Coherent properties of the stimulated emission from a three-level atom

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First-order correlation functions of the electric field and photon-number probabilities in the case of a three-level atom interacting with two-mode radiation field are obtained to investigate the coherent properties of the stimulated emission under different initial conditions. It is found that double stimulation will cause the field to approach its initial coherent state and the oscillation of the photon-number probabilities to collapse and revive.

I. INTRODUCTION

In this paper we investigate the coherent properties of the stimulated emission in the case of a three-level atom excited by two-mode field. We shall consider the case that initially a "Λ" structure three-level atom is in its excited state and comes into interaction with one- or two-mode radiation field, of which one mode is coherent but the other chaotic or both coherent or chaotic, and study the extent to which the field from stimulated emission will also be coherent through the comparison between $\langle E_1^- E_1^+ \rangle$ and $\langle E_1^- \rangle \langle E_1^+ \rangle$ (Ref. 1) and of photon-number probabilities under different conditions. In addition, the coherent behavior of the photon-number probabilities beyond the short-time regime is also studied.

II. FORMULATION AND COMPARISON

The "Λ" structure three-level atom[2–4] has a common upper level $|a\rangle$ and two lower levels $|b_1\rangle$ and $|b_2\rangle$. The transition between $|a\rangle$ and $|b_1\rangle$ ($|b_2\rangle$) is mediated by mode 1(2) with frequency Ω_1 (Ω_2). The Hamiltonian of this system in the rotating-wave approximation is[3,4]

$$\hat{H}=\hbar\omega_a\hat{s}_{11}+\hbar\sum_{i=1}^{2}(\omega_{bi}\hat{s}_{i+1,i+1}+\Omega_i\hat{a}_i^{\dagger}\hat{a}_i)+\hbar\sum_{i=1}^{2}\lambda_i(\hat{s}_{1,i+1}\hat{a}_i+\text{H.c.})\ . \tag{1}$$

The electric field operators for mode i are[5]

$$\hat{E}_i=\mathscr{E}_i(\hat{a}_i+\hat{a}_i^{\dagger})=\hat{E}_i^{+}+\hat{E}_i^{-}\quad(i=1,2)\ , \tag{2}$$

in which

$$\hat{E}_i^{+}=\mathscr{E}_i\hat{a}_i\ ,\quad \hat{E}_i^{-}=\mathscr{E}_i\hat{a}_i^{\dagger}\ . \tag{3}$$

In what follows we shall consider initially mode 1 in a coherent state $|\alpha_1\rangle$ with mean photon number $\bar{n}_1=|\alpha_1|^2$ and the atom in its upper state $|a\rangle$, i.e., the density operator for the atomic-field system is

$$\hat{\rho}(0)=\begin{bmatrix}\hat{\rho}_F(0) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\ , \tag{4}$$

that for the field is

$$\hat{\rho}_F(0)=\hat{\rho}_{F_1}(0)\otimes\hat{\rho}_{F_2}(0)\ , \tag{5}$$

and that for mode 1 is

$$\hat{\rho}_{F_1}(0)=\sum_{m_1 m_1'}\frac{\alpha_1^{m_1}(\alpha_1^*)^{m_1'}}{(m_1!m_1'!)^{1/2}}e^{-|\alpha_1|^2}|m_1\rangle\langle m_1'|\ . \tag{6}$$

On resonance $\Omega_i=\omega_a-\omega_{b_i}$, we can follow the same procedure as Ref. 2 or 3 and then obtain the first-order correlation function $\langle E_1^- E_1^+\rangle$ and the mean electric field of the positive frequency $\langle E_1^+\rangle$ and the negative $\langle E_1^-\rangle$ for mode 1, respectively.

When initially mode 2 does not exist, i.e.,

$$\hat{\rho}_{F_2}(0)=\sum_{n_2'}\delta n_{20}'\,|n_2'\rangle\langle n_2'|\ ,$$

we have

$$\langle E_1^- E_1^+\rangle_{\text{co}}(t)=\mathscr{E}_1^2\left[\bar{n}_1+\lambda_1^2\sum_{n_1}(n_1+1)\sin^2(\sqrt{\mu_0}t)W_c(n_1)/\mu_0\right]\ , \tag{7}$$

$$\langle E_1^-\rangle_{\text{co}}(t)=\mathscr{E}_1\alpha_1^* s_0(t)e^{i\Omega_1 t}\ , \tag{8}$$

