# Comparison between active- and passive-cavity interferometers

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Both active- and passive-cavity interferometers are considered at present for displacement sensing in broadband gravitational-radiation detectors. In spite of an apparent difference between the noise sources that limit their performance, we show that active- and passive-cavity interferometers are of the same strain (or displacement) sensitivity if identical resonators and stored fields of equal intensity are assumed.

## I. INTRODUCTION

There is a strong belief that long-base-line laser interferometers will sooner or later achieve sensitivities that will allow detection of gravitational radiation (GR). Accordingly, large interferometers have been built and are currently undergoing a process of testing and upgrading[1-3] while very large ones are seriously being considered.[4,5] In order to achieve as long an effective base line as possible, the arms of these interferometers contain either optical delay lines or optical cavities. These are passive optical systems, since they receive light from an external source, usually an argon-ion laser.

Active-cavity systems that take advantage of the very high sensitivity of laser frequency to changes in resonator length are an alternative approach to GR detection.[6,7] A prototype gravitational-radiation detector employing an active-cavity displacement sensor has recently been constructed, with a noise level equivalent to displacements of $3\times10^{-15}$ cm/Hz$^{1/2}$, above 2 kHz.[8]

In the early development stages, there was hope that active-cavity detectors could be made more sensitive than passive-cavity detectors.[6] On the other hand, since the phase noise of laser light, due to spontaneous emission, is higher than the one due to shot noise that limits the performance of passive interferometers, it has been argued that active-cavity systems are intrinsically less sensitive than passive ones. We show in what follows that for fields of equal intensities stored within identical resonators, active- and passive-cavity interferometers are of the same displacement sensitivity, although the types of noise which limit their performance are apparently of different nature.

## II. ACTIVE-PASSIVE COMPARISON

Consider the interferometer geometry shown in Fig. 1, consisting essentially of two perpendicular Fabry-Perot cavities and a beam splitter. If an active medium is added to the resonators, they become lasers operating in a heterodyne configuration characteristic of an active-cavity interferometer.[8] If, on the other hand, the active medium is removed and the system is fed light from an external laser, the geometry of Fig. 1 corresponds to a passive-cavity interferometer of the type used for the gravitational-radiation detector[3] at the California Institute of Technology. For the sake of comparison, we shall assume a passive- and an active-cavity interferometer with identical Fabry-Perot cavities.

A parameter crucial to interferometer performance is the amount of light handled by the system. For convenience, we chose to describe this parameter by $I_{st}$, the intensity of the light stored within the resonators, integrated over the cross section of the beam. Active and passive systems will thus be compared under the assumption that they employ fields with the same $I_{st}$.

In an active-cavity interferometer, the laser beams of frequencies $\omega_1$ and $\omega_2$ combine at the beam splitter (see Fig. 1) and provide, after photodetection, a beat signal of frequency $\omega_B=\omega_2-\omega_1$. When the mirror spacings in the two lasers change by $\Delta L_1$ and $\Delta L_2$, the beat frequency changes by an amount $\Delta\omega_B=\omega(\Delta L_2-\Delta L_1)/L$, where $\omega$ is the mean value of $\omega_1$ and $\omega_2$, while $L$ is the mean value of $L_1,L_2$, the optical length of the laser resonators. Changes in $\omega_B$ are monitored by frequency demodulation. The intrinsic noise that limits active-cavity interferometer performance consists of laser frequency fluctuations due

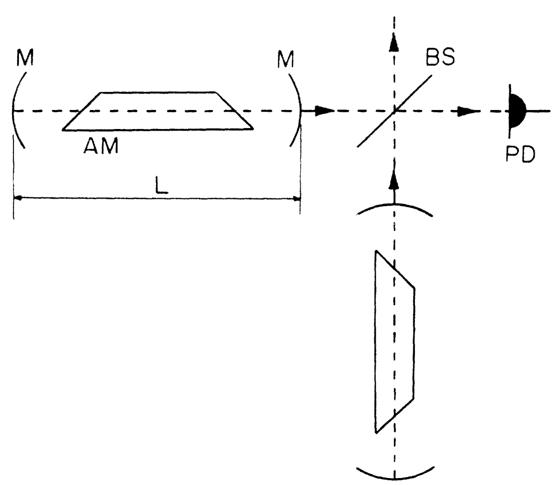


FIG. 1. Optical layout of the active-cavity interferometer. M, mirrors; AM, active medium; BS, beam splitter; PD, photodiode.

