Dissipation in a fundamental model of quantum optical resonance

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The fully quantum-electrodynamical model of a two-level atom interacting with a single-cavity mode predicts an atomic evolution whose form is dictated by the discrete nature of the field energy and its statistical distribution. We demonstrate that the revivals of atomic excitation which are the signature of the quantum nature of the evolution are strongly affected by field dissipation even when the damping hardly affects the underlying Rabi oscillations.

Rydberg atoms, with their long spontaneous lifetimes and large dipole moments are exceedingly sensitive to weak radiation fields. Recent experiments by Haroche and co-workers[1] have demonstrated that the Rydberg-atom maser (Rydberg atoms in a very-high-Q resonant cavity) is capable of detecting only a few photons. The atom—single-photon coupling energy itself is close to experimental resolution.[2] Such a system can not only discriminate between fields of differing photon statistics, but may even be capable of resolving effects entirely due to the quantized, *discrete* nature of the radiation field. In this, the Rydberg maser approaches that idealization of fundamental two-level-atom—single-mode-radiation-field interaction, the Jaynes-Cummings model (JCM).[3] In the JCM, the familiar Rabi oscillations in the evolution of the atomic inversion as the atom and field exchange energy are affected by the distribution of photon numbers which cause a dephasing or "collapse" as the range of possible Rabi frequencies interfere. In a coherent-state field, the Poisson distribution of photon numbers is responsible for this collapse and is a purely quantum effect. Eberly and co-workers[4] have demonstrated that the discrete nature of the photon number distribution leads to a further purely quantum effect as the dephased Rabi oscillations partially rephase or "revive." The revivals are governed by the single-photon Rabi frequency (effectively the "granularity" of the field).

Rydberg-atom maser experiments on the evolution of atoms in coherent and in thermal fields[5] have prompted renewed theoretical interest in the JCM.[6] In these experiments, the finite cavity Q introduces field dissipation into the system. The cavity losses introduce Langevin quantum noise sources which react back on the JCM and affect the collapses and revivals in a nontrivial way. Sachdev[7] has obtained an analytic solution for *spontaneous* emission in a damped single-mode cavity at zero temperature and approximate solutions exist for the evolution of an atom in such a cavity at finite temperatures.[1,7] The quantum theory of an atom interacting with a thermal field at temperature T in a high-Q cavity using a dressed-state formalism has been discussed by Haroche and co-workers.[1]

The central problem remains that of accounting for dissipation in a JCM driven by a coherent-state field. Fully quantum-electrodynamical features such as collapses and revivals need to be given proper account but bath Langevin forces must be equally accounted for in a fully quantum-electrodynamic way.

We have obtained an approximate analytic solution for the time development of a two-level atom in a quantized field mode which is resistively damped by the finite cavity Q. Unlike previous results, our solution is valid for arbitrary initial field states. Our method demonstrates for the first time to our knowledge the effects of damping on collapses and revivals. We report here results on the simplest case of a zero-temperature cavity excited by a coherent-state field sufficient to excite many Rabi oscillations in a cavity damping time. Our approach is based upon an inductive solution of the equations of motion of the atom-field reduced density operator, combined with a transformed representation of the density matrix. No decorrelation between atom and field operators is required or employed. Our results demonstrate that dissipation profoundly influences revivals by smearing out the discrete nature of the Poisson sum over Rabi oscillations; such effects appear well before cavity-field lifetimes damp out the Rabi oscillations.

The density matrix for the combined atom-field system in the interaction picture is obtained by standard master-equation techniques.[8] The resistive cavity coupling is ascribed to a bath of harmonic oscillators which when treated in Born-Markov approximation leads to a damping rate γ of photon number in the cavity. In the simplest case of a cavity of 0 K and a field mode exactly resonant with the two-level atomic transition frequency we have

$$\frac{d}{dt}\rho=-ig[(a^{\dagger}\sigma_{-}+\sigma_{+}a),\rho]+\gamma a\rho a^{\dagger}-\frac{\gamma}{2}a^{\dagger}a\rho-\frac{\gamma}{2}\rho a^{\dagger}a\ , \qquad (1)$$

where g is the atom—cavity-mode coupling constant, the field is represented by creation and annihilation operators $a^{\dagger}$ and a, and the atom by the usual Pauli spin-$\frac{1}{2}$ operators.

