Parity nonconservation in singly ionized helium

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We analyze the sensitivity of parity experiments in singly ionized helium to the weak neutral-current interaction between electrons and nucleons. Such experiments could eventually provide measurements of the radiative corrections to the weak neutral-current coupling constants and precision measurements of the Weinberg angle. Some new "quasioptical" techniques for measuring parity nonconservation are discussed, and these appear to be particularly suitable for parity experiments in He^+. The problem of systematic effects caused by unwanted electric fields is considered, and the effect of electric fields arising from motion of He^+ ions transverse to a magnetic field is analyzed in detail.

I. INTRODUCTION

The standard model[1] of the weak and electromagnetic interactions is in agreement with all existing experimental data, but it is still an important task of experimental physics to continue testing its predictions. Bouchiat and Piketty[2] stress the role of the atomic-physics experiments in testing the standard model. They point out that parity experiments in heavy atoms place the most restrictive constraints on one class of alternative models which predict a second massive neutral vector boson.[3] In addition, the existing neutral-current experiments do not rule out the existence of a very light neutral gauge boson which would be observable in atomic-physics experiments but not in high-energy scattering. Such a particle has been suggested in connection with supersymmetric theories.[4]

Atomic parity experiments measure the weak neutral-current interactions between electrons and nucleons. The parity-nonconserving parts of these interactions are parametrized by four coupling constants[5] which characterize the nucleon-spin-independent couplings (C_{1p},C_{1n}) and the nucleon-spin-dependent couplings (C_{2p},C_{2n}) for the proton and the neutron. Two of these couplings have been determined to an accuracy of about 10% from parity experiments in heavy atoms[6] and from the Stanford Linear Accelerator Center (SLAC) polarized-electron-scattering experiment.[7] The other two couplings (C_{2p},C_{2n}) are essentially undetermined. More precise measurements of C_{1p} and C_{1n} can be expected from experiments in heavy atoms, but uncertainties in the atomic-physics calculations will ultimately limit the precision which can be achieved. Further progress in the determination of these constants may depend on experiments designed to measure parity nonconservation in one-electron atoms where the atomic physics is known exactly and where all four couplings can be determined.

In addition to testing the lowest-order predictions of the standard model, experiments in simple atoms provide a unique opportunity for measuring higher-order electroweak radiative corrections.[8] These corrections are not easily measured in other experiments. The higher-order corrections to the electron-nucleon coupling constants have been calculated by Lynn[8] and by Marciano and Sirlin[9] and it is of considerable interest to verify these calculations.

Experiments[10,11] are currently in progress to measure the coupling constant C_{2p} in hydrogen using atomic beam techniques. Although preliminary results have been obtained,[12] these experiments have not yet reached the sensitivity necessary to test the standard model because of systematic errors associated with unwanted electric fields. It is of interest to consider whether similar experiments using hydrogenic ions of higher Z are possible. In an earlier publication,[13] we pointed out some advantages to the use of hydrogenic ions for measurement of the coupling constants C_{1n} and C_{1p}. We also suggested that the ions $^3He^+$ and $^4He^+$ might be ideal candidates.[14]

In this paper we consider parity experiments in ionized helium in more detail. In Sec. II, we summarize the weak interaction coupling constants that can be measured in ionized helium and give their magnitudes in the standard model. We also discuss the expected sizes of the electroweak radiative corrections to these constants. Then, in Sec. III, we give a general description of the atomic beam method for searching for parity nonconservation and we analyze the sensitivity of this method for measurement of the couplings C_{1n} and C_{1p} using singly ionized helium. In Sec. IV we suggest some specific experimental techniques which could be used in He^+ parity experiments. The emphasis will be on new techniques that are made possible by the use of He^+. Finally, in Sec. V we consider the sensitivity of the He^+ parity experiments to systematic errors caused by unwanted electric fields. Most of the discussion of systematic errors will concern motional electric fields.

II. PARITY MIXING IN He^+

The parity-nonconserving part of the weak neutral-current interaction causes a mixing between the $nS_{1/2}$ and $nP_{1/2}$ states of a hydrogenic ion with a matrix element given by

$$\langle nS_{1/2},m_J',m_I' \,|\, V \,|\, nP_{1/2},m_J,m_I \rangle = i\bar{V}\langle m_J',m_I' \,|\, -C_1+2C_2\sigma_e\cdot\mathbf{I} \,|\, m_J,m_I \rangle \;, \quad (1)$$

where the dimensional factor $\bar{V}$ is

$$\bar{V}=0.118Z^4\frac{(n^2-1)^{1/2}}{n^4}\text{ Hz} . \quad (2)$$

C_1 and C_2 are the two effective coupling constants which characterize the weak interaction for a given nucleus. These constants are defined in terms of the nucleon constants as follows:[11]

$$C_1 \equiv ZC_{1p}+NC_{1n} \;, \quad (3)$$

$$C_2\langle Im_I' \,|\, \mathbf{I} \,|\, I,m_I \rangle \equiv \frac{1}{4}\sum_j \langle I,m_I' \,|\, \sigma_j[C_{2p}(1+\tau_{3j}) + C_{2n}(1-\tau_{3j})] \,|\, I,m_I \rangle \;, \quad (4)$$

where the sum extends over all the nucleons in the nucleus. Here, τ_j and σ_j are the isospin and spin operators for the jth nucleon. Note that C_1 depends only on the number of protons (Z) and neutrons (N) in the nucleus whereas C_2 depends on a nuclear matrix element.

Lynn,[8] and Marciano and Sirlin,[9] have calculated the electroweak radiative corrections to these couplings. They find that the radiative corrections can be large. For example, the lowest-order prediction for C_{1p} is 0.046 while the corrected value is 0.067. Although the couplings C_{1n} and C_{1p} are reliably calculated with almost no strong interaction uncertainty, there is considerable strong interaction uncertainty in the couplings C_{2p} and C_{2n}. Marciano and Sirlin conclude that precise measurements of the couplings C_{1p} and C_{1n} in hydrogen and deuterium provide a probe of the electroweak radiative corrections which is insensitive to both strong interaction and atomic physics uncertainties. The same arguments could be applied to parity experiments in He^+, and so it is of interest to consider the size of the He^+ coupling constants and the radiative corrections to these constants.

