# Tests of the discrete symmetries C, P, and T in one-photon transitions of positronium

R. S. Conti, S. Hatamian, and A. Rich

Department of Physics, University of Michigan, Ann Arbor, Michigan 48109 (Received 8 July 1985; revised manuscript received 21 November 1985)

We examine one-photon transitions in positronium (Ps) as possible tests of the discrete symmetries of C (charge-conjugation) and/or P (parity) and/or T (time-reversal) invariance. We discuss two general classes of experiment. The first class consists of direct searches for transitions forbidden by a given symmetry. The second class is composed of experiments that search for an asymmetry in the rate of an allowed transition upon reversal of an externally controllable vector quantity, such as an applied magnetic field **B** or rf photon spin  $S_{\gamma}$ . For a given symmetry, we compare limits on symmetry-violating mixings, which can reasonably be expected from these Ps experiments, with limits placed by existing atomic experiments. We conclude from this analysis, that the most promising experiments in Ps are those that search for C violation with no P violation.

#### I. INTRODUCTION

Positronium (Ps) is the only accessible leptonic system that has eigenstates of C (charge conjugation) and CP(charge conjugation and parity) as well as of P (Fig. 1).<sup>1,2</sup> As such, it offers a unique opportunity to search for violations of these discrete symmetries.<sup>3</sup> Many tests of the discrete symmetries C, P, and T have been made in other systems,<sup>4</sup> but these tests do not preclude measurable symmetry violations in Ps, because comparison between experiments, especially those involving markedly different energy scales, is highly model dependent. In particular, models that exhibit symmetry violations predominantly or ex-



FIG. 1. The theoretical values for the n = 1 and n = 2 energy levels of positronium (in GHz). Each state is labeled by  $n^{2S+1}L_{JPC}$ , the usual spectroscopic notation with the parity, *P*, and charge-conjugation, *C*, eigenvalues appended.

clusively in the leptonic sector can usefully be investigated in Ps.

In this Brief Report we present a general analysis of onephoton optical and microwave transitions of the form  $Ps \rightarrow Ps' + \gamma$  (stimulated emission) or  $Ps + \gamma \rightarrow Ps'$  (absorption), as tests of the discrete symmetries C and/or P and/or T. The precision that can be attained in these experiments is compared with results already obtained in other lowenergy experiments. We conclude that the most promising experiments in Ps are those which search for C violation without P violation (CP nonconservation). Finally, these tests are contrasted with the observation of CP nonconservation in the  $K^0, \overline{K}^0$  system.

### II. THEORETICAL ANALYSIS OF SYMMETRY TESTS IN ONE-PHOTON TRANSITIONS OF Ps

We examine two general kinds of experiments to test for symmetry violations: (A) searches for symmetry-forbidden transitions in the absence of external electric and magnetic fields and (B) measurements of asymmetries, described below, each due to the interference of a symmetry-violating amplitude,  $\mathfrak{M}_x$ , with an electromagnetic, symmetry-conserving amplitude  $\mathfrak{M}_{y}$ . We consider three different *one*-photon transitions: (1)  $1^{3}S_{1} \rightarrow 2^{3}S_{1}$   $(H_{\mathcal{CF}})$ , (2)  $n^{3}S_{1} \rightarrow 2^{1}S_{0}$  $(H_{C^{\not\!p}})$ , and (3)  $n^{3}S_{1} \rightarrow 2^{1}P_{1}$   $(H_{\not\!QP})$ . Each of these transitions has a relatively long-lived initial  $n^{3}S_{1}$  state (n=1 or 2), and is sensitive to a different symmetry-violating Hamiltonian (H) which can mix an intermediate state into the initial or final state and cause the transition to become allowed. We have introduced here a notation in which the symmetry properties of H are specified by its subscripts C, P, and T. The Hamiltonian is odd (even) under a given discrete symmetry if there is a slash (no slash) through the corresponding symbol and it is composed of an unspecified linear combination of odd and even terms if the symbol is absent.