Numerical solution for components of Eq. (1) have been reported[1] and approximate analytical solutions for the case of few field excitations.[7] We proceed instead by transforming to a new "dissipation picture" which allows analytic solution and which stresses the departure from semiclassical Rabi time evolution.

We define the operators J and L which act to the right upon the density matrix:[9,10]

$$J\rho=\gamma a\rho a^{\dagger}\ , \tag{2a}$$

$$L\rho=-\frac{\gamma}{2}a^{\dagger}a\rho-\frac{\gamma}{2}\rho a^{\dagger}a\ . \tag{2b}$$

The density matrix in the dissipation picture is defined as

$$\chi=e^{-(J+L)t}\rho\ . \tag{3}$$

The evolution of this dissipation-picture density matrix is given by the transformed master equation

$$\frac{d}{dt}\chi=-ig\left[e^{\gamma t/2}a^{\dagger}\sigma_{-}\chi-2\sinh\left[\frac{\gamma t}{2}\right]\sigma_{-}\chi a^{\dagger}+e^{-\gamma t/2}\sigma_{+}a\chi-e^{\gamma t/2}\chi\sigma_{+}a+2\sinh\left[\frac{\gamma t}{2}\right]a\chi\sigma_{+}-e^{-\gamma t/2}\chi a^{\dagger}\sigma_{-}\right]\ . \tag{4}$$

This equation resembles the undamped quantum Jaynes-Cummings evolution equation except for the presence of terms in which χ is sandwiched between operators; these terms, together with those involving $\exp(+\gamma t/2)$, account for the change in loss rate when the atom makes a transition. In the semiclassical limit in which the field is taken as a classical entity unaffected by the atomic evolution, Eq. (4) becomes

$$\frac{d}{dt}\rho=-i\Omega e^{-\gamma t/2}[(\sigma_{+}+\sigma_{-}),\rho]\ , \tag{5}$$

where $\Omega=ga$ is the semiclassical Rabi frequency and $\rho=\chi$ in this approximation. In this limit we note that the resistive coupling merely damps the atom-field perturbation in an exponential form expected from a phenomenological viewpoint. Departures from this form are a consequence of the quantized nature of the atom-field interaction.

In terms of atom-field components, Eq. (4) becomes

$$\frac{d}{dt}\chi(n,m;+,+)=-ig(n+1)^{1/2}e^{-\gamma t/2}\chi(n+1,m;-,+)+ig(m+1)^{1/2}e^{-\gamma t/2}\chi(n,m+1;+,-)\ , \tag{6a}$$

$$\frac{d}{dt}\chi(n+1,m;-,+)=-ig(n+1)^{1/2}e^{\gamma t/2}\chi(n,m;+,+)+ig(m+1)^{1/2}e^{-\gamma t/2}\chi(n+1,m+1;-,-)+ig(m+1)^{1/2}2\sinh\left[\frac{\gamma t}{2}\right]\chi(n+1,m+1;+,+)\ , \tag{6b}$$

$$\frac{d}{dt}\chi(n,m+1;+,-)=-ig(n+1)^{1/2}e^{-\gamma t/2}\chi(n+1,m+1;-,-)+ig(m+1)^{1/2}e^{\gamma t/2}\chi(n,m;+,+)-ig(n+1)^{1/2}2\sinh\left[\frac{\gamma t}{2}\right]\chi(n+1,m+1;+,+)\ , \tag{6c}$$

$$\frac{d}{dt}\chi(n+1,m+1;-,-)=-ig(n+1)^{1/2}e^{\gamma t/2}\chi(n,m+1;+,-)+ig(m+1)^{1/2}e^{\gamma t/2}\chi(n+1,m;-,+)+(e^{\gamma t}-1)\frac{d}{dt}\chi(n+1,m+1;+,+)\ . \tag{6d}$$

The coefficient of $|n\rangle\langle m+1|\otimes|+\rangle\langle -|$ is $\chi(n,m+1;+,-)$ where $|m+1\rangle$ and $|n\rangle$ are field number states and $|\pm\rangle$ are atomic excited and ground states. We note that the dissipation-picture density-matrix elements are coupled only to elements with the same number of quanta and a single element of the set containing one more quantum in the bra and ket sides of the density matrix. It is this feature of the representation that allows us to solve for the atomic inversion. If we were to take the atom to be initially excited with the field in the vacuum state as did Sachdev,[7] the number of participating atom-field states shrinks enormously and the inhomogeneous terms (terms corresponding to different number of quanta in this representation) in Eq. (6) are then absent.