In Table I, we summarize the couplings that can be measured in H, D, $^3He^+$, and $^4He^+$ and give their values in lowest order and the values when the radiative corrections are included. On the basis of this table, a measurement of the coupling $C_1(^4He^+)$ appears to be the best candidate both for observation of parity nonconservation and for determination of the radiative corrections to the standard model. It is the largest coupling, an order of magnitude larger than the coupling (C_{2p}) that is being searched for in the present hydrogen parity experiments, and the radiative correction to it is about 8%. A further advantage of experiments utilizing $^4He^+$ is that this ion has no hyperfine structure to complicate interpretation of the data. This latter point is particularly relevant in a comparison with deuterium. Although C_1(D) is relatively large, the complicated and close-spaced hyperfine structure of this atom leads to difficult experimental problems. The lack of hyperfine structure in $^4He^+$ also makes it a good candidate for providing precision measurements of the Weinberg angle because there are no contributions from the nucleon-spin-dependent couplings to be sorted out.

Experiments in singly ionized helium ($^3He^+$) can also provide information on the nucleon-spin-dependent couplings (C_2). It is certainly important to measure these couplings because they have not been determined to date. On the other hand, $C_2(^3He^+)$ is more than an order of magnitude smaller than $C_1(^4He^+)$ and will be difficult to observe. In this paper we will focus on the prospects for measuring the couplings C_1 and we put off discussion of the measurement of C_2 until later.

TABLE I. Lowest-order predictions and radiative corrections for the weak interaction coupling constants in $^3He^+$, $^4He^+$, H, and D.

Coupling	Lowest-order prediction[a]	Value including radiative corrections[b]
$C_1(^3He^+)$	−0.41	−0.35
$C_2(^3He^+)$	−0.058	−0.069
$C_1(^4He^+)$	−0.91	−0.84
C_1(H)	0.046	0.067
C_2(H)	0.058	0.081
C_1(D)	−0.45	−0.42
C_2(D)	0.0	0.012

[a]Based on $\sin^2\theta_W=0.227$ and $g_A=1.25$.
[b]Based on Ref. 9.

III. SENSITIVITY OF He^+ PARITY EXPERIMENTS

The beam resonance method for measuring parity nonconservation in He^+ involves the detection of a pseudoscalar term in the rate of a microwave transition within the $n=2$ shell. The sensitivity of this type of experiment can be characterized by two parameters. One is the counting time T required to measure an effect, and the other is the asymmetry A. We define A to be the fraction of the transition rate corresponding to the pseudoscalar term. In this section we use an approximate solution for the rate of a typical transition which can be used to measure the coupling C_1 in He^+ and use this solution to calculate the asymmetry and estimate the counting time. To simplify the discussion, we specialize to the isotope $^4He^+$ so that we do not need to consider hyperfine structure. Most of our conclusions, however, will apply equally well to measurements of C_1 in $^3He^+$.

The energy levels of the $2S_{1/2}$ and $2P_{1/2}$ states of $^4He^+$ in the presence of an external magnetic field are shown in Fig. 1. The $2S_{1/2}$ states are labeled α and β and $2P_{1/2}$ levels are labeled e and f. The parity experiments we consider here involve the observation of the transition $\alpha\to\beta$ in a beam of metastable He^+. This transition normally proceeds via a parity conserving $M1$ amplitude but if α and β are mixed with e and f by the weak interaction, there will also be a parity nonconserving $E1$ amplitude. The interference term between these two amplitudes is first order in the weak interaction mixing and is the quantity which is to be measured.

The results of this section will be based on the specific

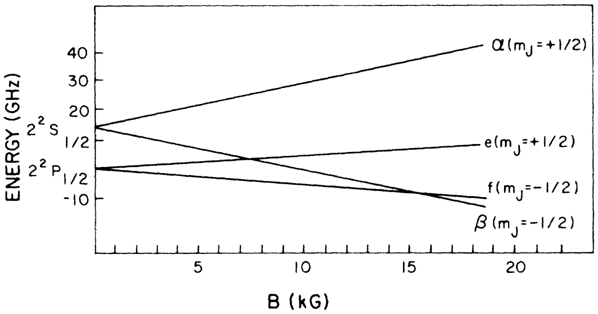

FIG. 1. Zeeman diagram for $^4He^+$. The $2S_{1/2}$ states are labeled α and β and the $2P_{1/2}$ states are labeled e and f.

configuration of interaction region fields, shown in Fig. 2, which might be used in a typical parity experiment. In the next section, we will discuss other possible field configurations. In the present case, the polarization vector $\boldsymbol{\epsilon}$ of an oscillating electric field $\boldsymbol{\epsilon}(t)=\boldsymbol{\epsilon}\cos(\nu t)$ defines the x axis, a static magnetic field $\mathbf{B}$ defines the z axis, and an oscillating magnetic field $\mathbf{m}(t)=\mathbf{m}\cos(\nu t)$ lies in the y-z plane. We neglect any spatial dependence of the oscillating fields and assume that in the rest frame of an ion passing through the transition region, these fields turn on at time $t=0$ and remain constant in amplitude[15] until the ion leaves the interaction region at time $t=\tau$. Then, given that the ion is in the α state at time $t=0$, we calculate the amplitude b for it to be in the β state at time $t=\tau$. An approximate solution to this problem can be obtained by the method described by Levy and Williams[12] which gives the amplitude

$$b=\Lambda\left[\frac{e^{-i(\omega_{\alpha'\beta'}-\nu)\tau-\gamma_{\alpha'}\tau/2}-e^{-\gamma_{\beta'}\tau/2}}{\omega_{\alpha'\beta'}-\nu-i(\gamma_{\alpha'}-\gamma_{\beta'})/2}\right]. \quad (5)$$

In this expression the parameter Λ is the second-order matrix element given by

$$\Lambda=i[\mu_0 m_y+p_2\epsilon_x(C_1\bar{V}\Delta_{\parallel})]/2\hbar\ , \quad (6)$$

where

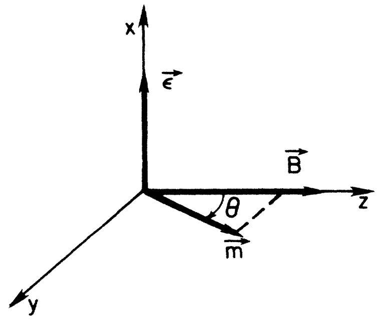

FIG. 2. Typical interaction region field configuration consisting of a static magnetic field $\mathbf{B}$, an oscillating electric field $\boldsymbol{\epsilon}(t)=\boldsymbol{\epsilon}\cos(\nu t)$, and an oscillating magnetic field $\mathbf{m}(t)=\mathbf{m}\cos(\nu t)$.