In a type-A measurement, a search is made for one of the above symmetry-violating transitions, resonant at the appropriate photon frequency. In analyzing the symmetry properties of these transitions, we note that the photon has

C = -1 and thus a given transition unambiguously does or does not conserve C. However, since the photon can bring in or carry away an arbitrary amount of angular momentum, the parity change depends upon the multipolarity of the transition. Thus, the observation of a given transition could be attributed to either of two interactions with opposite Pproperties and different intermediate states. For example, transition 3 could be due to  $H_{\ell P}$  mixing some of the  $2^{3}P_{1}$ intermediate state into the final state making an  $E_1$  transition possible, or  $H_{\ell \ell}$  could mix the  $2^{1}S_{0}$  state into the final state allowing an  $M_1$  transition. Since for plane-wave excitation, allowed  $E_1$  transitions have transition rates at least  $O(\alpha^{-2})$  higher than  $M_1$  and  $E_2$  transitions, a measurement of each of the transitions 1, 2, and 3 is most sensitive to the  $E_1$ -associated interaction Hamiltonian. We estimate that, with presently available technology, type-A measurements can yield values or limits for  $\langle 2^{3}S_{1}|H_{\ell P}|2^{3}P_{1}\rangle$ ,  $\langle 2^{1}S_{0}|H_{CP}|2^{3}P_{0}\rangle$ , and  $\langle 2^{1}P_{1}|H_{CP}|2^{3}P_{1}\rangle$  with uncertainties at the 10-, 3-, and 1-MHz levels, respectively.

In a type-*B* measurement an asymmetry  $A_{x,y}^{t} [A_{x,y}^{t} \equiv (\lambda_{+} - \lambda_{-})/(\lambda_{+} + \lambda_{-})]$  in the transition rate  $\lambda$  ( $\lambda \propto |\mathfrak{M}_{x}^{t} + \mathfrak{M}_{y}^{t}|^{2}$ ) of transition t (t = 1, 2, or 3) is measured. Here + (-) indicates that one of the externally controllable vectors in the experiment, i.e., the rf photon momentum **k**, the photon spin  $\mathbf{S}_{y}$ , the static electric field **E**, or the magnetic field **B**, is in the normal (reversed) direction. The symmetry-forbidden  $E_{1}$  transition amplitude  $\mathfrak{M}_{x}$  becomes allowed if  $H_{x}$  ( $x = \mathcal{OP}$ ,  $\mathcal{OP}$ , or  $\mathcal{OP}$ ) can mix an appropriate intermediate state into the initial or final state. For example, in perturba-

tion theory for transition 3 we get

$$\mathfrak{M}_{\ell P}^{3} = \frac{\langle 2^{1}P_{1} | H_{\ell P} | 2^{3}P_{1} \rangle \langle 2^{3}P_{1} | E_{1} | 2^{3}S_{1} \rangle}{E(2^{1}P_{1}) - E(2^{3}P_{1})}$$

The electromagnetic  $E_1$  or  $M_1$  amplitude  $\mathfrak{M}_y^t$  can exist either in the absence of external fields (y=0), e.g.,  $\mathfrak{M}_0^2 = \langle 2^1 S_0 | M_1 | 2^3 S_1 \rangle$ , through intermediate state mixing by a magnetic field (y=B), by an electric field (y=E), or by both (y=EB). Asymmetries  $A_{xy}^t$  arising from pairs of amplitudes  $\mathfrak{M}_x^t, \mathfrak{M}_y^t$ , are listed in terms of the externally controlled vectors in column 2 of Table I. In column 4 we present order of magnitude estimates of the precision  $(\sigma | \langle H_x \rangle |)$  attainable with presently available technology for each symmetry-violating amplitude.

