Brief Reports

Brief Reports are short papers which report on completed research which, while meeting the usual **Physical Review** *standards of scientific quality, does not warrant a regular article. (Addenda to papers previously published in the* **Physical Review** *by the same authors are included in Brief Reports.) A Brief Report may be no longer than 3½ printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Large-scale symmetry in the longitudinal correlation function of isotropic homogeneous fluid turbulence

Gerald Rosen
Department of Physics, Drexel University, Philadelphia, Pennsylvania 19104
(Received 13 January 1986)

It is observed that the Frenkiel-Klebanoff-Huang turbulence data for the longitudinal correlation function at grid Reynolds numbers from 12 800 to 81 000 are subsumed by the simple empirical expression $f=[1+(r/2L)]^{-3}$, where r denotes the separation distance between two points in the turbulent fluid flow and $L=L(t)$ is the integral scale. Hence, the longitudinal velocity correlation is rescaled by the self-similarity dilatation factor λ^{-3} under the separation-distance transformations $r\rightarrow\lambda r+(\lambda-1)2L$ at fixed t.

It has recently been shown that statistical-dynamical self-similarity must be featured in the free decay of inertia-dominated incompressible fluid turbulence, with the experimentally established decay law $u^2\propto t^{-6/5}$ and integral scale dependence $L\propto t^{2/5}$ following deductively and without any additive assumption from a Gaussian normal probability distribution over velocity fields at the initial instant $t=0$.[1] This symmetry property of incompressible fluid turbulence at large Reynolds numbers holds for values of the decay time t greater than the small time associated with the viscosity-dominant interval of time-evolution of an initially Gaussian normal probability distribution. Correspondingly, one expects the longitudinal correlation function[2] of isotropic homogeneous turbulence to manifest a related symmetry for values of the separation distance r greater than the small distance associated with viscosity-dominant dynamics. The purpose of the present paper is to point out that such a symmetry is indeed evident in the longitudinal correlation function according to the measurements of Frenkiel, Klebanoff, and Huang.[3] For grid-generated turbulence at Reynolds numbers UM/ν from 12 800 to 81 000 and decay times such that $L=0.65M$, the data reported by Frenkiel, Klebanoff, and Huang are subsumed by the simple empirical expression[4]

$$f=[1+(r/2L)]^{-3} \tag{1}$$

as shown by the comparison in Table I.

TABLE I. Comparison of experimental values for the longitudinal correlation function [Fig. 2 of Ref. 3 with the Taylor approximation $f=R(r/U)$] and values given by the empirical relation (1). Since the measurements in Ref. 3 were made at wind and water tunnel locations for which $L=0.65M$, the integral scale L in (1) is eliminated in favor of the mesh length M.

r/M	0	0.10	0.20	0.30	0.40	0.60
$f[=R(r/U)]$	1	0.80	0.65	0.52	0.45	0.32
f[by (1)]	1	0.800	0.651	0.536	0.447	0.320
r/M	1	1.60	2	2.40	2.80	3.20
$f[=R(r/U)]$	0.19	0.09	0.06	0.04	0.03	0.02–0.03
f[by (1)]	0.180	0.090	0.061	0.043	0.032	0.024

Under the self-similarity dilatation transformations[1] with $\mathbf{x}\rightarrow\lambda\mathbf{x}$ and $t\rightarrow\lambda^{5/2}t$ for all parameter values $\lambda>0$, one has $r\rightarrow\lambda r$, $u^2\rightarrow\lambda^{-3}u^2$, and $L\rightarrow\lambda L$; thus the correlation function (1) is invariant, while the two-point velocity correlation tensor[2] is rescaled by the factor λ^{-3}. The same rescaling of the longitudinal part of the velocity correlation tensor, i.e.,[2] $\langle u_1(\mathbf{x}+r\mathbf{e},t)u_1(\mathbf{x},t)\rangle=u^2(t)f(r,t)$ with $\mathbf{e}=(1,0,0)$, is realized by the purely spatial group of transformations at fixed t (hence, with u^2 and L invariant) under which

$$r\rightarrow\lambda r+(\lambda-1)2L \tag{2}$$

and therefore $f\rightarrow\lambda^{-3}f$ as a consequence of (1). This *separation-distance scaling symmetry* manifest in (1) is expressed equivalently for $f=f(r,t)$ by

$$f(\lambda r+(\lambda-1)2L,\ t)=\lambda^{-3}f(r,t)\quad\text{for all }\lambda>0\ . \tag{3}$$