 

to spontaneous emission. The spectral density $\delta L^2$ of the smallest displacements detectable with an active-cavity interferometer[8] can be written as

$$\delta L_A^2 = \frac{\hbar\omega L^2}{I_{st} Q^2 (1 - R_1 R_2)} , \tag{1}$$

where the subscript $A$ stands for the active cavity, $Q$ is the quality factor of the resonators, and $R_1, R_2$ are the power reflectivities of the laser mirrors.

The limit to the measurement of small displacements with a Michelson interferometer is determined by the shot noise generated by the photon flux impinging on the photodetector. The spectral density of displacements equivalent to the shot noise is[9,10]

$$\delta L_P^2 = \frac{\hbar\omega}{2\eta I (\phi')^2} , \tag{2}$$

where the subscript $P$ stands for the passive system, $\eta$ is the quantum efficiency of the photodetector, $I$ is the total intensity of the light leaving the interferometer, and $\phi' = d\phi/dL$ is the sensitivity of the phase shift in each arm to changes in arm length.

Optimization of Eq. (2) is now carried out for an interferometer that contains a passive Fabry-Perot cavity in each arm. The sensitivity of this configuration is compared in Sec. III with that of a Michelson interferometer which employs delay lines.

Assume that one of the resonator mirrors has zero transmittance and an amplitude reflectivity $r$ such that $R_1 = r^2$. The fractional loss the beam experiences for each reflection on this mirror thus is $1 - r^2$. Further, assume that the coupling mirror has amplitude reflectivity $r_c$, such that $R_2 = r_c^2$ and that the losses are the same as for the high reflector. If we choose $r$ and $r_c$ real and negative, the amplitude transparency coefficient $t$ has to be taken purely imaginary (see, e.g., Ref. 11). Coupler transmittance thus is $T_c = -t^2 = r^2 - r_c^2$. Under steady-state conditions, the incident amplitude $A$, the outcoming amplitude $B$, and the amplitude $C$ of the stored field (see Fig. 2) are related as follows:

$$B = r_c A + tr\Phi C , \tag{3}$$

$$C = tA + r_c r \Phi C , \tag{4}$$

where $\Phi = \exp(i4\pi L/\lambda)$ is the phase factor corresponding to a full round trip in the resonator. Solving Eqs. (3) and (4) for $B$ and $C$ yields the output-to-input power ratio $\Theta$, the sensitivity of the phase to changes in arm length $\phi'$, and the relation between the intensities of the output beam and the stored field for the resonator with losses[12] operated as a reflector:

$$\Theta = \frac{I}{I_0} = \frac{(r_c - r^3)^2}{(1 - r_c r)^2} , \tag{5}$$

$$\phi' = \left[ \frac{4\pi}{\lambda} \right] \frac{r(r_c^2 - r^2)}{(r_c - r^3)(1 - r_c r)} , \tag{6}$$

$$I = \frac{(r_c - r^3)^2}{2(r^2 - r_c^2)} I_{st} . \tag{7}$$

Equations (5)–(7) are evaluated for an optical cavity

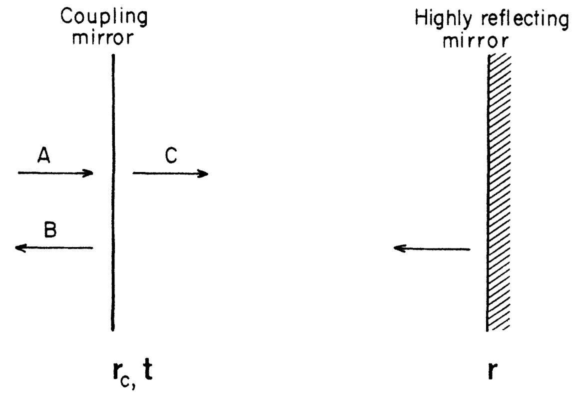


FIG. 2. Optical resonator geometry. $A$ and $B$ are the amplitudes of the incoming and outgoing fields, $C$ is the amplitude of the field transmitted by the coupling mirror, $t$ is the amplitude transmissivity of the coupling mirror, and $r_c, r$ are amplitude reflectivities.

operated at resonance ($\Phi = 1$), where $\phi'$ is the largest.