The C, P, and T properties of an interaction which cause a B-type asymmetry are specified by the symmetry properties of the external vectors and are listed in column 3. It is important to note that the T properties can also depend on the phase of the energy denominators in amplitudes  $\mathfrak{M}_x$  and  $\mathfrak{M}_y$ . The energy denominators for asymmetries  $A_{xy}^1$  or  $A_{xy}^2$ are all predominantly real and do not change the T properties derived from the external vectors. The asymmetry,  $A_{dP,B}^2$ , however, is evaluated near the  $2^1P_{10}$  and  $2^3P_{10}$  level crossing where the imaginary part of the energy difference (due to the width of the  $2^3P_{10}$  state) is comparable to the real part. To analyze this situation correctly,  $\mathfrak{M}_{dP}^2$  must be calculated using second-order time-dependent perturbation theory.<sup>5,6</sup> This calculation yields

$$A_{\ell P,B}^{3} = \frac{\lambda_{+} - \lambda_{-}}{\lambda_{+} + \lambda_{-}} = +2.3(\hat{\mathbf{B}} \cdot \hat{\mathbf{S}}_{\gamma}) \left\{ \frac{[\Gamma(2^{3}P_{1})/2] \mathrm{Im} \langle 2^{1}P_{10} | H_{\ell P f} | 2^{3}P_{10} \rangle}{[h\nu + E(2^{3}P_{1}) - E(2^{3}S_{1})]^{2} + [\Gamma(2^{3}P_{1})/2]^{2}} - \frac{[h\nu + E(2^{3}P_{1}) - E(2^{3}S_{1})] \mathrm{Re} \langle 2^{1}P_{10} | H_{\ell P f} | 2^{3}P_{10} \rangle}{[h\nu + E(2^{3}P_{1}) - E(2^{3}S_{1})]^{2} + [\Gamma(2^{3}P_{1})/2]^{2}} \right] .$$
(1)

where  $\Gamma(2^{3}P_{1})$  is the width of the  $2^{3}P_{1}$  state and +(-) indicates that  $\hat{\mathbf{B}}$  and  $\hat{\mathbf{S}}_{y}$  are parallel (antiparallel). Observe now,

TABLE I. Asymmetries of interest are listed in terms of the externally controlled vectors in each experiment; photon spin  $S_y$ , photon direction k, electric field E, and magnetic field B. A pair of amplitudes  $\mathfrak{M}_x^t$  and  $\mathfrak{M}_y^t$  interfere, producing an asymmetry  $A_{x,y}^t$  defined by  $A_{x,y}^t \equiv (\lambda_+ - \lambda_-)/(\lambda_+ + \lambda_-)$ , where  $\lambda_+$  ( $\lambda_-$ ) is the transition rate with one of the listed external vectors in the normal (reversed) direction. Also listed is a state-mixing interaction  $H_x$  which can give rise to the listed asymmetry, along with an estimate of the precision  $\sigma |\langle H_x \rangle|$  attainable in such a measurement using available technology.

Transition	Asymmetry	H <sub>x</sub>	Precision $(\sigma  \langle H_x \rangle )$
$1.  1^{3}S_{1} + \gamma \rightarrow 2^{3}S_{1}$	Advecs S · E	Hdat	~ 10 MHz
	$A_{\mathcal{C}\mathcal{P},B}^{1} \propto \mathbf{k} \cdot \mathbf{B}^{\mathrm{a},\mathrm{b}}$	Η <sub>¢FT</sub>	~ 10 MHz
$2.  2^{3}S_{1} + \gamma \rightarrow 2^{1}S_{0}$	$A_{CF,EB}^{2} \propto \mathbf{E} \cdot \mathbf{B}$	H <sub>C</sub> #1	~3 MHz
	$A_{C\not\!\!\!\!/}^2 \otimes \mathbf{S}_{\gamma} \cdot \mathbf{k}$	$H_{C \not = T}$	$\sim$ 3 MHz
3. $n^{3}S_{1} \rightarrow 2^{1}P_{1} + \gamma$	$A_{\not CP,B}^3 \propto \mathbf{S}_{\gamma} \cdot \mathbf{B}^{c}$	HQPT	$\sim 100  { m KHz}$
<i>n</i> = 1, 2	$A_{\mathcal{Q}P,B}^{1} \propto \mathbf{S}_{\gamma} \cdot \mathbf{B}^{c}$	H∉PT	$\sim 100 \text{ KHz}$

<sup>a</sup>This asymmetry measurement was first proposed in footnote b but was erroneously thought to reverse sign with circular polarization of the exciting radiation.

