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Line profiles of soft-x-ray laser gain coefficients

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Spectral profiles of gain coefficients at soft-x-ray wavelengths near 200 Å in laser-driven exploding foils and laser-produced magnetically confined plasma columns are shown not only to be influenced by thermal Doppler broadening, but very likely also by collisional narrowing of some lines and by Doppler effects from hydrodynamic turbulence. The multiply charged ions, $z = 24$ and 5, in these dense and relatively low-temperature plasmas are highly collisional, with ion-ion mean free paths of about 20 and 200 Å, respectively. They involve plasma flows with Reynolds numbers $\gtrsim 10^5$. Collisional narrowing and turbulent broadening may have combined to spoil the predicted gain of the $\lambda = 183$-Å selenium line in the foil experiment. Any turbulent flow must have a fundamental wavelength exceeding a critical size ranging from ~ 40 μm for the 206- and 209-Å selenium lines to ~ 90 μm for the 183-Å selenium line for observable gain to occur in the single-pass oscillator experiments.

The emphasis in theoretical evaluations[1] of various x-ray laser schemes based on transitions between bound states of multiply ionized atoms immersed in dense laser-produced plasmas has been on the kinetics of level populations caused by the different pumping schemes. It generally has been assumed that the spectral profiles of emission and absorption coefficients are essentially Gaussian with a width corresponding to the ion temperature, i.e.,

$$\Delta\omega_d = 2(2kT_i \ln 2/M_i)^{1/2} \frac{2\pi}{\lambda} , \quad (1)$$

where T_i and M_i are ion temperature and ion mass, respectively. (Natural line broadening tends to be negligible near line center.) This thermal Doppler broadening was assumed in the interpretations of the two recent experimental demonstrations[2,3] of amplification of stimulated soft-x-ray emission, and in the theoretical design[4] and in a different interpretation[5] of one of these successful experiments. The purpose of the present note is to point out that this assumption may have to be replaced by a more detailed evaluation of spectral line broadening, or even narrowing. Since the peak values of the emission and absorption cross sections are, at least approximately, inversely proportional to their profile widths, any modifications in the widths by some factor can be equally as important as a corresponding change in level populations.

The ion species, temperatures, and densities in the two experiments[2,3] are different. However, it is interesting to observe that not only are the wavelengths λ of the amplified lines about the same, i.e. (mainly), 206 and 209 Å in Ref. 2 and 182 Å in Ref. 3, but that also the mean free paths λ_{ii} of the ions, Se^{24+} and C^{5+}, are not all that different in the two experiments. These collisional mean free paths can be estimated following Spitzer[6] as

$$\lambda_{ii} \approx \frac{1}{8}\left(\frac{k_B T_i}{z^2 E_H}\right)^2 (N_i a_0^3)^{-1} (\ln\Lambda)^{-1} a_0 , \quad (2)$$

where $E_H = 13.6$ eV is the Rydberg, z the ionic charge, N_i the ion density, a_0 the Bohr radius, and $\ln\Lambda$ the Coulomb logarithm. For typical experimental parameters as listed in Table I and by use of $\ln\Lambda = 3$ and 2, respectively, this gives $\lambda_{ii} \approx 20$ and 190 Å in the selenium and carbon plasmas. (The sensitivity of λ_{ii} to temperature is less than one might think, e.g., doubling the temperature in the carbon plasma gives $\lambda_{ii} \approx 480$ Å.)

The actual values of Λ,

$$\Lambda = \frac{\rho_D}{r_L} \approx \left(\frac{k_B T_e}{4\pi N_e e^2}\right)^{1/2} \frac{3 k_B T_i}{e^2 z^2} , \quad (3)$$

where ρ_D is the electron Debye length and r_L the Landau length, are only ~ 14 and 9 for the conditions of the two experiments. Strictly speaking, Eq. (2) should therefore be replaced by a calculation including also nondominant terms.[6] It is also of interest that the ion-ion coupling parameters, $\Gamma = z^2 e^2 / r_i k_B T_i$, with $r_i = (4\pi N_i/3)^{-1/3}$, are ~ 0.9 and ~ 0.6 and that the ion Debye lengths are equal to or even smaller than the mean ion-ion separations, $r_i \approx 23$ and 62 Å. Inclusion of Debye screening by ions would therefore be inappropriate and it seems unlikely that the above estimates for λ_{ii} are off by much, making it fairly safe to conclude that the wavelengths of the amplified lines are larger than or about equal to the mean free paths of the ions involved.

This situation is conducive to collisional narrowing[7] of the

TABLE I. Plasma conditions in soft-x-ray laser experiments.

