}(\tau)$. However, for small intensities the time interval θ between two consecutive photopulses can be measured with good accuracy. Consequently, the time-interval probability $W(\theta)$ is an appropriate technique for very low intensities.[8-12]

In particular, light beams with a periodic intensity can be analyzed by this technique.[11] For a light beam whose intensity is periodic with a period P, $W(\theta)$ is a function which oscillates with the same period. This is the reason why a technique consisting of measuring the squared cosine transform $Q_C(S)$ of $W(\theta)$ was recently used to analyze light beams with a square-wave intensity[13] and to determine their period P. For values of S near π/P the transform $Q_C(S)$ passes through a sharp maximum. It was found that, for small intensities, the value of P can be obtained from this maximum with a relative error e_P which is smaller than the one involved in the determination of P from $g^{(2)}(\tau)$. Moreover, the advantage increases as the intensity of the light beam decreases. Later, a simpler technique consisting of measuring the integral $Q_R(X)$ of $W(\theta)R(X,\theta)$, where $R(X,\theta)$ is a square wave, was studied.[14]

For values of X near $1/P$ the transform $Q_R(X)$ passes through a sharp maximum that can be fitted to a Lorentzian curve to obtain P. It was found that the values of e_P involved in the determination of P from $Q_C(S)$ or $Q_R(X)$ are similar, while the experimental values of $Q_R(X)$ are obtained in a simpler way than the experimental values of $Q_C(S)$.

Since the measurement of $Q_C(S)$ or $Q_R(X)$ have proved to be advantageous methods of analyzing periodically modulated light beams of low intensity, we planned to use these techniques for signal processing in laser Doppler velocimetry (LDV) experiments where a low intensity is obtained. The aim of this paper is to verify that the velocity in a differential Doppler system can be determined from the measurement of $Q_C(S)$ or $Q_R(X)$, and compare the accuracy of these methods with the one based on the measurement of $g^{(2)}(\tau)$.

THEORY

Let us consider a light beam with a small periodic intensity whose period is P. In Ref. 13 the squared cosine transform of the time-interval probability $W(\theta)$ was defined as

$$Q_C(S)=\int_0^\infty W(\theta)\cos^2(S\theta)d\theta \ , \tag{1}$$

which can also be expressed in terms of the cosine Fourier transform as

$$Q_C(S)=\frac{1}{2}\left[1+\int_0^\infty W(\theta)\cos(2S\theta)d\theta\right] . \tag{2}$$

This function can be obtained experimentally by measuring repeatedly the time interval θ_i between two consecutive photopulses and using the relation

$$Q_C(S)=(1/N_\theta)\sum_{i=1}^{N_\theta}\cos^2(S\theta_i)$$
$$=0.5+(1/2N_\theta)\sum_{i=1}^{N_\theta}\cos(2S\theta_i)\ . \qquad (3)$$

In Ref. 14 the square-wave transform $Q_R(X)$ of $W(\theta)$ was defined as

$$Q_R(X)=\int_0^\infty W(\theta)R(X,\theta)d\theta\ , \qquad (4)$$

where

$$R(X,\theta)=\begin{cases}1 & \text{when } (4M-1)/4X\le\theta<(4M+1)/4X\ ,\\ 0 & \text{when } (4M+1)/4X\le\theta<(4M+3)/4X\ .\end{cases} \qquad (5)$$

The values of $Q_R(X)$ can be obtained experimentally from the measured values of θ_i by the relation

$$Q_R(X)=(1/N_\theta)\sum_{i=1}^{N_\theta}R(X,\theta_i) \qquad (6)$$

in a very simple way because the values of $R(X,\theta)$ are 0 or 1.

Let us consider a square-wave signal whose period is P. For this signal $Q_C(S)$ passes through a maximum[13] for S near π/P, and $Q_R(X)$ also passes through a maximum[14] for X near $1/P$. In these maxima Q_C and Q_R can be accurately approximated[14] by Lorentzian curves. Thus, once the experimental values for Q_C or Q_R near the maximum are obtained, the results can be fitted to a Lorentzian curve (the height, width, and position of the maximum being unknown) plus a background equal to 0.5.

In a LDV experiment with a differential Doppler system two laser beams interfere with each other and produce a set of fringes.[2] The fluid whose velocity is to be measured passes through the fringes. If seeded particles are carried by the fluid, as a particle crosses these fringes the intensity of the scattered light oscillates with a period P which is proportional to the inverse of the velocity v of the particle. If the period P is determined, v can be obtained.