$$\Delta_{\parallel}=(\omega_{\alpha e}+i\gamma_p/2)^{-1}-(\omega_{\beta f}+i\gamma_p/2)^{-1}\ , \quad (7)$$

$$p_2=\frac{\sqrt{3}\,|e|a_0}{Z}\ , \quad (8)$$

and $\gamma_p\equiv\Gamma_{2p}/\hbar$. The real parameters ω'_α, ω'_β, γ'_α, and γ'_β are given by

$$\omega'_\alpha-i\gamma'_\alpha/2=\omega_\alpha+\frac{(p_2\epsilon_x)^2/4}{\omega_{\alpha f}-\nu+i\gamma_p/2}\ , \quad (9)$$

$$\omega'_\beta-i\gamma'_\beta/2=\omega_\beta+\frac{(p_2\epsilon_x)^2/4}{\omega_{\beta e}-\nu+i\gamma_p/2}\ , \quad (10)$$

and we use the notation in which ω_{ij} is the circular frequency separation of state i from state j.

Taking the absolute square of Eq. (5), and neglecting the term quadratic in $\bar{V}$ we obtain the transition probability per unit time:

$$|b|^2=[\mu_0^2 m_y^2+2p_2\mu_0 C_1\bar{V}\,|\Delta_{\parallel}|\cos(\lambda)m_y\epsilon_x]\rho(\nu)\ . \quad (11)$$

Here, λ is the relative phase between the parity-conserving and the parity-nonconserving amplitudes and we have defined the function $\rho(\nu)$ which determines the line shape of the $\alpha\to\beta$ resonance. Equation (11) can be written in a more general form in which the rate is expressed in terms of invariant combinations of the perturbing fields:

$$|b|^2=[\mu_0^2(\mathbf{m}\times\hat{\mathbf{B}})^2+2p_2\mu_0C_1\bar{V}\,|\Delta_{\parallel}|\cos(\lambda)\boldsymbol{\epsilon}\times\mathbf{m}\cdot\hat{\mathbf{B}}]\rho(\nu)\ . \quad (12)$$

The $M1$ rate is proportional to the scalar $(\mathbf{m}\times\mathbf{B})^2$ and the interference term is proportional to the pseudoscalar $(\boldsymbol{\epsilon}\times\mathbf{m}\cdot\mathbf{B})$. The pseudoscalar term can be detected because it changes sign under a reversal of $\mathbf{B}$ or when the angle θ of Fig. 2 is changed to $-\theta$. The scalar term is unaffected by these operations. The asymmetry is easily calculated from Eq. (11) with the result

$$A=(4\sqrt{3}/Z\alpha)(\epsilon_x/m_y)C_1\bar{V}\,|\Delta_{\parallel}|\cos\lambda\ . \quad (13)$$

This quantity is useful for discussing systematic errors, and it gives an indication of how accurately the reversals need to be made. The factor (ϵ_x/m_y) depends on the angle θ, and we can make the asymmetry large by choosing θ to be small. The limit on how small θ can be will ultimately be set by experimental conditions such as detector background. In the next section, we will show how the factor $\cos(\lambda)$ can also be adjusted experimentally. Setting $\epsilon_x/m_y=10$ and $\cos(\lambda)=1$, we find that the asymmetry has the field dependence shown in Fig. 3. The maximum asymmetry ($A\approx10^{-6}$) occurs near the βf level crossing.

The counting time required to measure this asymmetry can be estimated if we assume that the noise is dominated by counting statistics. In this case, the counting time corresponding to a signal-to-noise ratio of one is given by

$$T=(1+\kappa)/|b|^2J\eta A^2\ , \quad (14)$$

where J is the number of α-state ions per second entering the interaction region, and η is the overall detection efficiency for ions in the β state. The parameter κ is the ratio of the detector background signal to the $\alpha\to\beta$ resonance

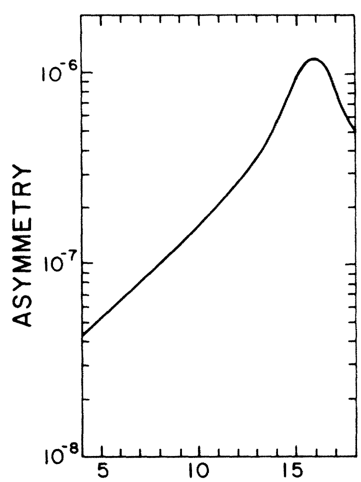

FIG. 3. Asymmetry as a function of magnetic field for $\epsilon_x/m_y=10$.

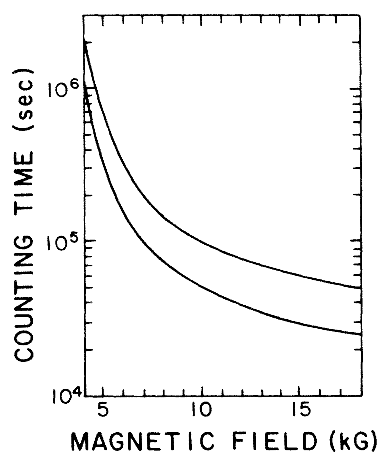

FIG. 5. Minimum counting time $T_{\min}$ vs magnetic field. The lower curve is for the case of no background ($\kappa=0$), and the upper curve is for the case where the background signal is equal to the $\alpha\to\beta$ resonance signal ($\kappa=1$).

signal. The counting time has a minimum $T_{\min}$ as a function of the product $\epsilon_x^2\tau$ which sets the optimum strength of the oscillating field $|\epsilon_x|$. A short derivation leads to the following form for the minimum counting time:

$$T_{\min}=(1+\kappa)/\gamma_p\tau C_1{}^2\cos^2(\lambda)J\eta\xi\ , \tag{15}$$

where ξ is a function depending only on the magnetic field strength. The value of $|\epsilon_x|$ required to minimize the counting time is dependent on the magnetic field as shown in Fig. 4, which is the result of a computer calculation.