<sup>b</sup>W. Bernreuther and O. Nachtmann, Z. Phys. C 11, 235 (1981).

<sup>c</sup>A level crossing experiment with externally variable T properties (see Sec. II).

ł

that if  $H_{\mathscr{L}P}$  is Hermitian (i.e., it does not result from a *CP* violation in an external decay channel), then

 $\langle 2^{1}P_{10}|H_{dPt}|2^{3}P_{10}\rangle$ 

is purely imaginary and

 $\langle 2^{1}P_{10}|H_{\ell PT}|2^{3}P_{10}\rangle$ 

is purely real. This is reflected in their symmetry assignments in Table I and in Eq. (1). The *T*-odd and *T*-even terms, respectively, can be distinguished by their resonant and dispersive dependences on  $\nu$ .

## III. LIMITS ON SYMMETRY VIOLATIONS FROM OTHER ATOMIC EXPERIMENTS

In this section we compare the Ps symmetry-violation experiments, detailed in Sec. II, with other experiments that test the same symmetries. Since the energy and distance dependences of interactions can be quite model dependent, we restrict our analysis here to atomic systems, which are all of the same approximate size. Moreover, we only consider symmetry violations manifested through the mixing of atomic states of opposite symmetry, rather than through a basic electron-photon interaction independent of formation of a bound state. We analyze three types of direct experiments: heavy-atom parity (HAP) tests, static electric dipole moments (EDM's), and Ps charge-conjugation tests, which

$$|\langle 2^{3}S_{1}'|\mathbf{d}|2^{3}S_{1}'\rangle| = |\langle 2^{3}S_{1}|\mathbf{d}|2^{3}P_{1}\rangle \frac{\langle 2^{3}P_{1}|H_{CPT}|2^{3}S_{1}\rangle}{E(2^{3}S_{1}) - E(2^{3}P_{1})} +$$

Although a completely model-independent comparison is not possible, we consider it unlikely that violations of the form  $\langle H_{\mathcal{CFT}} \rangle$  and  $\langle H_{\mathcal{CFT}} \rangle$  at the 10-MHz level in Ps could be consistent with the null result in Xe.

(iii) Ps C-violation tests. Direct atomic tests of interactions of the form  $H_{\not CP}$  (which includes  $H_{\not CPf}$  and  $H_{\not CPT}$ ) have only been made in Ps.<sup>13</sup> These were searches for the C-violating decays<sup>14, 15</sup> 1<sup>1</sup>S<sub>0</sub>  $\rightarrow$  3 $\gamma$  and 1<sup>3</sup>S<sub>1</sub>  $\rightarrow$  4 $\gamma$  which placed limits on the branching ratios

$$R_B(1^1S_0) = \lambda(1^1S_0 \to 3\gamma)/\lambda(1^1S_0 \to 2\gamma) < 2.8 \times 10^{-6},$$

and

$$R_B(1^{3}S_1) = \lambda(1^{3}S_1 \rightarrow 4\gamma)/\lambda(1^{3}S_1 \rightarrow 3\gamma) < 8 \times 10^{-6}.$$

Such C-violating transitions cannot occur as an admixture of the initial state with another Ps state via an  $H_{\not CP}$  mixing since there is no Ps state that has the opposite C and the same P and J as either the  $1^{3}S_{1}$  or the  $1^{1}S_{0}$  state. Although these branching-ratio experiments do test models which include C violation in the basic  $e -\gamma$  interaction,<sup>16,17</sup> they do not place any limit on an  $H_{\not CP}$  bound-state mixing. Moreover, since Ps is still the only available atom with eigenstates of C, no direct symmetry test limiting  $H_{\not CP}$  mixing has been made in any atomic system.

