Branching ratios for the decay of $n=3$ hydrogen atoms in axial and transverse electric fields

N. Rouze,* C. C. Havener,† W. B. Westerveld, and J. S. Risley
Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202
(Received 13 June 1985)

The branching ratios for the $n=3$ to $n=2$ Balmer-α decay of hydrogen atoms in axial and transverse electric fields in the range 0—1000 V/cm have been calculated with use of a density-matrix formalism to take into account the time evolution of the atomic states in the presence of an electric field. The branching ratios are useful when the production of $2s$ hydrogen atoms is measured with the use of an electric field and when it is desired to correct for cascade contributions from the $n=3$ level. The total $n=3$ to $n=2$ branching ratio is found to depend on each of the 14 independent quantities which determine the axially symmetric $n=3$ density matrix, thus emphasizing the need to determine the complete density matrix including the off-diagonal coherence terms. If the off-diagonal density-matrix elements are not known, it is preferable to use transverse electric fields since, in this configuration, the contributions to the branching ratios from the off-diagonal terms are less than with axial electric fields. For transverse fields of approximately 200 V/cm, the contribution from the off-diagonal terms are nearly zero.

I. INTRODUCTION

This paper presents the branching ratios for the $n=3$ to $n=2$ Balmer-α decay from hydrogen atoms in axial and transverse electric fields in the range 0—1000 V/cm. The branching ratios are useful when it is desired to measure the cross section for the production of metastable $2s$ hydrogen atoms by applying an electric field to mix the $2s$ and $2p$ states, allowing the atoms to decay by emitting Lyman-α radiation. To correct for cascade contributions to the $n=2$ level, branching ratios for the decay of higher-n levels to the $n=2$ level in an electric field are needed. This paper presents these branching ratios for the case of $n=3$. Examples of the use of these branching ratios are provided in the study of H + Ne collisions by Van Zyl, Gealy, and Neumann[1] and the study of H + He and D + He collisions by Grosser and Krüger.[2]

The key result presented below is that the total $n=3$ to $n=2$ branching ratio depends not only on the cross sections for producing the individual $n=3$ excited states but also on the coherent excitation of different states. This result emphasizes the need for a complete description of atoms formed in atomic collision processes including coherent excitations.

The branching ratios were calculated using a density-matrix formalism to account for the time evolution of the excited atomic states in the presence of an electric field. This formalism has been used by Havener *et al.*[3] to determine the density matrix which describes $n=3$ hydrogen atoms formed in $H^+ + He$ electron-transfer collisions. The density matrix gives a complete characterization of collisionally produced atoms and includes the diagonal elements which are proportional to the cross sections for producing the individual nlm_l sublevels and the off-diagonal elements which give the coherences between sublevels.

II. THEORY

It is assumed that the excited hydrogen atoms are formed in collisions which exhibit cylindrical symmetry about the projectile beam axis and reflection symmetry through any plane containing the beam axis. It is also assumed that LS coupling holds during the collision so that the collisionally produced hydrogen atoms can be described by a density matrix ρ^L in the (l,m_l) representation. As shown in Ref. 3, ρ^L is determined by 14 independent quantities, six diagonal elements corresponding to the cross sections for producing the $3s_0$, $3p_0$, $3p_{\pm1}$, $3d_0$, $3d_{\pm1}$, and $3d_{\pm2}$ states and the real and imaginary parts of the s_0p_0, s_0d_0, p_0d_0, and $p_{\pm1}d_{\pm1}$ off-diagonal elements. Thus, 14 branching ratios are given as a function of applied electric field. Summing these values with the appropriate weighting factors gives the branching ratio for any possible density matrix.

The branching ratio for a particular $n=3$ density matrix is given by the total probability for decay from the $n=3$ level to the $n=2$ level, summed over all polarizations of the emitted radiation and integrated over all emission angles. The instantaneous transition rate $A(\boldsymbol{\varepsilon}_i,t)$ for the emission of Balmer-α photons of polarization $\boldsymbol{\varepsilon}_i$ at a time t into a solid angle $\Delta\Omega$ is given by

$$A(\boldsymbol{\varepsilon}_i,t)=\frac{e^2\omega^3}{2\pi\hbar c^3}\Delta\Omega \sum_{\substack{lm_l,(l'm_l')\\ l_fm_{l_f}}} \langle l_fm_{l_f}\,|\,\boldsymbol{\varepsilon}_i^*\cdot\mathbf{r}\,|\,lm_l\rangle \times\rho^L_{lm_l,l'm_l'}(t) \times\langle l'm_l'\,|\,\mathbf{r}\cdot\boldsymbol{\varepsilon}_i\,|\,l_fm_{l_f}\rangle\ , \quad (1)$$

 ©1986 The American Physical Society

where the average over the initial states lm_l of the $n=3$ level is weighted by their time-dependent populations which are given by elements of the $n=3$ density matrix $\rho^L(t)$. The sum over the final states $l_f m_{l_f}$ includes only the $n=2$ states.