Since for low intensities the period P of a square wave[13,14] can be obtained more accurately from $Q_C(S)$ or $Q_R(X)$ than from $g^{(2)}(\tau)$, we applied these techniques to the determination of P in a LDV system where a stable velocity is obtained.

EXPERIMENT

In order to show that $Q_C(S)$ and $Q_R(X)$ are useful quantities in LDV, we used the experimental setup schematized in Fig. 1. A differential Doppler system with a 5-mW He-Ne laser was used to measure the velocity of a natural seeded water flow in a Plexiglass tube. Forward-scattered light was detected by a photomultiplier connected to a photon correlation system consisting of (Fig. 2) an amplifier-discriminator, a real-time digital correlator, a time-interval meter, and a computer that controlled the correlator and time-interval meter.

Since each particle which crosses the fringes originates scattered light whose intensity oscillates with a period P (if the velocity of the fluid does not change with time), $Q_C(S)$ can be expected to pass through a maximum for S near $S_M=\pi/P$ and $Q_R(X)$ to pass through a maximum for X near $X_M=1/P$. To verify this, the period P in our experiment was determined by measuring $g^{(2)}(\tau)$ (the usual procedure), which must oscillate[2] with a period equal to P. To obtain a good signal the filter F (Fig. 1) was selected to obtain a mean intensity $I\simeq10^5$ photopulses/s and a 64-channel Malvern correlator was used to obtain $g^{(2)}(\tau)$ from 3×10^6 samples of $n(t,T)$ for $T=3\ \mu$s. The value obtained was $P=31\ \mu$s, from which it was expected that we should obtain $S_M=101\,342\ \text{s}^{-1}$ and $X_M=32\,258\ \text{s}^{-1}$. Later, $Q_C(S)$ and $Q_R(X)$ were measured for low intensities and for values of S near S_M and X near X_M. Since the time-interval meter we used could only make 20 samples of θ per second, we obtained $Q_C(S)$ and $Q_R(X)$ from only 10^4 samples of θ in order to avoid a large measurement time. The results are shown in Figs. 3–6 for two values of $\overline{m}=\overline{I}P$. It can be observed that $Q_C(S)$ and $Q_R(X)$ behave as expected and that the values of S_M and X_M are in good agreement with the expected ones, especially if the fact that the fluid velocity was not steady in a long time interval is taken into account. It can also be observed that the values of $Q_C(S)$ and $Q_R(X)$ to the left of the maxima are larger than the ones to the right. A similar behavior can be found if the expressions of $Q_C(S)$ and $Q_R(X)$ for a square wave are analyzed.[13,14] Finally, if the half-widths of $Q_C(S)-0.5$ and $Q_R(X)-0.5$ (0.5 is the ex-

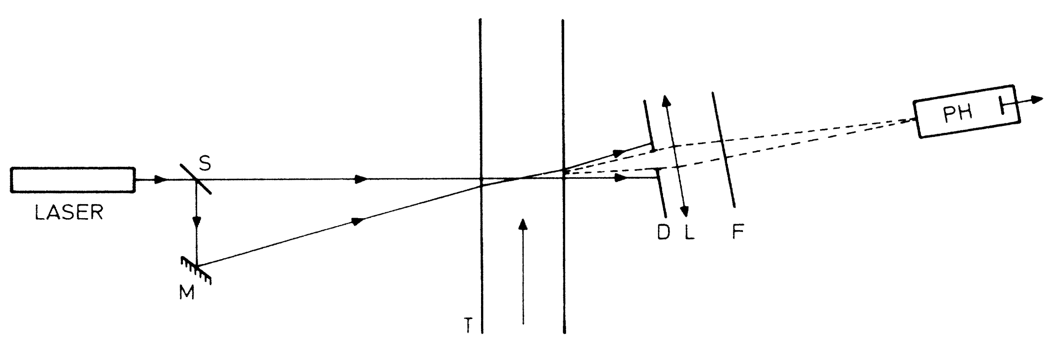

FIG. 1. Experimental setup: S, beam-splitter plate; M, mirror, T, Plexiglass tube; D, diaphragm to block the straight laser light; L, focusing lens; F, grey filter; PH, photomultiplier.