For given mirror losses, i.e., for given $r$, it is found that the coupler reflectivity that minimizes $\delta L_P^2$ is $r_c = r^3$. Using also the explicit form of the cavity quality factor $Q = 2\pi L/\lambda(1 - r_c r)$, the noise limit for an interferometer with arms consisting of passive optical resonators becomes

$$\delta L_P^2 = \frac{\hbar\omega L^2}{4\eta r^4 I_{st} Q^2 (1 - r^4)} . \tag{8}$$

Under the assumption that the active and the passive interferometer employ identical Fabry-Perot cavities and contain stored fields of the same intensity, comparison of Eqs. (1) and (8) yields

$$\frac{\delta L_A^2}{\delta L_P^2} = 2\eta , \tag{9}$$

where the fact that $r^4 \sim 1$ has been taken into account. In other words, the ultimate sensitivity of the interferometer is the same, irrespective of whether it employs active or passive optical resonators. The reason for this is that in both cases the noise level is determined by the stored energy, on one hand, and by the magnitude of the losses, on the other.

## III. COMPARISON BETWEEN PASSIVE CAVITY AND DELAY LINE

In an optical delay line, the light beam is repeatedly bounced back and forth between two highly reflecting mirrors of power reflectivity $R = r^2$. The noise level for an interferometer with arms consisting of delay lines is obtained from Eq. (2) by replacing $I = \Theta I_0$ and taking into account the fact that for $z$ reflections $\Theta = R^z$ and $\phi' = (2\pi/\lambda)(z+1)$:

$$\delta L_D^2 = \frac{\hbar\omega}{2\eta I_0 k^2 R^z (z+1)^2} , \tag{10}$$

where the subscript $D$ stands for the delay line and $k=2\pi/\lambda$.

For given $R$, the minimum value of $\delta L_D^2$ as a function of $z$ is[13]

$$\delta L_D^2 = \frac{1.85\hbar\omega(1-r^2)^2}{2\eta I_0 k^2} \ . \tag{11}$$

The optimum noise limit [Eq. (8)] for an interferometer with passive optical cavities is rewritten by using Eqs. (2), (5), and (6) and the optimum condition $r_c = r^3$:

$$\delta L_P^2 = \frac{\hbar\omega(1-r^2)^2}{2\eta I_0 k^2 r^6} \ . \tag{12}$$

Comparison between Eqs. (11) and (12) yields

$$\frac{\delta L_D^2}{\delta L_P^2} = 1.85 \ , \tag{13}$$

since $r^6 \sim 1$.

Equation (13) shows that the optimum noise limits are similar for interferometers using either passive Fabry-Perot cavities or optical delay lines.

We conclude this section by stressing that, for given mirror losses, the Fabry-Perot cavity is optimized by satisfying the condition $r_c = r^3$, while the delay line is optimized by an appropriate choice of $z$, the number of reflections.[13]

## IV. DISCUSSION

The way to obtain the high strain sensitivity required by gravitational radiation detection is highlighted by evaluating the spectral density of the smallest detectable strain $\delta L^2/L^2$ by use of Eq. (8) [or of the equivalent Eq. (1)]:

$$\frac{\delta L^2}{L^2} = \text{const} \times \frac{1-r^4}{L^2 I_{st}} \ , \tag{14}$$

where the constant has dimensions energy $\times$ length$^2$. Equation (14) follows e.g., from Eq. (8) by replacing the explicit form of $Q$ and the optimum coupler reflectivity $r_c = r^3$.