The probability for decay from the $n=3$ to the $n=2$ level $P(\varepsilon_i)$ with the emission of a photon of polarization ε_i into a solid angle $\Delta\Omega$ is given by the time integration of the transition rate,

$$P(\varepsilon_i)=\int_0^\infty A(\varepsilon_i,t)dt$$
$$=\frac{e^2\omega^2}{2\pi\hbar c^3}\frac{\Delta\Omega}{\Gamma_{\rm av}}\sum_{\substack{lm_l,(l'm_l')\\ l_f m_{l_f}}}\langle l_f m_{l_f}|\varepsilon_i^*\cdot\mathbf{r}|lm_l\rangle\tilde{\rho}^L_{lm_l,l'm_l'}\langle l'm_l'|\mathbf{r}\cdot\varepsilon_i|l_f m_{l_f}\rangle\ , \tag{2}$$

where

$$\tilde{\rho}^L_{lm_l,l'm_l'}=\Gamma_{\rm av}\int_0^\infty\rho^L_{lm_l,l'm_l'}(t)dt\ , \tag{3}$$

where $\Gamma_{\rm av}=99.852$ MHz is the average decay rate for the $n=3$ manifold.[4] The factor of $\Gamma_{\rm av}$ in Eqs. (2) and (3) is included so that the units of $\tilde{\rho}$ and ρ are the same.

By summing over the final $n=2$ states in Eqs. (1) and (2), only isotropic contributions to the $n=2$ populations are determined. Thus, it is assumed that cascade contributions to the polarization and angular distribution of the $n=2$ radiation are not needed. Similarly, by integrating over all time in Eq. (3), it is assumed that the $n=2$ atoms are observed for a long period of time compared to the $n=3$ lifetime. If these criteria are not applicable, the time-dependent contributions to the individual elements of the $n=2$ density matrix must be determined so that an integration can be performed over the observation time of the $n=2$ atoms.

The procedure which was used to obtain the time-integrated density matrix $\tilde{\rho}^L$ in Eq. (2) has been described in Ref. 3. Briefly, the initial density matrix $\rho^L(t=0)$ in the lm_l representation was transformed to $\rho^J(t=0)$ in the $lsjm_j$ representation by coupling in an unpolarized spin. The time-evolved density matrix $\rho^J(t)$ was determined by using the complex Hamiltonian which included terms to account for the fine structure and Lamb-shift splittings, the interaction with the electric field, and the radiative decay. The eigenvalues λ_i of the Hamiltonian matrix H^J were determined using the matrix S which diagonalized H^J. S and the λ_i were used to express the time dependence of $\rho^J(T)$ as exponential factors so that the time-integrated density matrix $\tilde{\rho}^J$ could be written as

$$\tilde{\rho}^J_{rs}=\Gamma_{\rm av}\sum_{k,l,m,n}S_{rk}(S^{-1})_{kl}\rho^J_{lm}(t=0)\times[(S^\dagger)^{-1}]_{mn}(S^\dagger)_{ns}\times\int_0^\infty e^{-i(\lambda_k-\lambda_n^*)t/\hbar}dt\ . \tag{4}$$

Performing the integration in Eq. (4) gives

$$\tilde{\rho}^J_{rs}=\Gamma_{\rm av}\sum_{k,l,m,n}S_{rk}(S^{-1})_{kl}\rho^J_{lm}(t=0)\times[(S^\dagger)^{-1}]_{mn}(S^\dagger)_{ns}\frac{\hbar}{i(\lambda_k-\lambda_n^*)}\ . \tag{5}$$

The time-integrated density matrix $\tilde{\rho}^L$ was found by taking the trace over the spin.