We turn now to indirect measurements, those for which an energy splitting  $\Delta \nu$  or decay rate  $\lambda$  is precisely calculable from QED and for which the presence of the  $H_{d'P}$  mixing would shift  $\Delta \nu$  or  $\lambda$  by an experimentally detectable amount. For example, in Ps, measurements and theoretical are sensitive to interactions of the form  $H_{pT}$ ,  $H_{pT}$ ,  $H_{pT}$ , and  $H_{pP}$ , respectively. Symmetry-violating interactions can also be manifested indirectly as deviations of energy splittings or decay rates from their values calculated from quantum electrodynamics (QED). Such effects are considered later in this section.

(i) Heavy atom parity tests. Matrix elements of the form  $\langle H_{FT} \rangle$  (which includes  $H_{EFT}$  and  $H_{CFT}$ ) have been measured in HAP tests.<sup>7,8</sup> In this case there exists a well verified theory, the standard electroweak theory, which predicts the measured results. From the calculations of Bernreuther and Nachtmann,<sup>9</sup> who use the standard electroweak model to compute the  $H_{EFT}$  mixing in Ps, we obtain a value for  $|\langle 2^{3}S_{1}|H_{EFT}|2^{3}P_{1}\rangle|$  of  $2.16 \times 10^{-4}$  Hz. This is far below the sensitivity listed in Table I. The HAP tests are predominantly sensitive to short-range electron-nucleon interactions (e - N), but are also sensitive to electron-electron interactions (e - e) (e.g., the effect of e - e is calculated to be about 2% of that of e - N in Cs).<sup>10</sup> We conclude that experiments in Ps cannot compete with HAP tests in searching for amplitudes of the form  $\langle H_{EFT} \rangle$  and  $\langle H_{CFT} \rangle$ , even in the context of a purely leptonic model.

(ii) Static electric dipole moments. Measurements of atomic EDM's<sup>11</sup> place limits on symmetry violations of the form  $H_{PT}$  (which includes  $H_{QPT}$  and  $H_{CPT}$ ). For example, a recent result<sup>12</sup> in Xe gives  $\langle d \rangle_{Xe} = (-0.3 \pm 1.1) \times 10^{-26} e$  cm. In comparison, if a measurement of  $A_{CP,E}^{1}$  (Table I) yielded  $|\langle 2^{3}P_{1}|H_{CPT}|2^{3}S_{1}\rangle| = 10$  MHz, the EDM in the  $2^{3}S_{1}'$  state would be

$$\frac{\langle 2^{3}S_{1}|H_{CPT}|2^{3}P_{1}\rangle}{E(2^{3}S_{1}) - E(2^{3}P_{1})} \langle 2^{3}P_{1}|\mathbf{d}|2^{3}S_{1}\rangle = 4 \times 10^{-11} \ e \ \mathrm{cm} \ . \tag{2}$$

calculations of  $E(1^{3}S_{1}) - E(1^{1}S_{0})$ ,  $E(2^{3}S_{1}) - E(2^{3}P_{2})$ ,  $\lambda(1^{3}S_{1})$ , and  $\lambda(1^{1}S_{0})$  have each been made, but none of the states involved has the same P and J but opposite C with respect to any other Ps state. Thus again, no  $H_{CP}$  state mixing can occur. The only other atoms where sufficiently precise theoretical calculations of energy splittings are available are H and He. Here the requirement that the mixed states have opposite C eigenvalue no longer applies, and states of the same P and the same angular momentum (J or F) are relevant. In He the appropriate states have such large energy splittings that no useful limit is placed. In H, however, the matrix element