The magnetic field dependence of $T_{\min}$ is shown in Fig. 5 which is based on the typical parameters[16] $J=6\times10^{11}$ s^{-1} (100 nA), $\tau=2$ μs, $\eta=0.05$, $\kappa=0$ and 1. We also take $\cos(\lambda)=1$ and $C_1=1$. This figure indicates that magnetic fields of 10 kG or larger are needed for viable experiments. The counting time which would be required for an experiment done at the βf level crossing (15.7 kG) is on the order of a half of a day and so an experiment just to observe parity nonconservation appears practical on signal-to-noise grounds. Measurement of the radiative corrections would require an improvement on the above "typical" parameters. This could involve developing larger metastable beams, more efficient metastable detectors, or increasing the transit time τ. The dependence of $T_{\min}$ on τ suggests the use of slow-ion beams and long transition regions.

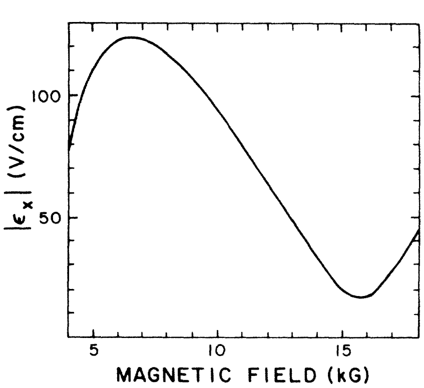

FIG. 4. Oscillating field strength $|\epsilon_x|$ (in V/cm) corresponding to the minimum counting time ($T_{\min}$) vs magnetic field.

IV. QUASIOPTICAL PARITY EXPERIMENTS

The main differences among the various hydrogen parity experiments which use a microwave resonance method are in the configurations of the transition regions.[10,11] The same configurations can, in principle, be used for experiments in He^+, but we will not discuss them here. Instead, we consider some new possibilities which are suggested by the higher microwave frequencies which would be used in the He^+ parity experiments. The wavelength of the microwave radiation required is much smaller than typical transition-region dimensions. For example, an experiment done at the βf crossing in He^+ would use a microwave frequency of 44 GHz for the $\alpha\to\beta$ resonance. The corresponding wavelength is about 7 mm, whereas typical transition-region dimensions would be 20 cm or more. This situation suggests the use of "quasioptical" microwave techniques for the He^+ parity experiments. These techniques provide for precise control of the microwave polarization and propagation direction, which can be used to good advantage in the design of parity experiments.

A. Experiments involving open resonators

The first technique we will discuss involves the use of a Fabry-Perot open resonator[17] consisting of two spherical mirrors facing each other. Consider the transition region depicted in Fig. 6. The ion beam travels along the magnetic field with velocity **v**, and microwave radiation is applied using an open resonator whose axis makes an angle

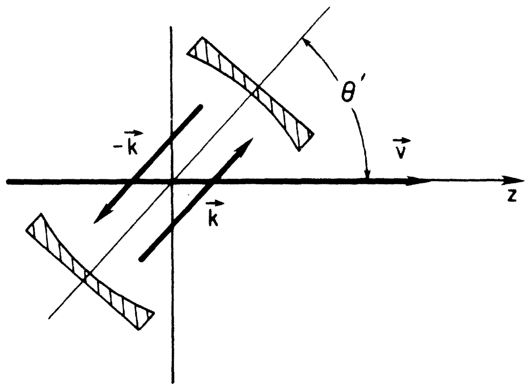

FIG. 6. Modes of a Fabry-Perot resonator can be expressed as a linear combination of two traveling waves. An ion traversing the resonator as shown will see two frequencies because of the Doppler effect. If the frequency separation is larger than the linewidth of the transition of interest, it is possible to separately resolve the two components. This provides a means for reversing the propagation vector **k** by tuning the microwave frequency.

θ' with the ion velocity. The normal modes of the resonator can be represented as a superposition of two plane waves traveling in opposite directions. In the rest frame of the ions the microwave frequency will be shifted by the Doppler effect and this shift will be of opposite sign for the two counterpropagating traveling waves. If the Doppler shift is larger than the linewidth of the $\alpha\rightarrow\beta$ transition, the two components will be separately resolved and the microwave propagation direction **k** can be reversed by switching between the two Doppler-shifted components. A reversal of **k** is useful for detecting parity nonconservation and in particular it could be used to measure the pseudoscalar $\boldsymbol{\epsilon}\cdot\mathbf{m}\times\mathbf{B}$ which was discussed in the example of Sec. III. For optical transitions, this pseudoscalar is equivalent to $\mathbf{B}\cdot\mathbf{k}$ which can be detected by a reversal of **k**. The switch between the two Doppler-shifted components can be accomplished by a small modulation of either the microwave frequency or of the magnetic field strength. The frequency modulation should be done symmetrically relative to the Q curve of the resonator. It is important to note that in the rest frame of the ions the reversal of the direction of **k** will not be perfect because of relativistic corrections. This problem requires that there be another means for reversing the pseudoscalar term. In the present case, both **k** and **B** could be reversed and the pseudoscalar picked out using a double-subtraction technique. Since the scalar term is even under both reversals, the requirement on the accuracy of each of the separate reversals is considerably reduced.

If the transit time of an ion through the resonance region is about 1 μs and the magnetic field is homogeneous to a part in 10^5 in this region, the linewidth of the $\alpha\rightarrow\beta$ transition would be about 1 MHz.[18] For a 44 GHz transition frequency and a beam energy of 300 eV, the resolution of the two Doppler-shifted components requires only that the resonator axis be tilted by three degrees or more from a line perpendicular to the beam velocity.

B. Linear polarization experiments

Another quasioptical microwave technique allows control of the polarization of microwaves. We will consider here a specific scheme depicted in Fig. 7. Two rectangular waveguides (R_1 and R_2) feed microwave power from the same source into a "dual orthomode transducer" which produces an output into a circular waveguide. The polarization of the microwaves in the circular waveguide is determined by the relative amplitude and phase of the orthogonal components ϵ_1 and ϵ_2 which can be controlled using the phase shifter and pin modulator in series with R_1. This system allows the production of a beam of microwaves with arbitrary polarization.