As shown in Eq. (29), Eq. (30), and Appendix B of Ref. 3, $P(\varepsilon_i)$ may be written as

$$P(\varepsilon_i)=\frac{\Delta\Omega}{\Gamma_{\rm av}}\varepsilon_i^*\cdot\overleftrightarrow{C}\cdot\varepsilon_i\ , \tag{6}$$

where the Cartesian tensor $\overleftrightarrow{C}$ gives the intensity and polarization of the radiation emitted in any direction. As shown in Appendix B of Ref. 3, $\overleftrightarrow{C}$ may be written in terms of a set of orthonormal 3×3 basis tensors $\overleftrightarrow{S}_{kq}$,

$$\overleftrightarrow{C}=\sum_{k,q}c_{kq}\overleftrightarrow{S}^\dagger_{kq} \tag{7}$$

where the c_{kq} are given by

$$c_{kq}=\frac{e^2\omega^3}{2\pi\hbar c^3}\sum_{l,l',l_f}(-1)^{l+l'+k+1}\langle l_f||\mathbf{r}||l\rangle\langle l'||\mathbf{r}||l_f\rangle\times\tilde{\rho}^L_{kq}(ll')\begin{Bmatrix}1&k&1\\ l&l_f&l'\end{Bmatrix}, \tag{8}$$

and the $\tilde{\rho}^L_{kq}$ are the multipole moments of the time-integrated density matrix $\tilde{\rho}^L$ and are given by Eq. (B9) of Ref. 3. Expressions for the $\overleftrightarrow{S}_{kq}$ are given by Eq. (B10) of Ref. 3 and by Carrington.[5]

The branching ratio B is determined by integrating Eq. (6) over all emission directions (described by the angles θ and ϕ) and summing over all polarizations of the emitted radiation,

$$B=\sum_i\int_0^{2\pi}\int_0^\pi\frac{1}{\Gamma_{\rm av}}\varepsilon_i^*\cdot\overleftrightarrow{C}\cdot\varepsilon_i\sin\theta\,d\theta\,d\phi\ . \tag{9}$$

The sum and integrals can be performed using Eq. (7), explicit expressions for the $\overleftrightarrow{S}_{kq}$,[5] and two polarization vectors which are orthogonal to the emission direction,

$$B=\frac{8\pi c_{00}}{\sqrt{3}\Gamma_{\rm av}}\ . \tag{10}$$

The branching ratios were calculated for axial electric fields both parallel and antiparallel to the projectile beam axis and for transverse electric fields perpendicular to the projectile beam axis. The procedure outlined above was applied to 14 9×9 basis matrices M_i which corresponded to the 14 independent elements of the $n=3$ density ma-