In an active-cavity system, $I_{st}$ can be increased by employing a high-gain amplifying medium and high pumping levels. For passive-cavity devices, one has to increase the power of the laser beam injected into the cavities. It is also desirable to have mirrors of lowest possible losses and a long cavity. In the case of a passive resonator, the resulting narrow bandwidth sets critical frequency stability requirements upon the laser which provides the light to the interferometer. On the other hand, a long laser resonator means close mode spacing. The resulting large number of modes that may simultaneously oscillate is not an attractive possibility for an active-cavity interferometer. Thus, the natural way to improve the sensitivity of an active-cavity system is to increase both $Q$ and $I_{st}$ by use of top quality optics and by choosing a high-gain active medium, while keeping the resonators reasonably short.

It should be kept in mind that the active-passive cavity comparison in Sec. II has been made under the implicit assumption that the active medium of the laser does not affect resonator $Q$ in any way. If very high quality mirrors and low output coupling are used, this requires the active medium itself to be of very low scattering and that virtually no absorption should take place, except for the laser transition itself. Moreover, worries have been expressed about the active medium contributing excess noise, e.g., gas pressure fluctuations and plasma noise in a He-Ne laser tube,[14] thus preventing operation at the spontaneous-emission noise limit. Nevertheless, we have been able to operate an active-cavity detector employing two low-power He-Ne lasers at the spontaneous-emission noise limit, whereas we measured a displacement noise level of $3\times10^{-15}$ cm/Hz$^{1/2}$ in the kilohertz range.[8] While gas-pressure fluctuations and plasma noise might become a problem with long and/or high-power gas lasers, we expect that it will be possible to improve displacement sensitivity by 2—3 orders of magnitude by employing a properly selected solid-state amplifying medium.[8] Finally we note that, in the case of passive Fabry-Perot cavities, it might prove difficult to match the reflectivities of the mirrors as required by the optimum condition $r_c = r^3$.

## V. CONCLUSIONS

It has been shown that for identical resonators and stored fields of equal intensity, active- and passive-cavity interferometers considered for gravitational-radiation detection are of the same sensitivity. However, the practical approach to high sensitivity is different for the two kinds of interferometers. Thus, high quality optics, high gain amplifying medium, and short resonators are the best way for active-cavity systems. For practical reasons, only a limited amount of optical power can presently be injected into passive-cavity systems. This is compensated for by an increase in arm length.

[1]H. Billing, K. Maischberger, A. Ruediger, R. Schilling, L. Schnupp, and W. Winkler, J. Phys. E **12**, 1043 (1979).

[2]R. Schilling, L. Schnupp, D. H. Shoemaker, W. Winkler, K. Maischberger, and A. Ruediger, Max Planck Institute for Quantum Optics Report No. MPQ 88, 1984 (unpublished).

[3]R. W. P. Drever, in *Proceedings of the NATO Advanced Study Institute on Gravitational Radiation, Les Houches, 1982,* edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983).

[4]P. Linsay, P. Saulson, and R. Weiss (unpublished).

[5]K. Maischberger, A. Ruediger, R. Schilling, L. Schnupp, D. Shoemaker, and W. Winkler, Max Planck Institute for Quantum Optics Report No. MPQ 96, 1985 (unpublished).

[6]M. Weksler, Z. Vager, and G. Neumann, Appl. Opt. **19**, 2717 (1980).

[7]S. N. Bagayev, V. P. Chebotayev, A. S. Dychkov, and V. G.

Goldort, Appl. Phys. **25**, 161 (1981).

[8]A. Abramovici, Z. Vager, and M. Weksler, J. Phys. E (to be published).

[9]R. L. Forward, Phys. Rev. D **17**, 379 (1978).

[10]W. A. Edelstein, J. Hough, J. R. Pough, and W. Martin, J. Phys. E **11**, 710 (1978).

[11]A. E. Siegman, *An Introduction to Lasers and Masers* (McGraw-Hill, New York, 1971).

[12]P. Giacomo, Rev. Opt. **35**, 442 (1956).

[13]R. Weiss, Quart. Prog. Rep. Res. Lab. Electronics MIT **105**, 54 (1972).

[14]A. Brillet and P. Tourrenc, in *Proceedings of the NATO Advanced Study Institute on Gravitational Radiation, Les Houches, 1982,* edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983).