Consider a configuration in the interaction region consisting of an ion beam traveling along the magnetic field (z axis) and a beam of microwaves incident from the negative y axis. We assume we have independent control over the amplitudes and phases along two orthogonal directions $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ so that the oscillating field is given by

$$\boldsymbol{\epsilon}(t)=\boldsymbol{\epsilon}_1\cos(\nu t)+\boldsymbol{\epsilon}_2\cos(\nu t+\varphi)\ , \qquad (16)$$

and $\mathbf{m}(t)=\hat{\mathbf{k}}\times\boldsymbol{\epsilon}(t)$. If $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ lie along the x and z axes, respectively, the matrix element Λ' connecting the states α' and β' has the form

$$\Lambda'=(\mu_0/2\hbar)e^{i\varphi}m_{2x}+i(p_2C_1\bar{V}\Delta_{\parallel}/2\hbar)\epsilon_{1x}\ . \qquad (17)$$

If $\varphi=0$ or π, the polarization is linear with the plane of polarization making an angle θ'' with the x axis. The angle θ'' is set by adjusting the magnitude of ϵ_1. The transition probability for this case is proportional to the square of (17)

$$|\Lambda'|^2=(\mu_0^2/4\hbar^2)(m_{2x})^2 +2(\mu_0p_2C_1\bar{V}/4\hbar^2)\,|\Delta_{\parallel}|\cos(\lambda')m_{2x}\epsilon_{1x}\ , \qquad (18)$$

where the phase factor is given by

$$\cos(\lambda')=\mathrm{Im}(\Delta_{\parallel})/|\Delta_{\parallel}|\ . \qquad (19)$$

The interference term in Eq. (18) changes sign under the operation $\theta''\rightarrow-\theta''$ which is accomplished by changing the phase φ from 0 to π.

The asymmetry for the linear polarization experiment can be obtained from Eq. (13) by making the substitutions $m_y\rightarrow m_{2x}$, $\epsilon_x\rightarrow\epsilon_{1x}$ and $\lambda\rightarrow\lambda'$. The asymmetry is thus proportional to the ratio (ϵ_{1x}/m_{2x}). A more general form for the transition probability is obtained by expressing Eq. (18) in invariant form as

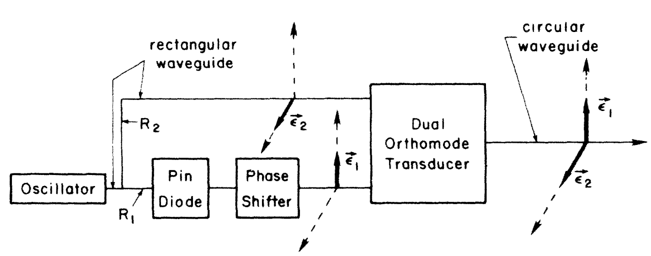

FIG. 7. Scheme allowing for complete control of the polarization of microwaves.

$$|\Lambda'|^2=(\mu_0^2/4\hbar^2)(\mathbf{m}\times\hat{\mathbf{B}})^2 +2(\mu_0 p_2 C_1 \overline{V}/4\hbar^2)\,|\Delta_{\parallel}|\cos(\lambda')(\mathbf{m}\cdot\hat{\mathbf{B}})(\boldsymbol{\epsilon}\cdot\hat{\mathbf{B}}) . \tag{20}$$

The pseudoscalar interference term is odd under polarization reversal ($\theta''\to-\theta''$) and reversal of **k**, and is even under reversal of the magnetic field.

The counting time required to measure the pseudoscalar $(\mathbf{m}\cdot\hat{\mathbf{B}})(\boldsymbol{\epsilon}\cdot\hat{\mathbf{B}})$ is given by making the replacements $\lambda\to\lambda'$ and $\xi\to\xi'$ in Eq. (15). The function ξ must be modified for the linear polarization experiment because the oscillating electric field $\epsilon(t)$ has components along both the x and z axes. The component along the z axis gives rise to quenching $\alpha\to e$ and $\beta\to f$ that was not included in the calculation of the last section. If the angle θ'' is small, these new couplings will have little effect on the counting time except near the magnetic fields (7.5 and 3.8 kG) where $\omega_{\alpha e}$ or $\omega_{\beta f}$ are equal to the transition frequency $\omega_{\alpha\beta}$. At these fields there would be an increase in the counting time caused by increased quenching of the metastable states. The main difference in the counting times required to measure the pseudoscalars $(\mathbf{m}\cdot\hat{\mathbf{B}})(\boldsymbol{\epsilon}\cdot\hat{\mathbf{B}})$ and $\hat{\mathbf{B}}\cdot\hat{\mathbf{k}}$, however, comes from the different magnetic field dependence of $\cos(\lambda)$ and $\cos(\lambda')$. In Fig. 8, $\cos(\lambda)$ and $\cos(\lambda')$ are plotted as a function of magnetic field. Away from the βf crossing $\cos(\lambda')$ is small and so the linear polarization experiment is probably only feasible near the crossing. This is a consequence of the fact that measurement of $(\mathbf{m}\cdot\hat{\mathbf{B}})(\boldsymbol{\epsilon}\cdot\hat{\mathbf{B}})$ depends on the $i\Gamma/2$ term in the energy denominator of the weak mixing parameter, and this becomes important only near the crossing. By contrast, the magnitude of $\cos(\lambda)$ is nearly one except near the crossing so that $\mathbf{B}\cdot\mathbf{k}$ can be measured away from the crossing.

C. Elliptical polarization experiments

In the more general case of arbitrary φ the microwave polarization will be elliptical. Equation (17) shows that adjustment of φ allows complete control over the relative phase between the parity-conserving and the parity-nonconserving amplitudes and so in the case of elliptical polarization we are justified in assuming that the interference term can be maximized with respect to this phase. In addition, this maximization can be done at any magnetic field so that we are no longer restricted to working near the crossing as in the case of the linear polarization experiment. The transition rate for elliptical polarization assuming ϵ_1 and ϵ_2 lie along the x and z axes is proportional to

$$|\Lambda''|^2=(\mu_0^2/4\hbar^2)(m_{2x})^2 -2(\mu_0 p_2 C_1 \overline{V}/4\hbar^2)\,|\Delta_{\parallel}|\cos(\lambda'')m_{2x}\epsilon_{1x} , \tag{21}$$

where $\cos(\lambda'')$ satisfies the relation

$$|\Delta_{\parallel}|\cos(\lambda'')=\sin\varphi\,\mathrm{Re}(\Delta_{\parallel})-\cos\varphi\,\mathrm{Im}(\Delta_{\parallel}) . \tag{22}$$