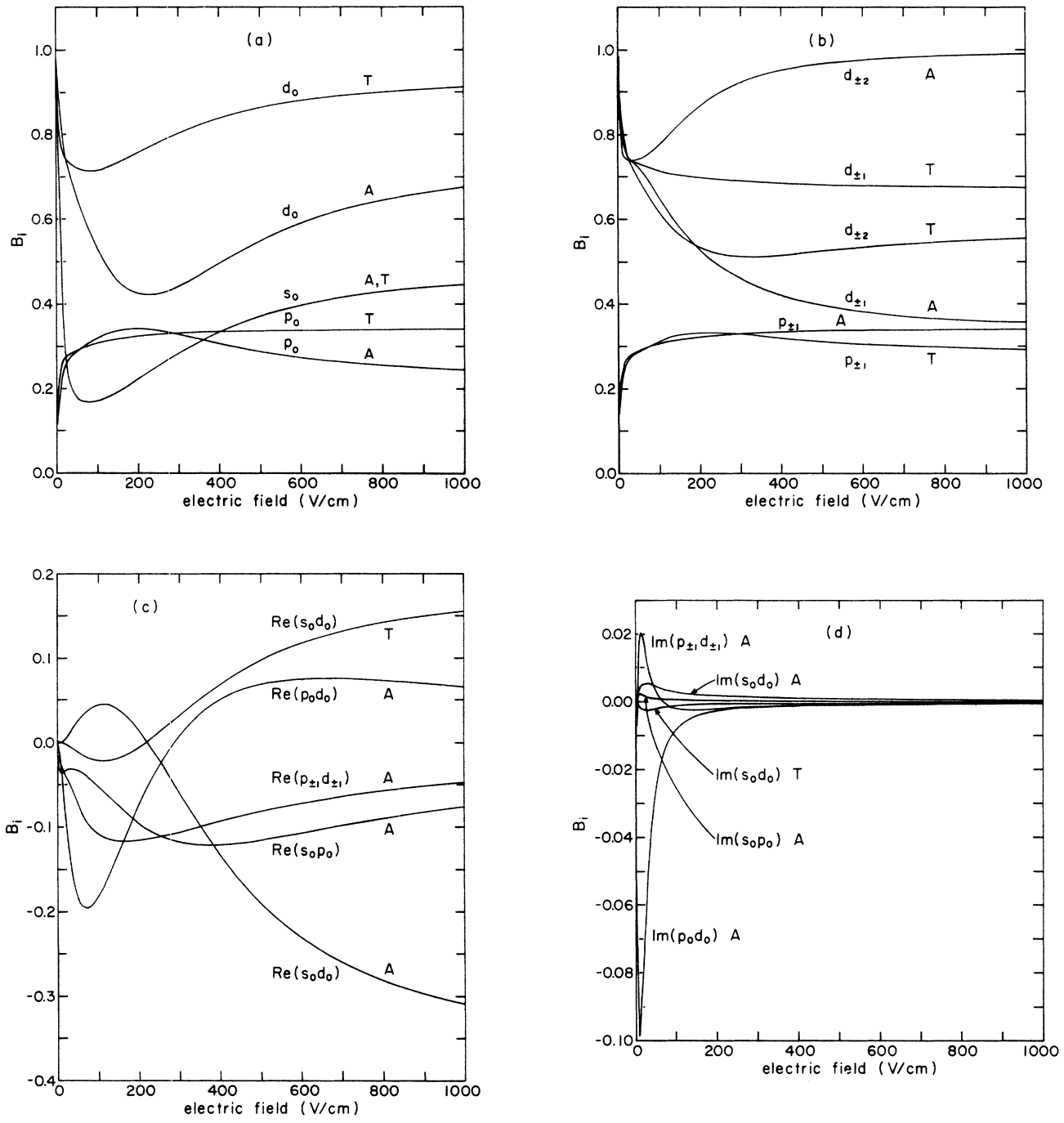

FIG. 1. $n=3$ to $n=2$ branching ratios B_i for $n=3$ hydrogen atoms in axial (A) electric fields directed parallel to the projectile beam axis and transverse (T) electric fields directed perpendicular to the projectile beam axis in the range 0—1000 V/cm. Included are the branching ratios for (a) $i=s_0$, p_0, and d_0; (b) $i=p_{\pm 1}$, $d_{\pm 1}$, and $d_{\pm 2}$; (c) $i=\mathrm{Re}(s_0p_0)$, $\mathrm{Re}(s_0d_0)$, $\mathrm{Re}(p_0d_0)$, and $\mathrm{Re}(p_{\pm 1}d_{\pm 1})$; and (d) $i=\mathrm{Im}(s_0p_0)$, $\mathrm{Im}(s_0d_0)$, $\mathrm{Im}(p_0d_0)$, and $\mathrm{Im}(p_{\pm 1}d_{\pm 1})$. For axial electric fields directed antiparallel to the projectile beam axis, $B_i(-E)=B_i(E)$ for $i=s_0$, p_0, $p_{\pm 1}$, d_0, $d_{\pm 1}$, $d_{\pm 2}$, $\mathrm{Re}(s_0d_0)$, and $\mathrm{Im}(s_0d_0)$ and $B_i(-E)=-B_i(E)$ for $i=\mathrm{Re}(s_0p_0)$, $\mathrm{Im}(s_0p_0)$, $\mathrm{Re}(p_0d_0)$, $\mathrm{Im}(p_0d_0)$, $\mathrm{Re}(p_{\pm 1}d_{\pm 1})$, and $\mathrm{Im}(p_{\pm 1}d_{\pm 1})$. For transverse electric fields, $B_i=0$ for all off-diagonal terms except $\mathrm{Re}(s_0d_0)$ and $\mathrm{Im}(s_0d)$.

trix. For example, the basis matrix $M_{p_{\pm 1}}$ corresponding to the $p_{\pm 1}$ states had two nonzero elements, $(M_{p_{\pm 1}})_{p_{+1}p_{+1}}=1$ and $(M_{p_{\pm 1}})_{p_{-1}p_{-1}}=1$. Similarly, the basis matrix corresponding to the imaginary parts of the $p_{\pm 1}d_{\pm 1}$ coherence terms had four nonzero elements, $(M_{\mathrm{Im}(p_{\pm 1}d_{\pm 1})})_{p_{+1}d_{+1}}=i$, $(M_{\mathrm{Im}(p_{\pm 1}d_{\pm 1})})_{p_{-1}d_{-1}}=i$, $(M_{\mathrm{Im}(p_{\pm 1}d_{\pm 1})})_{d_{+1}p_{+1}}=-i$, and $(M_{\mathrm{Im}(p_{\pm 1}d_{\pm 1})})_{d_{-1}p_{-1}}=-i$. Any density matrix ρ may be written as a superposition of