Element	Ion charge	Ion density	Electron density	Ion temp. (eV)	Electron temp. (eV)
Selenium[a]	24	2×10^{19} cm^{-3}	5×10^{20} cm^{-3}	400	900[b]
Carbon[c]	5	1×10^{18} cm^{-3}	5×10^{18} cm^{-3}	10	10

[a]From Refs. 2 and 4.
[b]The interpretation of Ref. 5 would suggest much lower temperatures of ~ 200 eV brought about by rapid radiative cooling.
[c]From Ref. 3, except that there is a temperature range of 10–20 eV given.

thermal Doppler profiles as had been pointed out before,[8] because the emitting ions may no longer be assumed as free streaming. In the limit $\lambda >> \lambda_{ii}$, the peak intensity of the normalized line profile is enhanced by a factor[7] $\lambda/2.8\lambda_{ii}$, and the peaks of the selenium-line profiles could therefore be higher by a factor ~ 3. However, only a small enhancement, if any, would be expected for the carbon line, especially if the temperature should be near the upper end of the range stated in Ref. 3, 10–20 eV.

A more accurate estimate of the collisional narrowing and central profile enhancement for $\lambda \approx \lambda_{ii}$ can be made following Rautian and Sobel'man,[9] who used kinetic theory methods, albeit assuming velocity-independent cross sections. They predict for weak collisions and small enhancements a factor $1+4(\ln 2)^{1/2}\nu_{ii}/3\sqrt{\pi}\Delta\omega_d$, where $\nu_{ii} = (3k_BT_i/M_i)^{1/2}/\lambda_{ii}$ is the ion-ion collision frequency. With $\nu_{ii} = 2.0\times10^{13}$ sec^{-1} and 0.8×10^{12} sec^{-1} for the selenium and carbon plasmas, respectively, and with use of Eq. (1), this factor is ~ 1.8 and ~ 1.07. (Note that $\Delta\omega_D$ in Ref. 9 is $\Delta\omega_d/2\sqrt{\ln 2}$.) Figure 1 in Ref. 9 suggests for the selenium plasma a factor 2.5 or even 2.7 if the weak collision model were replaced by a strong collision model.

Besides being subject to improved estimates of λ_{ii}, the conclusion regarding a significant narrowing of the selenium lines and a corresponding increase in their gain coefficients by a factor 2–3 must also be supported by some consideration of any collisional broadening associated with ion-ion collisions. While the velocity changes responsible for the collisional narrowing were estimated above by accounting only for the monopole-monopole term in the multipole expansion for the ion-ion interaction, any collisional broadening would have to come from monopole (perturbing ions in the ground state)–quadrupole (excited ions in $3p$ state) interactions and higher-order terms. The corresponding phase shifts can be estimated from the instantaneous frequency shifts by applying Eq. (60) of Ref. 10 to perturbations caused by ions of charge z, namely,

$$\eta \approx \frac{\Delta\omega\rho}{v} \approx \frac{e^2 z}{\hbar\rho^2 v}\frac{36a_0^2}{(z+1)^2}(2-3m^2) , \tag{4a}$$

where m is the orbital magnetic quantum number of the $3p$ electron ($n=3$, $l=1$). For the 90° Coulomb scattering impact parameter, $\rho = r_L = e^2z^2/3k_BT$ and by use of $v \approx (3k_BT/M_i)^{1/2}$, these phase shifts are

$$\eta_{90^\circ} \approx \left(\frac{M_i}{m_e}\right)^{1/2}\left(\frac{3k_BT_i}{2E_H}\right)^{3/2}\frac{36(2-3m^2)}{z^3(z+1)^2} , \tag{4b}$$

i.e., ~ 1 and -0.5 for $m=0$ and $m=\pm1$, respectively. Although actual phase shifts are somewhat smaller because of the Coulomb repulsion, by a factor $\lesssim 2$ for $\rho \gtrsim r_L$, they are nevertheless large enough to compromise the collisional narrowing. (See Ref. 9, Sec. 7 for a discussion of this interesting effect.) However, when the spin-orbit interactions are included, states with different m values are coupled such that for $3p_{1/2}$ electrons the quadrupole interaction phase shifts cancel, while no substantial modifications occur for the $3p_{3/2}$ levels. (The phase shifts are all equal in magnitude, which can be shown by use of appropriate Clebsch-Gordan coefficients.)