The asymmetry is proportional to (ϵ_{1x}/m_{2x}) as in the linear polarization case and it changes sign under the operation $\varphi\to-\varphi$. Other properties of the interference term can be illustrated by expressing Eq. (21) in invariant form. We first define the complex polarization vector

$$\boldsymbol{\epsilon}\equiv\boldsymbol{\epsilon}_1+\boldsymbol{\epsilon}_2 e^{i\varphi} , \tag{23}$$

in terms of which the rate is proportional to

$$|\Lambda''|^2=(\mu_0^2/4\hbar^2)\,|\mathbf{m}\times\hat{\mathbf{B}}|^2 +(\mu_0 p_2 C_1 \overline{V}/4\hbar^2)\{\mathrm{Im}(\Delta_{\parallel})[\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}}\times\hat{\mathbf{B}}\boldsymbol{\epsilon}^*\cdot\hat{\mathbf{B}}+\mathrm{c.c.}]+\mathrm{Re}(\Delta_{\parallel})[-2i\boldsymbol{\epsilon}^*\times\boldsymbol{\epsilon}\cdot\mathbf{k}+i(\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}}\times\hat{\mathbf{B}}\boldsymbol{\epsilon}^*\cdot\hat{\mathbf{B}}-\mathrm{c.c.})]\} . \tag{24}$$

This expression shows that the pseudoscalars in the interference term are all odd under a reversal of **k** but even under a reversal of **B**.

If the microwave polarization technique is used with a tilted Fabry-Perot resonator, both the microwave polarization and **k** can be reversed, and we have a powerful method for picking out the pseudoscalar term in the polarization dependent rate. In this case, the axis of the Fabry-Perot must be tilted relative to a line perpendicular to the magnetic field in order to resolve the two Doppler-shifted components. To generalize to an arbitrary orientation of **k**, the following two terms must be added to Eq. (24):

$$-i(\mu_0^2/4\hbar^2)(\boldsymbol{\epsilon}^*\times\boldsymbol{\epsilon}\cdot\hat{\mathbf{k}}\hat{\mathbf{B}}\cdot\hat{\mathbf{k}}) +[\mu_0 p_2 C_1 \overline{V}\mathrm{Re}(\Delta_{\parallel})/4\hbar^2]\,|\boldsymbol{\epsilon}|^2\mathbf{B}\cdot\hat{\mathbf{k}} . \tag{25}$$

The first term represents the dependence of the $M1$ rate on the component of the microwave circular polarization along the magnetic field. For the case of pure circular polarization, this is smaller than the total $M1$ rate by the factor $\hat{\mathbf{B}}\cdot\hat{\mathbf{k}}$, which for a three-degree tilt angle is 0.05. This term is odd under a magnetic field reversal and under $\varphi\to\varphi+\pi$ but even under the **k** reversal. The second term in Eq. (25) is the pseudoscalar discussed in Sec. IV A.

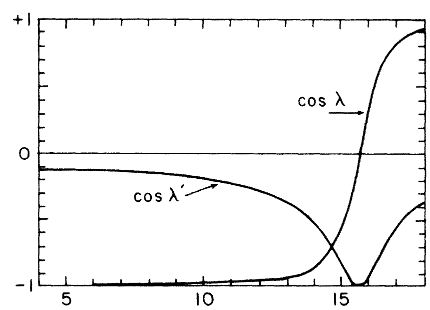

FIG. 8. Magnetic field dependence of the factors cos(λ) and cos(λ′).

V. SYSTEMATIC ERRORS FROM MOTIONAL ELECTRIC FIELDS

The sensitivity of hydrogenic atoms to electric fields poses a major problem for parity experiments in these atoms. Any unwanted electric field will cause mixing of the $2S$ and $2P$ levels which could mimic the mixing by the weak interaction. We will not give a complete discussion of the problems here since they depend on experimental details that are beyond the scope of the present discussion. A number of techniques for handling systematic errors from stray electric fields have been discussed in connection with the various hydrogen experiments[10,12] and many of these ideas can be applied to the He^+ experiments as well. Our main purpose in this section is to consider the effects of motional electric fields. These are electric fields associated with the velocity of the ion transverse to the static magnetic field. The interference terms induced by these fields have a different character in the He^+ experiments than they do in the hydrogen experiments because the ions follow helical trajectories and so the motional electric field rotates at the ion cyclotron frequency. Before analyzing this interesting situation, however, we will make a few general comments about the effects of electric fields on the amplitude for the transition $\alpha \rightarrow \beta$.

A. Amplitude induced by static electric fields

The second-order matrix element for the $\alpha \rightarrow \beta$ transition in the presence of microwave fields $\boldsymbol{\epsilon}(t) = \boldsymbol{\epsilon}\cos\nu t$, $\mathbf{m}(t) = \mathbf{m}\cos\nu t$, and an arbitrary static electric field $\mathbf{E}$ is given by

$$\Lambda_S = -(\mu_0/2)(m_x + i m_y) + (p_2^2/2)(E_x + iE_y)\epsilon_z \Delta_\perp + (p_2/2)(p_2 E_z + i\overline{V}C_1)(\epsilon_x + i\epsilon_y)\Delta_\parallel , \quad (26)$$

where $\Delta_\perp$ is defined by

$$\Delta_\perp = (\omega_{\beta e} + i\gamma_p/2)^{-1} - (\omega_{\alpha f} + i\gamma_p/2)^{-1} . \quad (27)$$

We will assume here that the transition is primarily $M1$ and the electric field is an unwanted stray field. Note that the E_z-induced amplitude and the weak-induced amplitude have the same dependence on magnetic field so the electric field strength $(E_z)_{\text{eq}}$ for which these two amplitudes are equal in magnitude is independent of B. From Eq. (26) we find this field is

$$(E_z)_{\text{eq}} = \overline{V}C_1/p_2 . \quad (28)$$

In the standard model, $\overline{V}C_1/p_2 \approx 3\times 10^{-9} Z^6$ V/cm and for He^+, it is about 0.2 μV/cm. Note that this is a factor of 64 larger than the corresponding $(E_z)_{\text{eq}}$ in the case of hydrogen. Fortunately, one can tolerate a much larger stray electric field than $(E_z)_{\text{eq}}$ since, under reversal of the magnetic field, the amplitude induced by E_z changes sign while the weak-induced amplitude does not. So there is a means for distinguishing the E_z-induced interference term. In addition, it is possible to suppress this term if the relative phase between the $E1$ and $M1$ amplitudes can be precisely controlled.[19] In this case, one can adjust the phase to zero the interference term associated with E_z. Since the weak amplitude and the E_z-induced amplitude are 90° out of phase, this also maximizes the pseudoscalar interference term.