Conversely, the scaling symmetry property (3) and the normalization condition[2] $f(0,t)=1$ imply that the longitudinal correlation function takes the form (1) with $L=L(t)$ identified as the integral scale.[4]

It should be observed that this separation-distance scaling symmetry appears to be distinct and not directly related to a possible Kolmogorov invariance[5] in the fine-scale (inertial subrange of wave numbers) at high Reynolds numbers.[6] Changing the distance between the two spatial points in the longitudinal velocity correlation in the indicated inhomogeneous fashion that brings in the integral scale additively, the effect of the separation-distance transformations (2) is identical to that of the self-similarity dilatation transformations,[1] which also rescale $\langle u_1(\mathbf{x}+r\mathbf{e},t)u_1(\mathbf{x},t)\rangle$ by the factor λ^{-3}.

[1]G. Rosen, Phys. Rev. A **32**, 2549 (1985). The initial value of the probability distribution over velocity fields involves the single constant c^2 with physical units of $(\text{length})^5/(\text{time})^2$; it follows that $u^2 = (\text{numerical constant})c^{4/5}t^{-6/5}$ and $L = (\text{numerical constant})\, c^{2/5}t^{2/5}$. Empirically one has [e.g., K. R. Sreenivasan, S. Tavoularis, R. Henry and S. Corrsin, J. Fluid Mech. **100**, 597 (1980)] $u^2 = 0.04U^{4/5}M^{6/5}t^{-6/5}$ and $L = 0.13U^{2/5}M^{3/5}t^{2/5}$, relations consistent with $c^2 = (\text{numerical constant})\,U^2M^3$.

[2]The longitudinal correlation function $f = f(r,t)$ is the scalar quantity which generates the two-point velocity correlation tensor of isotropic homogeneous incompressible turbulence,

$$\langle u_i(\mathbf{x}+\mathbf{r},t)u_j(\mathbf{x},t)\rangle = u^2\left[\left(f+\frac{1}{2}r\frac{\partial f}{\partial r}\right)\delta_{ij} - \frac{1}{2r}r_ir_j\frac{\partial f}{\partial r}\right] ,$$

where $u^2 = u^2(t) \equiv \frac{1}{3}\langle |u(\mathbf{x},t)|^2\rangle$ and $r \equiv |\mathbf{r}|$. In particular, for $\mathbf{r} = r\mathbf{e}$ with $\mathbf{e} \equiv (1,0,0)$, one obtains the longitudinal velocity correlation as

$$\langle u_1(\mathbf{x}+r\mathbf{e},t)u_1(\mathbf{x},t)\rangle = u^2 f .$$

[3]F. N. Frenkiel, P. S. Klebanoff, and T. T. Huang, Phys. Fluids **22**, 1606 (1979).

[4]Observe that formula (1) features the integral scale in a manner required by the general definition: $L = L(t) \equiv \int_0^\infty f(r,t)dr$. It is also noteworthy that formula (1) is consistent with the asymptotic dependence for large r, $\lim_{r\to\infty} r^3 f(r,t) = (\text{function of } t)$, predicted by G. Birkhoff [Commun. Pure Appl. Math. **7**, 19 (1954)] and P. G. Saffman [J. Fluid Mech. **27**, 581 (1967)].

[5]For example, F. H. Champagne, J. Fluid Mech. **80**, 67 (1978).

[6]It is clear however that the one-parameter separation-distance scaling symmetry is rooted in the inertia-dominated Navier-Stokes dynamics, and in this sense it shares a common origin with a possible Kolmogorov invariance. Maintaining the dilatation invariance symmetry which underlies self-similarity, the essentially correct analyses of nonlinear inertial transfer [R. H. Kraichnan, J. Fluid Mech. **47**, 525 (1971); **83**, 349 (1977); S. A. Orszag, *ibid.* **41**, 363 (1970); R. G. Deissler, Phys. Fluids **22**, 185, 1852 (1979)] should also imply the large-scale longitudinal correlation function symmetry described here.