Of the three selenium lines of interest, only the λ 183-Å line has an upper level involving a $3p_{1/2}$ electron. It is therefore the only line subject to the collisional narrowing discussed above. (Resonance broadening due to dipole-dipole interactions between its lower state and the neonlike ground-state ions is negligible.) To summarize, for the 183-Å line the original Doppler width of 1.6×10^{13} sec^{-1} from Eq. (1) is effectively reduced to $\sim 6\times10^{12}$ sec^{-1}, to be compared with a radiative width[4] of $\sim 3\times10^{12}$ sec^{-1} and an electron collisional width[4] (including deexcitation) of $\sim 1\times10^{12}$ sec^{-1}. After convolution of the (near) Gaussian and Lorentzian profiles one thus finds a ratio of the peak intensities of 2:1 for the normalized line profiles of the 183 Å and the 206- or 209-Å lines. Any previously calculated gain coefficients for the 183-Å line should therefore be multiplied by this factor, relative to the values for the other two selenium lines, and the disagreement for this line with experiment[2] becomes even larger if only the above processes are considered.

However, the plasmas in question have high flow velocities and small viscosities so that hydrodynamic turbulence cannot be ignored. In the selenium experiment, flow speeds are about 1×10^7 cm/sec so that the Reynolds number

$$R = \frac{vd}{\eta_k} \approx \frac{\nu_{ii}vd}{v_i^2} \approx \frac{d}{\lambda_{ii}}\frac{v}{v_i} , \tag{5}$$

where η_k is the viscosity and ν_{ii} the ion-ion collision frequency, is extremely large, namely, $R \gtrsim 1.5\times10^5$ for characteristic macroscopic dimensions $d \gtrsim 100$ μm. Turbulence must therefore be expected, leading to random velocities of the order of the flow speed $v \approx 3v_i$, where v_i is the characteristic ion thermal speed $v_i = (2k_BT_i/M_i)^{1/2}$. As to possible reasons for turbulence, imperfections in the foils and inhomogeneities in the driving laser come to mind which, especially if reinforced by self-focusing and filamentation[11] of the laser beam, would cause shear in the flow-velocity field. This would in turn give rise to Kelvin-Helmholtz instabilities growing on a time scale of order

$\Delta x/\Delta v \gtrsim 100$ psec for microscopic spatial scales $\gtrsim 10$ μm and velocity differences $\lesssim 10^7$ cm/sec. This is fast enough for turbulence to develop, especially since the flow velocity used here is an average value. It is also interesting to note that the electron mean free path is ~ 1 μm, which should set a lower limit for the spatial scale.

In the carbon experiment, irradiance and geometry would suggest even larger flow speeds, namely, to judge by experience[12] with freely expanding, laser blow-off plasmas, $v \approx 3 \times 10^7$ cm/sec. However, interactions with the 90-kG magnetic field are likely to reduce this speed until the kinetic energy density and magnetic pressure are about equal. This argument gives $v \approx 6 \times 10^6$ cm/sec. For $d \approx 1$ mm, the Reynolds number is then $R \approx 2.5 \times 10^5$ in this case (for $v_t \approx 1.2 \times 10^6$ cm/sec). One might think that the strong magnetic field would smooth this flow, but the ion Larmor radius, $r = M_i v_i c/eB \approx 1.5 \times 10^{-2}$ cm, is almost 10^4 times the ion-ion mean free path. Therefore, the ion motion is not much affected by the field, at least not for low-frequency turbulence in which local charge neutrality can be assumed. (The electrons are strongly magnetized in this plasma.) Again, hydrodynamic turbulence must therefore be expected, with fluctuating velocities probably approaching the flow speed, i.e., $v \approx 5 v_t$ in this case.

In both experiments, Doppler shifts between lines emitted by various eddies can therefore be considerably larger than the profile widths of the local gain coefficients. In accordance with the estimates presented here, the inclusion of turbulence would lead to reductions in calculated average gains by factors 3–5. On the other hand, the larger widths of the resonance lines caused by the turbulence would reduce the trapping of the lower laser levels. This effect should restore some of the calculated gain lost by the turbulent Doppler broadening of the gain coefficient profiles.