$$\langle E_1^+\rangle_{\text{co}}(t)=\mathscr{E}_1\alpha_1 s_0(t)e^{-i\Omega_1 t}\ , \tag{9}$$

$$\langle E_1^-\rangle_{\text{co}}\langle E_1^+\rangle_{\text{co}}(t)=\mathscr{E}_1^2\bar{n}_1 s_0^2(t)\ , \tag{10}$$

where

$$s_0(t)=\sum_{n_1}[\cos\sqrt{\mu_0}t\cos\sqrt{\mu_{01}}t+(\mu_{01}/\mu_0)^{1/2}\times\sin(\sqrt{\mu_0}t)\sin(\sqrt{\mu_{01}}t)]W_c(n_1)\ , \tag{11}$$

$$\mu_0=\lambda_1^2(n_1+1)+\lambda_2^2\ , \tag{12}$$

$$\mu_{01}=\lambda_1^2(n_1+2)+\lambda_2^2\ , \tag{13}$$

$$W_c(n_1)=\exp(-\bar{n}_1)\bar{n}_1^{\,n_1}/n_1!\ . \tag{14}$$

Equations (7) and (10) can be analyzed by means of nu-

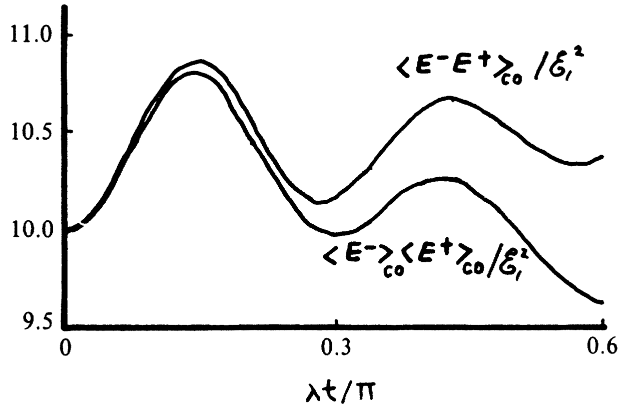

FIG. 1. $\langle E_1^- E_1^+\rangle_{\rm co}/\mathscr{E}_1^2$ and $\langle E_1^-\rangle_{\rm co}\langle E_1^+\rangle_{\rm co}/\mathscr{E}_1^2$ as functions of λt are compared for the case of one-mode stimulation with $\bar{n}_1=10$.

merical summation. Figure 1 shows the two quantities $\langle E_1^- E_1^+\rangle_{\rm co}/\mathscr{E}_1^2$ and $\langle E_1^-\rangle_{\rm co}\langle E_1^+\rangle_{\rm co}/\mathscr{E}_1^2$ as a function of λt, for the case $\lambda_1=\lambda_2=\lambda$ and $\bar{n}_1=10$. For a crude estimate in Fig. 1, the two quantities have the same value in the "time" (λt) range between 0 and 0.06π. This means that in the three-level single-excitation case, a field initially in a coherent state will stimulate emission which is also coherent to first order for "times" $\lambda t \le 0.06\pi$.

When initially mode 2 is in a coherent or thermal state with mean photon number $\bar{n}_2$, i.e.,

$$\hat{\rho}_{F_2}(0)=\sum_{m_2,m_2'} \alpha_2^{m_2}(\alpha_2^*)^{m_2'}e^{-|\alpha_2|^2}\,|m_2\rangle\langle m_2'|/(m_2!m_2'!)^{1/2} \quad (|\alpha_2|^2=\bar{n}_2)\,,$$

or

$$\hat{\rho}_{F_2}(0)=\sum_{m_2}[\bar{n}_2^{m_2}/(\bar{n}_2+1)^{m_2+1}]\,|m_2\rangle\langle m_2|\,,$$

we have

$$\langle E_1^- E_1^+\rangle_{\substack{cc\\ct}}(t)=\mathscr{E}_1^2\left[\bar{n}_1+\lambda_1^2\sum_{n_1,n_2}(n_1+1)\frac{\sin^2\sqrt{\mu}t}{\mu}W_{\substack{cc\\ct}}(n_1,n_2)\right], \tag{15}$$

$$\langle E_1^-\rangle_{\substack{cc\\ct}}=\mathscr{E}_1\alpha_1^* s_1(t)e^{i\Omega_1 t}\,, \tag{16}$$