For electric fields perpendicular to the magnetic field, the size of field which gives an amplitude equal in magnitude to that of the weak-induced amplitude is given by

$$| E_\perp^{\text{eq}} | = (\overline{V}C_1/p_2)(\epsilon_\perp/\epsilon_z) | \Delta_\parallel/\Delta_\perp | , \quad (29)$$

where we use the notation $E_\perp \equiv (E_x^2 + E_y^2)^{1/2}$. It is possible to increase the size of the electric field ($E_\perp$) which is tolerable by taking $(\epsilon_\perp/\epsilon_z)$ and $|\Delta_\parallel/\Delta_\perp|$ to be large. In Fig. 9 we give a plot of the ratio $|\Delta_\parallel/\Delta_\perp|$ versus magnetic field which shows a maximum near the βf level crossing. An important signature for the systematic errors arising from transverse electric fields is their dependence on the microwave electric field component ϵ_z. If the microwave polarization can be controlled, one could intentionally make ϵ_z large in order to study the systematic effect and then reduce it to zero to eliminate the undesirable interference term.

B. Motional electric fields

The most important source of transverse electric field comes from the motion of the ions transverse to the magnetic field. In order to study the nature of the systematic effects arising from this field, we will present an approximate solution for the $\alpha \rightarrow \beta$ transition amplitude for the experimental situation illustrated in Fig. 10. The ions travel in helical trajectories in a uniform magnetic field $\mathbf{B}$ which defines the z axis and microwave radiation, polarized parallel to the magnetic field, is incident from the positive y axis. The trajectory for a given ion is described by the position vector

$$\mathbf{x}(t) = R_c[\hat{\mathbf{x}}\cos(\omega_m t + \varphi_m) - \hat{\mathbf{y}}\sin(\omega_m t + \varphi_m)] + \hat{\mathbf{z}}v_z t + \mathbf{x}_0 . \quad (30)$$

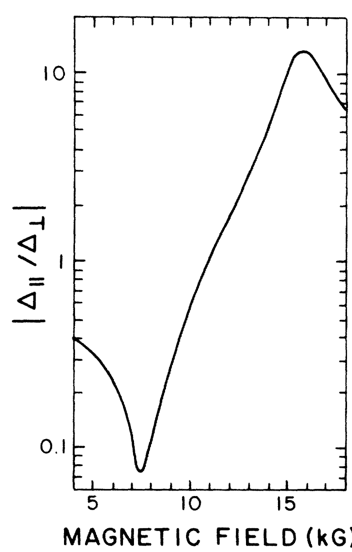

FIG. 9. Plot of the ratio $|\Delta_\parallel/\Delta_\perp|$ as a function of magnetic field.

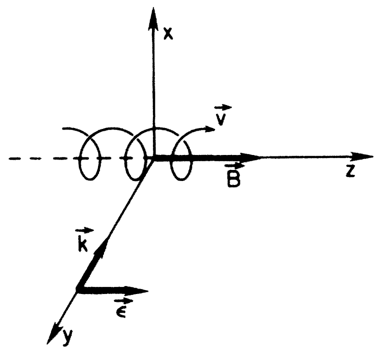

FIG. 10. Effect of a motional electric field is illustrated by considering an ion traveling in a helical motion along the static magnetic field. The ion passes through a beam of microwaves which is polarized along the static magnetic field.

The parameters R_c and ω_m are the cyclotron radius and the cyclotron frequency, respectively. The phase factor φ_m is determined by the position and velocity of the ion at time $t=0$. The motional electric field seen in the rest frame of the ion is given by

$$\mathbf{E}_m=(\mathbf{v}/c)\times\mathbf{B}=E_m[-\hat{\mathbf{x}}\cos(\omega_m t+\varphi_m)+\hat{\mathbf{y}}\sin(\omega_m t+\varphi_m)] , \tag{31}$$

which rotates at frequency ω_m. The magnitude of the motional field is given by

$$E_m=(R_c\omega_m B/c) . \tag{32}$$

Because of the Doppler effect, the transverse motion will also cause a modulation of the microwave frequency observed in the rest frame of the ion. To account for this effect, we assume that the radiation field has the time dependence

$$\epsilon(t)=\epsilon\cos[\Omega\sin(\omega_m t+\varphi_m)-\nu t] , \tag{33}$$

where

$$\Omega=2\pi R_c/\lambda_m , \tag{34}$$

and λ_m is the wavelength of the microwave radiation in the laboratory frame.

To further simplify the problem, we neglect the effect of the f state, and solve the three-level problem of the states α, β, and e. Using the approach of Levy and Williams[12] we first solve the two-level problems for the couplings α-e and β-e. This reduces the problem to an approximate two-level problem in the subspace a''-β''. For the solution of the mixing of α to e and β to e we can to sufficient approximation neglect the dependence on ω_m because it is small compared to the width of the $2P$ states. For example, in a field of 16 kG, the ion cyclotron frequency is about 6 MHz while $\Gamma_{2p}=1600$ MHz. On the other hand, ω_m is important for the $\alpha\rightarrow\beta$ transition where the linewidth is on the order of 1 MHz.

It is useful to expand the time dependence of the microwave fields in a Fourier series using the formula:[20]

$$\exp[i\Omega\sin(\omega_m t+\varphi_m)]=\sum_{n=-\infty}^{\infty}J_n(\Omega)e^{in(\omega_m+\varphi_m)} . \tag{35}$$