For the $\lambda = 183$-Å line in the selenium experiment the local profile, due to collisional narrowing and turbulent Doppler broadening, may be as much as a factor of 6 narrower than the average (along the line of sight) profile, to be compared with a factor 3 for the other selenium lines. Initially, regions with gain at a certain frequency near the 183-Å line (within the width of the local line profile) will therefore be small, say of order l_1, and far apart, say, by $L \approx 6 l_1$, where L corresponds to the fundamental wavelength of the turbulent flow and where the factor 6 is required to ensure a nearly constant frequency of maximum gain in the region of size l_1. On assumption of a pill box of diameter and height l_1, a diffraction-limited beamlet emerging along its axis would suffer a reduction in intensity $\Delta I_d/I \approx -4\lambda_1 L/l_1^2$ before reaching the next region with substantial gain at this frequency. (The beam diameter after traversing L is $\sim l + 2\lambda L/l$, i.e., the fractional increase in the cross-sectional area is $\sim 4\lambda L/l^2$.) If this loss is more than the relative increase $\Delta I_g/I \approx + g_1 l_1$ in the region of nearly constant gain at the particular frequency, such a beamlet cannot grow to eventually coalesce with other beamlets to form a more macroscopic beam no longer subject to these diffraction losses and describable by an average gain coefficient based on the turbulent Doppler broadening. (This description would of course also apply for a laser amplifier, if its input-beam diameter is larger than L.)

A necessary condition for observable gain is $\Delta I_d + \Delta I_g > 0$ or

$$g l > 4\lambda L/l^2 , \tag{6}$$

which with $L = 6 l_1$ results in

$$L > 12(6\lambda_1/g_1)^{1/2} \tag{7a}$$

for the 183-Å line. With $g_1 = 20$ cm^{-1}, i.e., the value predicted in Ref. 4 multiplied by 2 to account for collisional narrowing, one obtains $L > 89_0$ μm. The corresponding condition for the 206- and 209-Å selenium lines, i.e., with $L = 3 l_2$, is

$$L > 6(3\lambda_2/g_2)^{1/2} . \tag{7b}$$

Here the local gain coefficient should be ~ 3 times the measured value,[2] because of the turbulence, i.e., ~ 15 cm^{-1}. This value is in very good agreement with more recent calculations,[13] in which population of the upper laser levels by dielectronic recombination is explicitly accounted for, as is the above value for g_1, unless the neonlike ground state is severely depleted. Using $g_2 = 15$ cm^{-1}, one has $L > 39$ μm as a necessary condition for observable gain for the 206- and 209-Å lines.

For turbulent structures with, say, a vortex period near 50 μm, the combination of collisional narrowing for some selenium lines and significant Doppler shifts or broadening due to random flow velocities for all lines therefore can explain why the originally predicted best candidate for lasing was not observed.[2] On the other hand, should smoother driving laser energy distributions and more ideal foils result in larger scale turbulence, say, $L > 89$ μm, the 183-Å line should also lase, albeit with an observable gain that does not benefit from the collisional narrowing as long as turbulent Doppler broadening is dominant. According to Ref. 13, this observable gain would at most be equal to that for the other lines, and according to Ref. 5 be significantly smaller, should population of the upper laser levels by recombination be even more important because of significantly reduced temperatures.

On return to the carbon experiment, the conclusion is that collisional narrowing is probably not important, whereas turbulent Doppler broadening is likely to be very large. As to other line-broadening processes in this experiment, broadening caused by electron-ion collisions for the C VI line can be estimated[10,14] to be near 0.5×10^{12} sec^{-1}, whereas ion-produced fields give a Stark width[14] of about 3×10^{12} sec^{-1}, to be compared with a thermal Doppler width of $\Delta\omega_d \approx 7 \times 10^{12}$ sec^{-1} at 10 eV. The Stark width (which was obtained from the calculation[14] for C VI $n = 3 \rightarrow 1$ transitions by assumption of the same scaling as for He II $n = 3 \rightarrow 2$ to $n = 3 \rightarrow 1$ transitions[10]) may actually be somewhat larger because ion-dynamical corrections[15] to the quasistatic profiles were neglected. But then the fine-structure splitting was neglected as well, which amounts to $\sim 5 \times 10^{13}$ sec^{-1} and is mostly due to splitting of the $n = 2$ levels. The $n = 3$ interval for the strongest line is an order of magnitude smaller so that the Stark width of its components should not be much reduced by this splitting. On consideration of all of this, ion-ion interactions (mostly monopole-dipole in this case) would permit for the carbon experiment a collisional narrowing by at most a factor 2, if the actual mean free path were much shorter than estimated above. Since the temperature can hardly be lower, the density would have to be almost ten times higher than assumed in the estimate of ion broadening. Collisional narrowing is therefore not likely for the carbon line in any case.

However, should the turbulence indeed be saturated, as estimated above, regions with gain at a given frequency would have a size $l_3 \approx L/5$, where L is the fundamental wavelength of the turbulence. For observable gain, L must fulfill

$$L > 10(5\lambda/g)^{1/2} , \tag{7c}$$

or $L > 55$ μm, by use of a local gain coefficient of 30 cm^{-1}, i.e., five times the measured gain coefficient.[3]

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