$$\langle E_1^+\rangle_{\substack{cc\\ct}}=\mathscr{E}_1\alpha_1 s_1(t)e^{-i\Omega_1 t}\,, \tag{17}$$

$$\langle E_1^-\rangle_{\substack{cc\\ct}}\langle E_1^+\rangle_{\substack{cc\\ct}}(t)=\mathscr{E}_1^2\bar{n}_1 s_1^2(t)\,, \tag{18}$$

where upper (lower) symbols $cc\,(ct)$ refer to mode 2 in the coherent (thermal) state, and

$$s_1(t)=\sum_{n_1,n_2}[\cos(\sqrt{\mu}t\cos(\sqrt{\mu_1}t)+(\mu_1/\mu)^{1/2}\times\sin(\sqrt{\mu}t)\sin(\sqrt{\mu_1}t)]W_{\substack{cc\\ct}}(n_1,n_2)\,, \tag{19}$$

$$\mu=\lambda_1^2(n_1+1)+\lambda_2^2(n_2+1)\,, \tag{20}$$

$$\mu_1=\lambda_1^2(n_1+2)+\lambda_2^2(n_2+1)\,, \tag{21}$$

$$W_{cc}(n_1,n_2)=\exp(-\bar{n}_1-\bar{n}_2)\bar{n}_1^{n_1}\bar{n}_2^{n_2}/n_1!n_2!\,, \tag{22}$$

$$W_{ct}(n_1,n_2)=\exp(-\bar{n}_1)\bar{n}_1^{n_1}\bar{n}_2^{n_2}/n_1!(\bar{n}_2+1)^{n_2+1}\,. \tag{23}$$

Equations (15) and (18) can also be analyzed by means of numerical calculation. Figures 2(a) and 2(b) show the "time" range in which curves of $\langle E_1^- E_1^+\rangle_{\substack{cc\\ct}}$ and $\langle E_1^-\rangle_{\substack{cc\\ct}}\langle E_1^+\rangle_{\substack{cc\\ct}}$ are superposed. By comparing Figs. 1, 2(a), and 2(b), we can see that the stimulation of the other mode (mode 2 here), no matter whether it is coherent or chaotic, will cause the stimulated emission in mode 1 to remain in the coherent state for longer times. This will also be shown in the following.

The photon-number probabilities of mode 1 in single and double stimulation can be found to be

$$p_{\rm co}(n_1)=\{\cos^2(\sqrt{\mu_0}t)+[(\lambda_1^2 n_1^2/\bar{n}_1+\lambda_2^2)/\mu_0]\times\sin^2(\sqrt{\mu_0}t)\}W_c(n_1)\,, \tag{24}$$

and

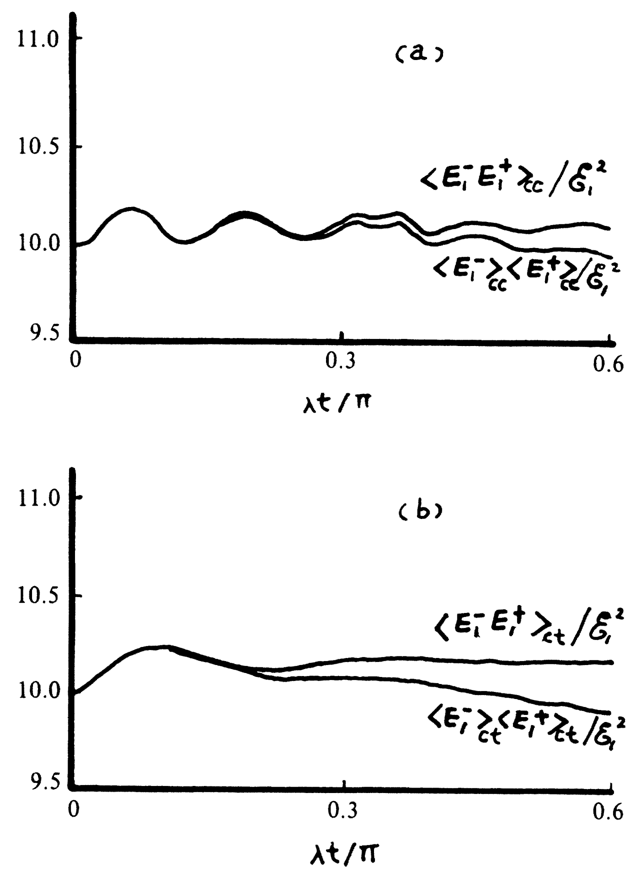

FIG. 2. Mode 1 initially in a coherent state with $\bar{n}_1=10$, and mode 2 (a) in a coherent state, (b) in a thermal state, with mean photon number $\bar{n}_2=50$.