The general solution of Eq. (6) is complicated. If we are interested only in the atomic inversion, it is sufficient to trace over the photon numbers after calculating $\chi(n,n;+,+)$. We proceed by eliminating $\chi(n,n+1;+,-)$, $\chi(n+1,n;-,+)$, and $\chi(n+1,n+1;-,-)$ from Eq. (6) to generate a third-order inhomogeneous ordinary differential equation for $\chi(n,n;+,+)$,

$$\frac{d^3}{dt^3}\chi(n,n;+,+)+\frac{3}{2}\gamma\frac{d^2}{dt^2}\chi(n,n;+,+)+\left[4g^2(n+1)+\frac{\gamma^2}{2}\right]\frac{d}{dt}\chi(n,n;+,+)+2\gamma g^2(n+1)\chi(n,n;+,+)$$
$$=2g^2(n+1)\left[\gamma\chi(n+1,n+1;+,+)+2(1-e^{-\gamma t})\frac{d}{dt}\chi(n+1,n+1;+,+)\right] . \quad (7)$$

This is soluble, but considerably simplifies in the underdamped limit[11] $n^2\gamma \ll (n+1)^{1/2}g$ (and no further approximations are made). We first solve the homogeneous equation [Eq. (7) without the right-hand side driving terms]. We take the atom to be initially inverted and the field to have a photon number distribution $p(n)$, then in the underdamped regime

$$\chi(n,n;+,+;\text{homog})=\tfrac{1}{2}p(n)e^{-\gamma t/2}\{1+\cos[2g(n+1)^{1/2}t]\} . \quad (8)$$

At this stage we assume (artificially) that the number distribution $p(n)$ is such that at some cutoff μ, $p(\mu+1)=0$ and at the end of the calculation let $\mu\to\infty$. Given this cutoff, the full solution for $\chi(\mu,\mu;+,+)$ coincides with the $\chi(\mu,\mu;+,+;\text{homog})$ simply because of the absence of driving terms involving $p(\mu+1)$. This full solution for $\chi(\mu,\mu;+,+)$ can then be used in Eq. (7) to derive in turn $\chi(\mu-1,\mu-1;+,+)$. The general solution is found by repeating this process inductively and finally taking $\mu\to\infty$, where we obtain

$$\chi(n,n;+,+)=\tfrac{1}{2}e^{-\gamma t/2}\left[\sum_{l=0}^{\infty}p(l)\cos[2g(l+1)^{1/2}t](e^{-\gamma t}-1)^{l-n}\frac{l!}{n!(l-n)!}\right.$$
$$\left.+\sum_{l=n}^{\infty}\frac{[2(l-n)]!}{2^{2(l-n)}[(l-n)!]^2}(1-e^{-\gamma t})^{l-n}p(l)\right] . \quad (9)$$

The untransformed density-matrix elements are found from $\rho=\exp[J(e^{\gamma t}-1)/\gamma]\exp(Lt)\chi$ so that

$$\rho(m,m;+,+)=\sum_{n=0}^{\infty}\frac{n!}{m!(n-m)!}(e^{\gamma t}-1)^{n-m}e^{-n\gamma t}\chi(n,n;+,+)$$
$$=\tfrac{1}{2}e^{-\gamma t/2}\left[p(m)\cos[2g(m+1)^{1/2}t]e^{-m\gamma t}\right.$$
$$\left.+\sum_{n=0}^{\infty}\sum_{l=n}^{\infty}e^{-l\gamma t}(e^{\gamma t}-1)^{l-m}\frac{n!}{m!(n-m)!}\frac{[2(l-n)]!}{2^{2(l-n)}[(l-n)!]^2}p(l)\right] . \quad (10)$$