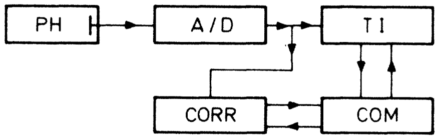

FIG. 2. Photon correlation system: PH, photomultiplier; *A-D*, amplifier-discriminator; TI, time-interval meter; CORR, correlator; COM, computer to control CORR and TI.

pected background of Q_C and Q_R) are estimated in Figs. 3–6, it is found that they are proportional to $\bar{I}$ as occurs for square-wave signals.[13,14] Furthermore, the relative half-widths (the half-width divided by the position of the maximum) of $Q_C(S)$ and $Q_R(X)$ for a fixed value of $\bar{I}$ are similar (approximately equal to 0.2 for $\bar{m} \simeq 0.3$ and approximately equal to 0.35 for $\bar{m} \simeq 0.5$).

Since the behavior of $Q_C(S)$ and $Q_R(X)$ in a LDV experiment was found to be as expected, we can proceed to study the accuracy of these techniques and compare it with the one obtained when measuring $g^{(2)}(\tau)$. Since the intensity scattered by a particle that crosses the fringes oscillates with a period P, we can study the relative error e_P involved in the determination of P when the velocity of the fluid is constant. To do this the change in the fluid velocity during the measurement time must be negligible. Since the measurement time that corresponds to a series of 10^4 samples of θ (in our experimental setup) is about 8 min, and several series are required in order to obtain e_P from several measured values of P, the velocity instabilities are not negligible during the total measurement time. Since we did not have a faster time-interval meter, to avoid this problem we decided to use a computer-simulation method to obtain e_P, as will be explained in the next section.

STUDY OF THE ERRORS AND CONCLUSIONS

In differential LDV experiments low intensities are generally obtained when the number of seeded particles that scatter laser light is so small that the mean distance between two consecutive particles is much larger than the width of the set of interference fringes. In such a LDV experiment the scattered intensity from each particle[15] can be expressed as a function of time as

$$I_j(t) = I_j \exp[-2(t-t_j)^2/\tau_c^2] \times \{1 + V\cos[\delta + 2\pi(t-t_j)/P]\} , \quad (7)$$

where t_j is the instant when the jth particle is at the center of the fringes, τ_c is the time that the particle takes to cross the radius of the laser beam, and V is the fringe visibility.

To simulate values of t_j we used an exponential random-number generator.[5] The values of I_j were assumed to be equal for all particles. We used the values $\tau_c = 10P$, $P = 10^{-3}$ s, $V = 1$, and $\bar{m} = 0.1$. The mean time interval between two consecutive particles was selected to be long enough to ensure that there is only one particle in the fringes. With these data the detected intensity $I(t)$ could be calculated as a function of time. To obtain values of the time interval θ between two consecutive photopulses we simulated the instants when photopulses were obtained when detecting $I(t)$ with a photon-counting system. This was achieved by dividing each period P into subintervals of length $\Delta \ll P$. Consequently, the variation of $I(t)$ in a subinterval was negligible and a random-number Poisson generator could be[3] used to simulate the number of detected photopulses in each subinterval. Furthermore, if Δ is chosen so that $\bar{I}\Delta \ll 1$ the probabili-

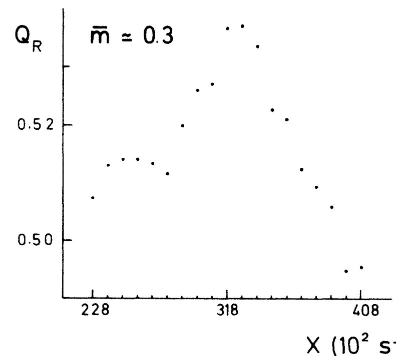

FIG. 4. Experimental results for $Q_R(X)$ obtained from 10^4 samples of the time interval θ, when $\bar{I} \simeq 9700$ photopulses/s and $P = 31$ μs ($\bar{m} = \bar{I}P \simeq 0.3$).

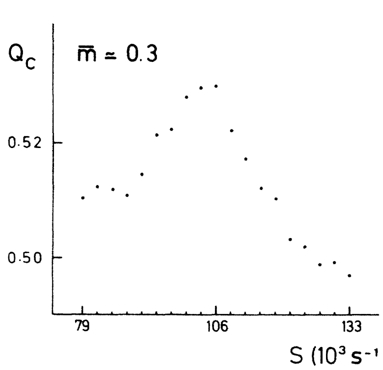

FIG. 3. Experimental results for $Q_C(S)$ obtained from 10^4 samples of the time interval θ, when $\bar{I} \simeq 9700$ photopulses/s and $P = 31$ μs ($\bar{m} = \bar{I}P \simeq 0.3$).