The coefficients $J_n(\Omega)$ are Bessel functions. The approximate two-level equations of motion can then be written as

$$i\begin{bmatrix}\dot{a}''\\ \dot{b}''\end{bmatrix}=\begin{bmatrix}\omega_\alpha''-i\gamma_\alpha''/2 & \sum_{n=-\infty}^{\infty}\tilde{\Lambda}_n e^{+i[\omega_m(n-1)t+n\varphi_m-\nu t]}\\ \sum_{n=-\infty}^{\infty}\Lambda_n e^{-i[\omega_m(n-1)t+n\varphi_m-\nu t]} & \omega_\beta''-i\gamma_\beta''/2\end{bmatrix}\begin{bmatrix}a''\\ b''\end{bmatrix} , \tag{36}$$

where

$$\Lambda_n=\tilde{\Lambda}_n=(\mu_0 m_x/2)J_n(\Omega)-(p_2^2\epsilon_z E_m/2)\Delta_\perp J_{n-1}(\Omega) , \tag{37}$$

$$\omega_\alpha''-i\gamma_\alpha''/2=\omega_\alpha+\frac{(p_2\epsilon_z)^2/4}{\omega_{\alpha e}-\nu+i\gamma_p/2} , \tag{38}$$

$$\omega_\beta''-i\gamma_\beta''/2=\omega_\beta+\frac{(p_2E_m)^2/4}{\omega_{\beta e}+i\gamma_p/2} , \tag{39}$$

and we have kept only the terms which are capable of resonance. If we turn off all but the nth term in the off-diagonal matrix element in (36) we can solve for the amplitude b_n for the ion to be in the β state at $t=\tau$. Given the initial conditions $a=1$, $b_n=0$ at $t=0$ we obtain the approximate expression

$$b_n=\Lambda_n e^{in\varphi_m}\left[\frac{e^{-i(\omega_{\alpha\beta}''+n\omega_m-\nu)\tau-\gamma_\alpha\tau/2}-e^{-\gamma_\beta''\tau/2}}{\omega_{\alpha\beta}''+n\omega_m-\nu-i(\gamma_\alpha''-\gamma_\beta'')/2}\right] . \tag{40}$$

Assuming the perturbations are very weak the full solution is given by

$$b=\sum_n b_n . \tag{41}$$

The relative sizes of the terms in this expression depend on the argument Ω of the Bessel functions in Eq. (37). For the $\alpha\rightarrow\beta$ resonance in He^+, $\Omega\approx7\times10^3(v_\perp/c)$. A typical value for $v_\perp/c$ is 2×10^{-5} (corresponding to an energy of about 1 eV), for which $\Omega\approx0.14$. As $\Omega\rightarrow0$, $J_n(\Omega)$ is approximately[20]

$$J_n(\Omega) \approx (\Omega/2)^n/n! \,. \tag{42}$$

This drops rapidly with n and so only a few of the terms in Eq. (41) need to be considered.

The term b_0 which is resonant at $\nu=\omega''_{\alpha\beta}$ is the main $M1$ resonance. It is proportional to

$$\Lambda_0=(\mu_0 m_x/2)J_0(\Omega)-(p_2^2\epsilon_z E_m/2)\Delta_\perp J_{-1}(\Omega) \,. \tag{43}$$

An interesting feature of this $M1$ resonance is that there is no Doppler broadening from the transverse motion. The Doppler effect gives rise to a pattern of lines spaced at intervals of ω_m but, providing we can separately resolve these lines, they are not broadened. In the parity experiments under discussion, the linewidth would be of order 1 MHz and the ion cyclotron frequency would be about 6 MHz, so these lines should be easily resolved. The second term in Eq. (43) in induced by the motional electric field. It is suppressed by the Bessel function $J_{-1}(\Omega)$ and since both E_m and the Bessel function coefficient [assuming the approximate form of Eq. (42)] are proportional to $v_\perp/c$, this term is quadratic in $v_\perp/c$. Thus, one way to eliminate possible systematic errors arising from the motional electric field is to make $v_\perp/c$ sufficiently small. It is of interest to find the value $(v_\perp/c)_{\rm eq}$ for which the magnitude of the motional-field-induced amplitude and the weak-induced amplitude are equal. Using Eq. (42) for the coefficient of the motional field term we find

$$(v_\perp/c)_{\rm eq}=[2(\overline{V}C_1/p_2B)(\Delta_\parallel/\Delta_\perp)(\epsilon_\perp/\epsilon_z)]^{1/2} \,. \tag{44}$$

If we assume $(\epsilon_\perp/\epsilon_z)=20$, and that the magnetic field is set at the β-f crossing point, we find $(v_\perp/c)_{\rm eq}\approx 3\times 10^{-5}$ which corresponds to a transverse energy of a little more than 1 V. It is not unreasonable to require that an ion beam developed for a He^+ parity experiment have transverse velocity less than this, but it does indicate that care must be taken in the design of a suitable ion source.

The main motional-field-induced amplitude is contained in the term b_1 which is proportional to

$$\Lambda_1=(\mu_0 m_x/2)J_1(\Omega)-(p_2^2\epsilon_z E_m/2)\Delta_\perp J_0(\Omega) \,. \tag{45}$$

This term is resonant at $\nu=\omega''_{\alpha\beta}+\omega_m$, and it should be well resolved from the main $M1$ resonance at $\nu=\omega''_{\alpha\beta}$. There can be an interference term between the main $M1$ amplitude and the tail of the main motional field amplitude, but this will tend to cancel in an average over the metastable beam because of the factor $e^{i\varphi_m}$ in Eq. (40). The phase φ_m depends on the initial direction of the transverse velocity, and this will be random. The fact that the main motional field resonance can be separately resolved means that this potential systematic effect can be studied in an amplified form. In addition, the weak-induced amplitude which is resonant at $\nu=\omega''_{\alpha\beta}+\omega_m$ would be suppressed by the factor $J_1(\Omega)$ so the motional field resonance provides a null experiment that can be useful in searching for possible systematic effects in the measurement.

VI. CONCLUSION

We have considered the prospects for microwave parity experiments in singly ionized helium which would measure the weak interaction coupling constants C_{1p} and C_{1n}. Such experiments appear to be feasible on signal-to-noise grounds and would typically involve the measurement of an asymmetry of 10^{-6} and a minimum counting time of half a day. The experiments require a large (>10 kG) magnetic field and a slow, intense metastable beam. The relatively high microwave frequencies that would be required in these experiments (mm waves) lead naturally to the use of so called quasioptical techniques and we have discussed some ideas that appear promising. Perhaps the most difficult aspect of the experiments involves systematic errors involving unwanted electric fields. We considered one such effect in some detail: the systematic error caused by motional electric fields. We showed that the cyclotron motion of the ions in the magnetic field leads to a supression of the unwanted effects and this problem appears to be tractable in the He^+ parity experiments.

It has not been possible to discuss all the details which would have to be covered in a specific proposal for a viable parity experiment in He^+ but we feel that an experiment of this type should be given serious consideration. There is no question that it would be quite difficult and would require considerable effort and commitment in order to produce results. On the other hand, the potential for such an experiment to test radiative corrections to the electroweak theory is real, and this is a matter of great importance which will be the motivation for large scale efforts in high-energy physics in the coming years.

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