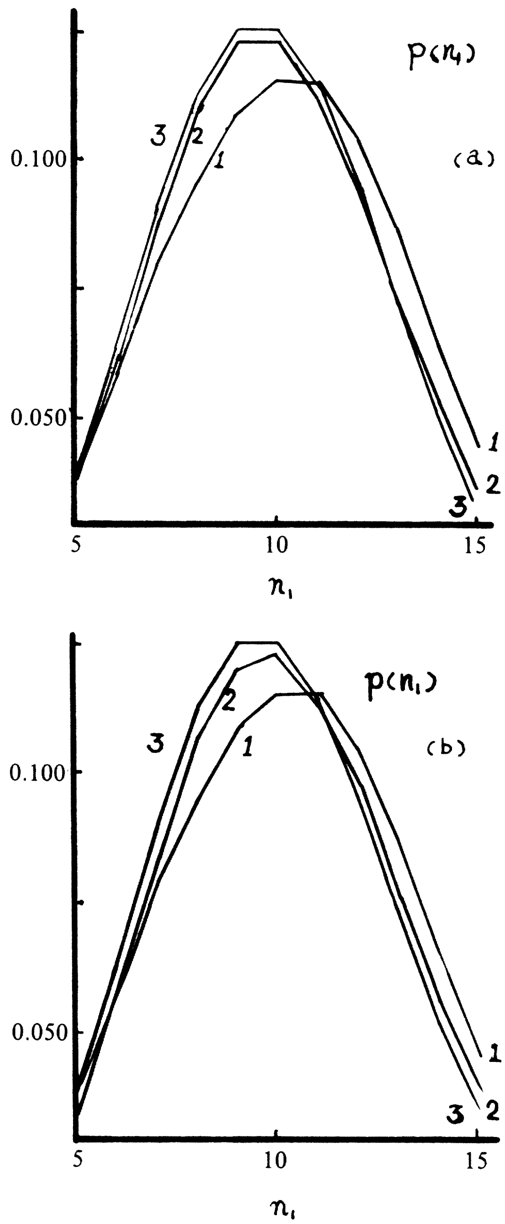

FIG. 3. Photon-number probabilities against photon number of mode 1. Curve 1 represents Poisson distribution of initial mode 1 with mean photon number $\bar{n}_1=10$; curve 2 represents double stimulation, when $t=0.4\pi/\lambda$, mode 2 initially in a (a) coherent state, (b) thermal state; curve 3 represents single stimulation, when $t=0.4\pi/\lambda$.

$$p_{\substack{cc\\ct}}(n_1)=\sum_{n_2}\left[\cos^2(\sqrt{\mu}t)+\lambda_1^2 n_1^2\sin^2(\sqrt{\mu_2}t)/\bar{n}_1\mu_2 + \lambda_2^2(n_2+1)\sin^2(\sqrt{\mu}t)/\mu\right]W_{\substack{cc\\ct}}(n_1,n_2)\;, \quad (25)$$

respectively, where

$$\mu_2=\lambda_1^2 n_1+\lambda_2^2(n_2+1)\;. \quad (26)$$

The curves numerically calculated form Eqs. (24) and (25) are shown in Figs. 3(a) and 3(b). Through comparing these curves we can see that the effects of the stimulation of the other mode (mode 2 here) will cause mode 1 to approach the coherent state. The above effects are stronger by coherent stimulation than by thermal one. This is in agreement with the above results (see Figs. 1 and 2).

As for the case when mode 1 is initially in a thermal state, it is easy to show

$$\langle E_1^-\rangle\langle E_1^+\rangle\equiv 0\;, \quad (27)$$

no matter what state mode 2 is initially in. It turns out that similar to the two-level case,[1] the stimulated field is never coherent in this case.