TABLE I. $n=3$ to $n=2$ branching ratios B_i for $n=3$ hydrogen atoms in axial electric fields directed parallel to the projectile beam axis. For axial electric fields directed antiparallel to the projectile beam axis, $B_i(-E)=B_i(E)$ for $i=s_0$, p_0, $p_{\pm1}$, d_0, $d_{\pm1}$, $d_{\pm2}$, $\mathrm{Re}(s_0d_0)$, and $\mathrm{Im}(s_0d_0)$ and $B_i(-E)=-B_i(E)$ for $i=\mathrm{Re}(s_0p_0)$, $\mathrm{Im}(s_0p_0)$, $\mathrm{Re}(p_0d_0)$, $\mathrm{Im}(p_0d_0)$, $\mathrm{Re}(p_{\pm1}d_{\pm1})$, and $\mathrm{Im}(p_{\pm1}d_{\pm1})$.

Electric field (V/cm)	B_{s_0}	B_{p_0}	$B_{p_{\pm1}}$	B_{d_0}	$B_{d_{\pm1}}$	$B_{d_{\pm2}}$	$B_{\mathrm{Re}(s_0p_0)}$	$B_{\mathrm{Im}(s_0p_0)}$	$B_{\mathrm{Re}(s_0d_0)}$	$B_{\mathrm{Im}(s_0d_0)}$	$B_{\mathrm{Re}(p_0d_0)}$	$B_{\mathrm{Im}(p_0d_0)}$	$B_{\mathrm{Re}(p_{\pm1}d_{\pm1})}$	$B_{\mathrm{Im}(p_{\pm1}d_{\pm1})}$
0	1.000	0.118	0.118	1.000	1.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.578	0.196	0.244	0.867	0.840	0.764	−0.037	0.002	0.000	0.003	−0.020	−0.099	−0.027	0.016
20	0.317	0.250	0.271	0.766	0.766	0.743	−0.035	0.002	0.002	0.005	−0.076	−0.079	−0.038	0.019
30	0.231	0.270	0.280	0.719	0.742	0.739	−0.032	0.001	0.008	0.005	−0.124	−0.055	−0.048	0.012
40	0.196	0.280	0.285	0.686	0.732	0.739	−0.032	0.001	0.015	0.005	−0.158	−0.039	−0.059	0.007
50	0.180	0.287	0.289	0.657	0.722	0.741	−0.034	0.001	0.022	0.005	−0.179	−0.028	−0.070	0.004
60	0.172	0.293	0.293	0.629	0.711	0.745	−0.036	0.001	0.028	0.004	−0.191	−0.021	−0.081	0.001
70	0.169	0.300	0.297	0.602	0.698	0.750	−0.039	0.001	0.033	0.004	−0.196	−0.016	−0.090	0.000
80	0.168	0.306	0.301	0.576	0.684	0.757	−0.043	0.001	0.038	0.003	−0.196	−0.013	−0.098	−0.001
90	0.169	0.312	0.304	0.553	0.668	0.765	−0.047	0.000	0.041	0.003	−0.191	−0.010	−0.103	−0.002
100	0.171	0.318	0.307	0.531	0.653	0.774	−0.051	0.000	0.043	0.003	−0.185	−0.009	−0.108	−0.002
150	0.192	0.337	0.317	0.457	0.584	0.823	−0.075	0.000	0.039	0.002	−0.134	−0.004	−0.117	−0.003
200	0.222	0.341	0.323	0.426	0.531	0.867	−0.096	0.000	0.015	0.002	−0.079	−0.003	−0.116	−0.002
250	0.253	0.337	0.327	0.425	0.491	0.899	−0.110	0.000	−0.020	0.001	−0.032	−0.002	−0.112	−0.002
300	0.284	0.328	0.331	0.443	0.461	0.922	−0.118	0.000	−0.059	0.001	0.005	−0.002	−0.107	−0.002
350	0.311	0.317	0.333	0.468	0.438	0.939	−0.121	0.000	−0.098	0.001	0.031	−0.001	−0.101	−0.002
400	0.334	0.306	0.335	0.496	0.421	0.951	−0.122	0.000	−0.133	0.001	0.049	−0.001	−0.094	−0.001
450	0.354	0.296	0.337	0.524	0.407	0.960	−0.119	0.000	−0.163	0.001	0.061	−0.001	−0.088	−0.001
500	0.372	0.287	0.338	0.549	0.397	0.967	−0.116	0.000	−0.190	0.001	0.068	−0.001	−0.083	−0.001
550	0.386	0.280	0.339	0.571	0.388	0.972	−0.112	0.000	−0.212	0.001	0.072	−0.001	−0.077	−0.001
600	0.398	0.273	0.339	0.590	0.382	0.976	−0.108	0.000	−0.231	0.000	0.074	−0.001	−0.073	−0.001
650	0.408	0.268	0.340	0.607	0.376	0.980	−0.103	0.000	−0.247	0.000	0.075	0.000	−0.068	−0.001
700	0.416	0.263	0.340	0.621	0.372	0.982	−0.099	0.000	−0.260	0.000	0.074	0.000	−0.064	−0.001
750	0.424	0.259	0.341	0.634	0.369	0.984	−0.095	0.000	−0.272	0.000	0.073	0.000	−0.061	−0.001
800	0.430	0.255	0.341	0.645	0.366	0.986	−0.091	0.000	−0.282	0.000	0.072	0.000	−0.058	−0.001
850	0.435	0.252	0.341	0.654	0.363	0.988	−0.087	0.000	−0.290	0.000	0.070	0.000	−0.055	−0.001
900	0.440	0.249	0.341	0.662	0.361	0.989	−0.083	0.000	−0.297	0.000	0.068	0.000	−0.052	−0.001
950	0.444	0.247	0.341	0.669	0.359	0.990	−0.080	0.000	−0.304	0.000	0.066	0.000	−0.050	−0.001
1000	0.447	0.245	0.342	0.675	0.358	0.991	−0.077	0.000	−0.309	0.000	0.064	0.000	−0.048	−0.001