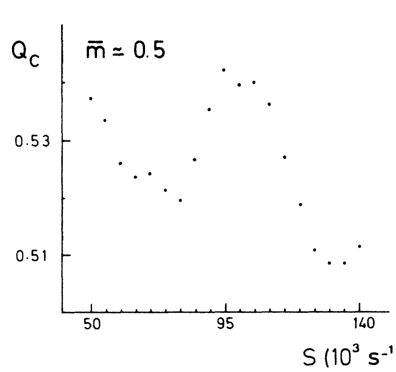

FIG. 5. Experimental results for $Q_C(S)$ obtained from 10^4 samples of the time interval θ, when $\bar{I} \simeq 16\,100$ photopulses/s and $P = 31$ μs ($\bar{m} = \bar{I}P \simeq 0.5$).

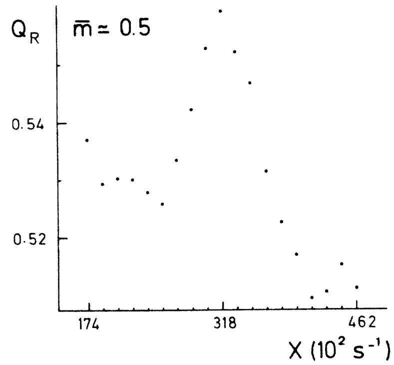

FIG. 6. Experimental results for $Q_R(X)$ obtained from 10^4 samples of the time interval θ, when $\overline{I} \simeq 16\,100$ photopulses/s and $P = 31$ μs ($\overline{m} = \overline{I}P \simeq 0.5$).

ty of obtaining more than one photocount in a subinterval is negligible[3] and only zeros and ones will be obtained by using the Poisson random-number generator. So, the distance between two consecutive ones give us a time interval θ. To simulate values of θ with good precision we used $\Delta = 3 \times 10^{-4} P$.

We simulated ten series of samples of θ with the above-mentioned data. In each series the values of θ that correspond to the passage of 2000 seeded particles through the interference fringes were simulated and the values of $Q_C(S)$ and $Q_R(X)$ were obtained (for 36 values of S and 36 values of X) by using Eqs. (3) and (6). To obtain P from $Q_C(S)$ and $Q_R(X)$ the peaks of these functions were fitted to Lorentzian curves with a background equal to 0.5, the other parameters (height, width, and position of the maximum) being unknown. Figures 7 and 8 show the results obtained in one of the ten simulated series. It can be observed that the relative half-widths of the two peaks are similar. Moreover, they are approximately equal to the corresponding ones for square-wave signals[13,14] and $\overline{m} = 0.1$ multiplied by 2.5. As occurs for square-wave signals and $\overline{m} = 0.1$, either in Q_C and Q_R, no rise appears to the left of the peak. The values of P were obtained from the maxima of fitted Lorentzians by using the relations $S_M = \pi / P$ and $X_M = 1/P$. By using the ten

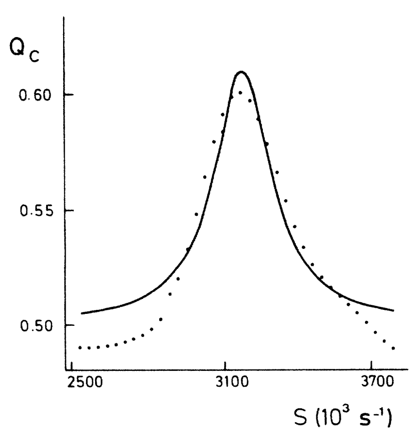

FIG. 7. Simulated values of $Q_C(S)$ (●) and fitted Lorentzian curve with a background equal to 0.5, for $\overline{m} = 0.1$.

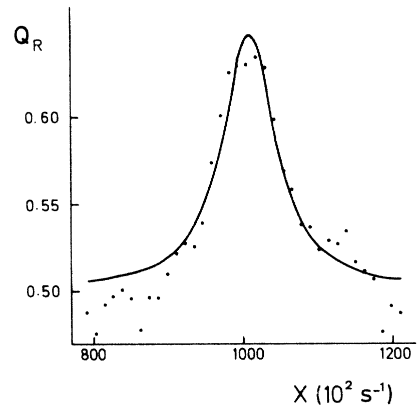

FIG. 8. Simulated values of $Q_R(X)$ (●) and fitted Lorentzian curve with a background equal to 0.5, for $\overline{m} = 0.1$.

simulated series of values of $Q_C(S)$ and $Q_R(X)$, the corresponding values of e_P were calculated.