$$\langle 2^2 P_{1/2}(F=1) | H_{dPT} | 2^2 P_{3/2}(F=1) \rangle$$

must be less than approximately 20 MHz if the experimental value<sup>18</sup> for the  $2^2S_{1/2} \rightarrow 2^2P_{3/2}$  transition frequency  $\nu = 9911.117(41)$  MHz is to agree with the QED theoretical value<sup>19</sup>  $\nu = 9911.160(13)$  MHz.<sup>20</sup> This limit only applies if the  $H_{\ell P}$  mixing is semileptonic in nature, since the mixing requires a proton spin flip. We conclude that no atomic experiment gives better limits than those which could be set by measurements of transition 3 or asymmetry  $A_{\ell P,B}^3$  especially in terms of a purely leptonic model of  $H_{\ell P}$ .

### IV. CP NONCONSERVATION: Ps COMPARED TO $K^0, \overline{K}^0$

In the 20 years since its discovery,<sup>21</sup> CP nonconservation has been observed only in the  $K^0, \overline{K}^0$  system and even there it has not been explained in any fundamental sense.

Several models<sup>22-24</sup> have been developed to explain the observed CP violations but evidence in favor of one model to the exclusion of all others is lacking. A second system that exhibits CP nonconservation would be invaluable in clarifying the nature of the interaction. Ps shares with the  $K^0, \overline{K}^0$ system the property of having (in the absence of CP violation) two adjacent states with the same J, the same P, and the opposite C. For Ps these are, as noted above, the  $2^{1}P_{1}$ and  $2^{3}P_{1}$  states with  $\Delta E = 1.8$  GHz, while in the  $K^{0}, \overline{K}^{0}$  system they are  $K_L$  and  $K_S$  with  $\Delta m - i\Delta\Gamma/2 = (0.85 + 0.89i)$ GHz. On the other hand, Ps differs from the  $K^0, \overline{K}^0$  system in that (i) it is a purely leptonic system, (ii) it is "diagonal," that is, the electron and positron are direct antiparticles to each other, and (iii) it is electromagnetically bound system. These differences, if CP nonconservation is seen in Ps, would place strong constraints on any CP-violating model. Finally, we note that, although the hadronic nature of currently popular models designed to explain CP nonconservation in  $K^0, \overline{K}^0$  implies that such models predict essentially unobservable CP-violating effects in Ps, no experimental result (e.g., from g-2 of the electron,  $\eta \rightarrow 3\gamma$ , or the  $K^0, \overline{K}^0$  system) interpreted in a model-independent fashion, precludes Ps from exhibiting a measurable CP violation.

#### V. SUMMARY

We have examined tests of the discrete symmetries C, P, and T in single-photon transitions of Ps. Limiting our analysis to interactions which are manifested as mixings between atomic states, we have compared these possible tests in Ps to existing tests in other atoms. We conclude that only in the case of interactions of the form  $H_{\ell P}$  can experiments in Ps (i.e., measurements of the transition matrix element  $\mathfrak{M}_{\ell P}^{3}$  or the asymmetry  $A_{\ell P,B}^{3}$ ) place more stringent limits than previous experiments. We are presently pursuing an experimental effort to search for the C-forbidden transition  $2^{3}S_{1} \rightarrow 2^{1}P_{1} + \gamma$  described by  $\mathfrak{M}_{\ell P}^{3}$ .

### ACKNOWLEDGMENTS

We would like to thank E. Fischbach, J. M.-Frere, W. L. Williams, and especially R. R. Lewis for useful discussions relating to *CP* violations in Ps. This work has been supported by National Science Foundation Grants No. PHY-8107573 and No. PHY-8403817 and a grant from the Office of the Vice President for Research of the University of Michigan.