TABLE II. $n=3$ to $n=2$ branching ratios B_i for $n=3$ hydrogen atoms in transverse electric fields perpendicular to the projectile beam axis. For all off-diagonal terms except $\mathrm{Re}(s_0d_0)$ and $\mathrm{Im}(s_0d_0)$, $B_i=0$.

Electric field (V/cm)	B_{s_0}	B_{p_0}	$B_{p_{\pm1}}$	B_{d_0}	$B_{d_{\pm1}}$	$B_{d_{\pm2}}$	$B_{\mathrm{Re}(s_0d_0)}$	$B_{\mathrm{Im}(s_0d_0)}$
0	1.000	0.118	0.118	1.000	1.000	1.000	0.000	0.000
10	0.578	0.244	0.220	0.790	0.802	0.841	0.000	−0.002
20	0.317	0.271	0.261	0.748	0.754	0.763	−0.001	−0.003
30	0.231	0.280	0.275	0.734	0.741	0.733	−0.004	−0.003
40	0.196	0.285	0.282	0.726	0.735	0.715	−0.007	−0.003
50	0.180	0.289	0.288	0.720	0.732	0.700	−0.011	−0.002
60	0.172	0.293	0.293	0.716	0.728	0.684	−0.014	−0.002
70	0.169	0.297	0.299	0.713	0.724	0.668	−0.017	−0.002
80	0.168	0.301	0.304	0.712	0.720	0.653	−0.019	−0.002
90	0.169	0.304	0.308	0.712	0.717	0.637	−0.021	−0.001
100	0.171	0.307	0.313	0.713	0.714	0.623	−0.022	−0.001
150	0.192	0.317	0.327	0.732	0.704	0.566	−0.019	−0.001
200	0.222	0.323	0.332	0.756	0.699	0.534	−0.008	−0.001
250	0.253	0.327	0.332	0.781	0.695	0.518	0.010	−0.001
300	0.284	0.331	0.329	0.802	0.692	0.512	0.030	−0.001
350	0.311	0.333	0.325	0.821	0.689	0.512	0.049	0.000
400	0.334	0.335	0.321	0.837	0.686	0.515	0.066	0.000
450	0.354	0.337	0.317	0.851	0.684	0.520	0.082	0.000
500	0.372	0.338	0.313	0.862	0.682	0.525	0.095	0.000
550	0.386	0.339	0.309	0.872	0.680	0.530	0.106	0.000
600	0.398	0.339	0.306	0.880	0.679	0.534	0.115	0.000
650	0.408	0.340	0.304	0.886	0.678	0.538	0.123	0.000
700	0.416	0.340	0.302	0.892	0.677	0.542	0.130	0.000
750	0.424	0.341	0.300	0.897	0.677	0.545	0.136	0.000
800	0.430	0.341	0.298	0.901	0.676	0.548	0.141	0.000
850	0.435	0.341	0.297	0.904	0.675	0.550	0.145	0.000
900	0.440	0.341	0.295	0.907	0.675	0.552	0.149	0.000
950	0.444	0.341	0.294	0.910	0.675	0.554	0.152	0.000
1000	0.447	0.342	0.293	0.912	0.674	0.556	0.155	0.000