To compare these errors with the ones involved in the determination of P from the intensity correlation function, we simulated values for $g^{(2)}(\tau)$. This was made by a program that counted the number $n(t,T)$ of ones generated by the above-mentioned Poisson random-number generator in successive time intervals $(t, t+T)$ and by using the well-known relation

$$g^{(2)}(lT) = \frac{(1/N) \sum_{i=1}^{N} n(t_i,T) n(t_i + lT, T)}{\left[(1/N) \sum_{i=1}^{N} n(t_i,T) \right]^2} , \quad l = 1,2, \ldots, 36 . \tag{8}$$

In this relation we used $T = 0.1P$ and a value of N that corresponded to the passage of 2000 seeded particles through the interference fringes. In each series the simulated values of $g^{(2)}(\tau)$ for 36 channels were fitted to the well-known function[15]

$$g^{(2)}(\tau) = A \exp(-\tau^2 / t_c^2)[1 + (V^2/2)\cos(2\pi\tau/P)] , \tag{9}$$

A being a constant to be fitted. From the values of P corresponding to the ten simulated series, e_P was evaluated.

The results for e_P from Q_C, Q_R, and $g^{(2)}$ are shown in Table I. It can be observed that the results from Q_C and Q_R are similar, whereas the results from $g^{(2)}$ are less accurate. This advantage of the techniques consisting of measuring Q_C or Q_R increases as $\overline{I}$ decreases. There are two reasons which allow us to make this assertion. First, the widths of Q_C and Q_R are proportional to $\overline{I}$ (they decrease as $\overline{I}$ does), as was seen from experimental results,

TABLE I. Values of e_P when P is obtained from $Q_C(S)$, $Q_R(X)$, or $g^{(2)}(\tau)$ under the same experimental conditions.

Technique	e_P (%)
$Q_C(S)$	0.50
$Q_R(X)$	0.52
$g^{(2)}(\tau)$	0.64

whereas the width of $g^{(2)}(\tau)$ [see Eq. (9)] does not depend on $\bar{I}$. Secondly, we can consider the fact that $W(\theta)$ is an oscillating probability, with a period P, which satisfies the normalization relation

$$\int_0^\infty W(\theta)d\theta=1 \tag{10}$$

for all values of $\bar{I}$. Bearing this in mind, it can be expected (as was verified for square-wave signals[13]) that

$$Q_C(S_M)=\int_0^\infty W(\theta)\cos^2(\pi\theta/P)d\theta \tag{11}$$

varies slowly as $\bar{I}$ varies. Thus, taking into account the expression[13] for the variance of $Q_C(S)$,

$$\mathrm{Var}Q_C(S)=(1/4N_\theta)[-1+4Q_C(S)-4Q_C^2(S)+Q_C(2S)]\ , \tag{12}$$

and the fact that the number of samples, N_θ, of the time interval θ that can be obtained in a fixed measurement time is proportional to $\bar{I}$, we obtain that $\mathrm{Var}Q_C(S_M)$ is approximately proportional to $\bar{I}^{-1}$. However, it is a well-known result[3] that for small intensities $\mathrm{Var}g^{(2)}(\tau)$ is proportional to $\bar{I}^{-2}$. Therefore, as the value of $\bar{I}$ decreases, noise in $g^{(2)}(\tau)$ increases more quickly than in $Q_C(S)$. A similar behavior can be expected for $Q_R(X)$ as occurs for square-wave signals.[14]

Bearing the above considerations in mind, we can conclude that for low-intensity LDV experiments where the velocity of a fluid is studied, the measurement of $Q_C(S)$ or $Q_R(X)$ may well constitute a more reliable technique than the usual intensity correlation technique. Furthermore, the measurement of $Q_R(X)$ can be made in a simpler way than the measurement of $g^{(2)}(\tau)$. In addition, we think that the techniques based on the measurement of Q_C or Q_R can be improved. For one thing, if the decay of $W(\theta)$ as θ increases is eliminated by using an appropriate filter function, the width of its Fourier transform must decrease. In this case the results obtained from Q_C and Q_R can be expected to improve. Further improvement could be obtained by finding a function which is more appropriate than a Lorentzian for fitting theoretical and experimental values of $Q_C(S)$ or $Q_R(X)$.

Finally, let us consider the fact that for small intensities the widths of $Q_C(S)$ and $Q_R(X)$ can be made smaller than the width of the Fourier transform of $g^{(2)}(\tau)$. Consequently, we can expect to obtain a better resolution of velocities in a turbulent flow by measuring $Q_C(S)$ or $Q_R(X)$ than when measuring $g^{(2)}(\tau)$.

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