It is also of interest to study the mean photon numbers at arbitrary time t. Here we may consider there is initially two-mode coherent field and $\lambda_1=\lambda_2=\lambda$. In the same way, when $\bar{n}_1, \bar{n}_2 \gg 1$, they can be found to be

$$\langle n_1(t)\rangle=\bar{n}_1+\lambda^2\sum_{n_1,n_2}(n_1+1)\sin^2(\sqrt{\mu}t)W_{cc}(n_1,n_2)/\mu$$
$$\approx\bar{n}_1+[\lambda^2(\bar{n}_1+1)/\bar{\mu}]\sum_{n_1,n_2}\sin^2(\sqrt{\mu}t)W_{cc}(n_1,n_2)\;, \quad (28)$$

$$\langle n_2(t)\rangle=\bar{n}_2+\lambda^2\sum_{n_1,n_2}(n_2+1)\sin^2(\sqrt{\mu}t)W_{cc}(n_1,n_2)/\mu$$
$$\approx\bar{n}_2+[\lambda^2(\bar{n}_2+1)/\bar{\mu}]\sum_{n_1,n_2}\sin^2(\sqrt{\mu}t)W_{cc}(n_1,n_2)\;, \quad (29)$$

where

$$\bar{\mu}=\lambda_1^2(\bar{n}_1+1)+\lambda_2^2(\bar{n}_2+1)\;. \quad (30)$$

It is clear from Eqs. (28) and (29) that stronger stimulation in mode 1 (i.e., larger $\bar{n}_1$) will result in smaller $\langle n_2(t)\rangle$ and vice versa. This phenomenon can be called "mode competition," which is the competition of the atomic transition probabilities. However, this "competition" does not cause the stimulated field to deviate from the coherent state (as is in the laser), but causes the field to approach it. This is because the stimulation in the other mode (mode 2) will decrease the atomic transition probability from $|a\rangle$ to $|b_1\rangle$ and then weaken the interaction of mode 1 field with the atom, thus causing less change in this (mode 1) field, i.e., causing the field to approach its initial coherent state.

III. COHERENT BEHAVIOR OF THE PHOTON-NUMBER PROBABILITIES

Now we proceed to study the photon-number probabilities beyond the short-time regime.

For simplicity, we may consider mode 1 under the condition that mode 2 is coherent initially. When $\bar{n}_2 \gg 1$ and $\lambda_1=\lambda_2=\lambda$, we can neglect the difference between μ and μ_2 and then obtain

$$p_{\substack{cc\\tc}}(n_1)=1-\sum_{n_2}A_{\substack{c\\t}}(n_1,n_2)\sin^2[(n_1+n_2+1)^{1/2}\lambda t]W_{\substack{cc\\tc}}(n_1,n_2)\;, \quad (31)$$

where

$$W_{tc}(n_1,n_2)=W_{ct}(n_2,n_1)\ , \tag{32a}$$

$$A_c(n_1,n_2)=\frac{n_1+1+\bar{n}_1^{-1}n_1^2}{n_1+n_2+1}\ ,$$

$$A_t(n_1,n_2)=\frac{n_1+1+n_1(1+\bar{n}_1^{-1})}{n_1+n_2+1}\ . \tag{32b}$$

Using the same method as Ref. 6, we obtain

$$p_{\substack{cc\\tc}}(n_1)\simeq 1-\tfrac{1}{2}A_{\substack{c\\t}}(n_1\bar{n}_2)\times\{1-f(t)\cos[\phi(t)]e^{-\psi(t)}\}W_{\substack{c\\t}}(n_1)\ , \tag{33}$$

where

$$W_t(n_1)=\bar{n}_1^{n_1}/(\bar{n}_1+1)^{n_1+1}\ , \tag{34}$$

$$f(t)=[1+(\bar{n}_2\lambda t)^2/4\Omega^6(n_1)]^{-1/4}\ , \tag{35}$$

$$\phi(t)=2\Omega(n_1)\lambda t+\bar{n}_2\sin[\lambda t/\Omega(n_1)]-\bar{n}_2\lambda t/\Omega(n_1)-\tfrac{1}{2}\tan^{-1}[\bar{n}_2\lambda t/2\Omega^3(n_1)]\ , \tag{36}$$

$$\psi(t)=2\bar{n}_2 f^4(t)\sin^2[\lambda t/2\Omega(n_1)]\ , \tag{37}$$

$$\Omega(n_1)=(n_1+\bar{n}_2+1)^{1/2}\ . \tag{38}$$

Comparing (24) with (33), we can see that different from single stimulation, double stimulation will cause coherence[6–8] in the time evolution of the photon-number probabilities, of which the initial oscillation period $t_0=\pi/\lambda\Omega(n_1)$, collapse time $t_c=\Omega(n_1)/\lambda(\bar{n}_2/2)^{1/2}$, and revival time $t_R=2\pi\Omega(n_1)/\lambda$.

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