TABLE III. Illustration of the use of Eq. (12) to calculate the total $n=3$ to $n=2$ branching ratio for $n=3$ hydrogen atoms in a 500-V/cm axial electric field. The $n=3$ density matrix is normalized so that $\mathrm{Tr}(\rho)=1$. See Table I or Fig. 1 for B_i.

Element i	Density matrix Value (example) ρ_i	Number of nonzero elements in basis matrix N_i	Branching ratio B_i	Weighted contribution $\rho_iN_iB_i$
s_0	0.45	1	0.372	0.167
p_0	0.1	1	0.287	0.029
$p_{\pm1}$	0.1	2	0.338	0.068
d_0	0.05	1	0.549	0.027
$d_{\pm1}$	0.05	2	0.397	0.040
$d_{\pm2}$	0.05	2	0.967	0.097
$\mathrm{Re}(s_0p_0)$	0.15	2	−0.116	−0.035
$\mathrm{Im}(s_0p_0)$	−0.1	2	0.000	0.000
$\mathrm{Re}(s_0d_0)$	0.1	2	−0.190	−0.038
$\mathrm{Im}(s_0d_0)$	0.05	2	0.001	0.000
$\mathrm{Re}(p_0d_0)$	0.06	2	0.068	0.008
$\mathrm{Im}(p_0d_0)$	0.03	2	−0.001	0.000
$\mathrm{Re}(p_{\pm1}d_{\pm1})$	−0.05	4	−0.083	0.017
$\mathrm{Im}(p_{\pm1}d_{\pm1})$	0.04	4	−0.001	0.000
			Sum B	=0.380

these 14 basis matrices, weighted by the values ρ_i of the density-matrix elements,

$$\rho = \sum_{i=1}^{14} \rho_i M_i \ . \tag{11}$$

For each basis matrix, the value determined by Eq. (10) was divided by the number of nonzero elements N_i in that matrix, yielding the branching ratio B_i. Thus, the values B_i presented below correspond to the branching ratios for individual density-matrix elements. The total branching ratio for the density matrix is given by

$$B = \sum_{i=1}^{14} \rho_i N_i B_i \ . \tag{12}$$

III. RESULTS AND DISCUSSION

Figure 1 shows the $n=3$ to $n=2$ branching ratios B_i for hydrogen atoms in axial and transverse electric fields in the range 0—1000 V/cm. The branching ratios for specific values of electric field are given in Tables I and II for axial and transverse fields, respectively. Table III demonstrates the use of Eq. (12) to weight and sum the individual values to determine the total branching ratio.

When the applied electric field is zero, the branching ratios demonstrate the well-known decay scheme, i.e., atoms in the $3s$ and $3d$ states decay only to the $n=2$ level and 11.8% of the $3p$ atoms decay to the $n=2$ level. When the electric field is applied, the branching ratios change dramatically. The $3s$ and $3d$ states are mixed with the $3p$ states by the electric field causing some atoms to decay to the $n=1$ level, decreasing the branching ratio to the $n=2$ level. Similarly, the $3p$ states mix with the $3s$ and $3d$ states, increasing the branching ratio to the $n=2$ level.

The branching ratios for axial and transverse electric fields are not equal, except for the $3s$ state. This inequality is expected since the projectile beam defines a z axis and states with different magnetic quantum numbers are not equivalent. Notice, however, that the sums of the branching ratios, $B_{p_{+1}}+B_{p_0}+B_{p_{-1}}$ and $B_{d_{+2}}+B_{d_{+1}}+B_{d_0}+B_{d_{-1}}+B_{d_{-2}}$, are equal for the two orientations of electric field since, when the magnetic sublevel populations are equal, the electronic probability distribution is spherically symmetric.