- <sup>1</sup>T. Fulton and P. C. Martin, Phys. Rev. 95, 811 (1954).
- <sup>2</sup>T. Fulton, Phys. Rev. A 26, 1794 (1982).
- <sup>3</sup>R. S. Conti, R. R. Lewis, and A. Rich, Bull. Am. Phys. Soc. 28, 692 (1983).
- <sup>4</sup>Partial surveys of C, P, and T tests can be found in J. W. Cronin, Science 212, 1221 (1981); E. D. Commins and P. H. Bucksbaum, Weak Interactions of Leptons and Quarks (Cambridge Univ. Press, New York, 1983).
- <sup>5</sup>For similar calculations, see J. S. Bell, in Proceedings of the International Workshop on Neutral Current Interactions in Atoms, Cargese, 1979, edited by W. L. Williams (unpublished), p. 288; M. Sargent, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974), p. 28.
- <sup>6</sup>An accurate expression for m<sup>3</sup><sub>8</sub> was derived using an energy matrix diagonalization technique taken from M. L. Lewis and V. W. Hughes, Phys. Rev. A 8, 625 (1973); S. M. Curry, *ibid.* 7, 447 (1973).
- <sup>7</sup>M. A. Bouchiat and L. Pottier, in *Atomic Physics 9*, edited by R. S. Van Dyck, Jr. and E. N. Fortson (World Scientific, Singapore, 1984), p. 246.
- <sup>8</sup>P. S. Drell and E. D. Commins, Phys. Rev. Lett. 53, 968 (1984).
- <sup>9</sup>W. Bernreuther and O. Nachtmann, Z. Phys. C 11, 235 (1981).
- <sup>10</sup>M. A. Bouchiat and C. Bouchiat, J. Phys. (Paris) 35, 899 (1974).
- <sup>11</sup>D. A. Wilkening and N. F. Ramsey, Phys. Rev. A 29, 425 (1984).
- <sup>12</sup>T. G. Vold, F. J. Raab, B. Heckel, and E. M. Fortson, Phys. Rev. Lett. **52**, 2229 (1984).

- <sup>13</sup>Tests in normal atoms of interactions of the form  $H_{Pf}$  (includes  $H_{\ell Pf}$  as well as  $H_{CPf}$ ) are possible, but none has as yet been performed.
- <sup>14</sup>A. P. Mills and S. Berko, Phys. Rev. Lett. 18, 420 (1967).
- <sup>15</sup>K. Marko and A. Rich, Phys. Rev. Lett. 33, 980 (1974).
- <sup>16</sup>H. S. Mani and A. Rich, Phys. Rev. D 4, 122 (1971).
- <sup>17</sup>A. P. Mills, thesis, Brandeis University, 1967 (unpublished).
- <sup>18</sup>K. A. Safinya, K. K. Chan, S. R. Lundeen, and F. M. Pipkin, in *Precision Measurements and Fundamental Constants II*, edited by B. N. Taylor and W. O. Phillips (U.S. Government Printing Office, Washington, 1984), p. 127.
- <sup>19</sup>P. J. Mohr, Phys. Rev. Lett. 34, 1050 (1975).
- <sup>20</sup>This assumes a second-order energy shift in the  $2^{3}P_{3/2}$  state. For the interaction  $H_{CPT}$  (which violates *CPT*) a first-order energy shift is possible resulting in a correspondingly better limit:

 $\langle 2^2 P_{3/2}(F=1) | H_{CPT} | 2^2 P_{3/2}(F=1) \rangle \leq 50 \text{ KHz}$ .

- <sup>21</sup>J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Lett. 13, 138 (1964).
- <sup>22</sup>M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
- <sup>23</sup>L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964).
- <sup>24</sup>L. Wolfenstein and D. Chang, in *Intense Medium Energy Sources of Strangeness*, University of California at Santa Cruz, 1983, edited by T. Goldman, H. E. Haber, and H. F. W. Sadrozinski, AIP Conf. Proc. No. 102 (AIP, New York, 1983).