An interesting result is that the $n=3$ to $n=2$ branching ratio depends on the off-diagonal elements of the $n=3$ density matrix. For the case of axial electric fields, each off-diagonal element contributes to the branching ratio. The contribution from elements between states with opposite parity changes sign for electric fields applied parallel and antiparallel to the projectile axis. For transverse electric fields, only the $s_0 d_0$ density-matrix element has a nonzero branching ratio and the magnitude of this ratio is one-half the ratio for an axial electric field. Thus, if the off-diagonal density-matrix elements are not known, it is preferable to use transverse electric fields since the contributions to the $n=3$ to $n=2$ branching ratio from these elements is less. In particular, the contributions from both $\mathrm{Re}(s_0 d_0)$ and $\mathrm{Im}(s_0 d_0)$ are nearly zero for transverse fields of approximately 200 V/cm. Alternatively, with axial electric fields, the contributions from the off-diagonal density-matrix elements between states with opposite parity can be found by reversing the polarity of the field.

As indicated in the review of Thomas,[6] the cross sections for producing the $3s$ state are larger than the $3p$ and $3d$ cross sections in many collisions. For example, for $H(n=3)$ atoms produced in electron-transfer collisions of H^+ on He, Hughes *et al.*[7] found the $3s$ cross section to be larger than the $3p$ and $3d$ cross sections for collision energies greater than 30 keV. Similarly, Van Zyl, Gealy, and Neumann[1] found that in H + Ne collisions the cross sections for exciting the hydrogen $3p$, $3d$, $4p$, and $4d$ states were unusually small compared to the $3s$ cross section. In these cases, it is often valid to neglect the contributions to the branching ratios from these sparsely populated states. Additionally, the contribution to the branching ratio from the off-diagonal density-matrix elements is restricted since the magnitude of these elements is limited,

$$|\rho^L_{lm_l,l'm_l'}| \le (\rho^L_{lm_l,lm_l}\rho^L_{l'm_l',l'm_l'})^{1/2} \ . \tag{13}$$

Thus, if particular diagonal elements are small, the corresponding off-diagonal elements will be small and it may be possible to neglect contributions from these elements.

IV. CONCLUSION

To accurately determine the population of $n=2$ hydrogen atoms which have been produced in various atomic collision processes, cascade contributions from the $n=3$ level are needed. These contributions can be determined if the $n=3$ populations are known along with the $n=3$ to $n=2$ branching ratios. When an electric field is applied to measure the population of metastable $2s$ hydrogen atoms, the branching ratios for $n=3$ atoms in an electric field are needed. This paper gives these branching ratios for electric fields in the range 0—1000 V/cm.

The key result presented is that, when an electric field is applied, the total $n=3$ to $n=2$ branching ratio depends on each independent element of the density matrix which completely describes the $n=3$ hydrogen atoms. This result emphasizes the need to determine the off-diagonal coherence terms as well as the diagonal cross-section terms when studying atomic collision processes. If the values of the off-diagonal density-matrix elements are not known, it is preferable to use transverse electric fields since contributions to the total branching ratio from these elements are less than with axial fields.

ACKNOWLEDGMENTS

This work was supported in part by the Atomic, Molecular, and Plasma Physics Program of the National Science Foundation, under Grant No. PHY-82-8905.

*Present address: Department of Physics, Hope College, Holland, MI 49423.
†Present address: Oak Ridge National Laboratory, Oak Ridge, TN 37831.

[1]B. Van Zyl, M. W. Gealy, and H. Neumann, Phys. Rev. A **31**, 2922 (1985).
[2]J. Grosser and W. Krüger, Z. Phys. A **318**, 25 (1984).
[3]C. C. Havener, N. Rouze, W. B. Westerveld, and J. S. Risley, preceding paper, Phys. Rev. A **33**, 276 (1986).
[4]H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Plenum, New York, 1977).
[5]C. G. Carrington, J. Phys. B **4**, 1222 (1971).
[6]E. W. Thomas, *Excitation in Heavy Particle Collisions* (Wiley-Interscience, New York, 1972).
[7]R. H. Hughes, C. A. Stigers, B. M. Doughty, and E. D. Stokes, Phys. Rev. A **1**, 1424 (1970).