The oscillating term in Eq. (10) is due to the sudden switch-on of the atom-field interaction at time $t=0$. The sums in Eq. (10) represent the slow and incoherent change in the number of quanta in the cavity. The departure from purely exponential decay in the components of this sum is due to the difference in the decay when the atom makes a transition, changing the number of quanta in the mode. The probability $P_+(t)$ that the atom is excited at time t is given by the sum over m of the terms in Eq. (10). We find the particularly simple result

$$P_+(t)=\tfrac{1}{2}e^{-\gamma t/2}\left[\sum_{m=0}^{\infty}e^{-m\gamma t}p(m)\cos[2g(m+1)^{1/2}t]+\sum_{n=0}^{\infty}\sum_{l=n}^{\infty}\frac{[2(l-n)]!}{2^{2(l-n)}[(l-n)!]^2}(1-e^{-\gamma t})^{l-n}p(l)\right] . \quad (11)$$

Note that there is no damping modification of the Rabi frequency in this underdamped limit. The simplicity of this result is a consequence of the very weak cavity damping. Many Rabi oscillations occur in a cavity lifetime. In an untransformed density-matrix treatment in a dressed-state basis, the secular approximation[8] enables us to neglect the couplings between coherences belonging to different manifolds (that is, off-diagonal density-matrix elements that do not correspond to the same total number of quanta in the atom-field system). This much simplifies the dynamics of the "cascade" through dressed states as the cavity Q resistively damps out the atom-field excitations.

The atomic inversion W is given in terms of $P_+(t)$ by

$$W(t)=2P_+(t)-1 . \quad (12)$$

In Fig. 1, the atomic inversion is plotted as a function of time for an initially excited atom interaction with a coherent field of mean photon number equal to 5 for three values of cavity damping. As the damping increases the revivals rapidly diminish in amplitude. The collapse of the initial Rabi oscillations is much less sensitive to cavity damping. The quiescent periods in which the inversion remains at a quasi-steady state slowly decay to values below zero. The revivals depend upon the discrete nature

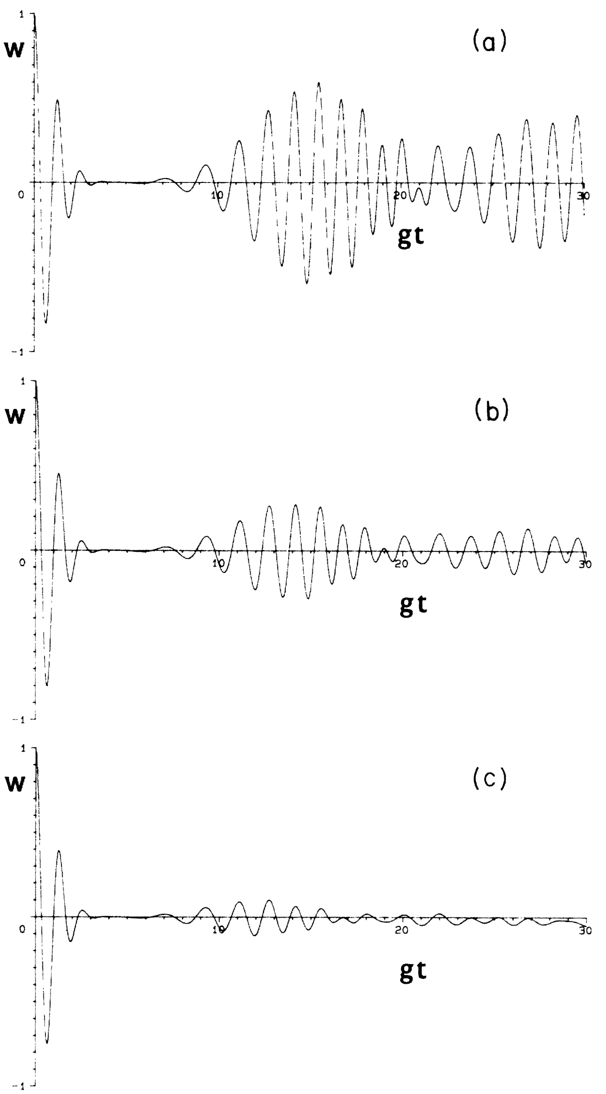

FIG. 1. Evolution of the atomic inversion W for an initially excited atom in a coherent field with mean photon number of 5 for (a) $\gamma = 0$, (b) $\gamma = 0.01g$, (c) $\gamma = 0.03g$.

of the photon number distribution; this "granularity" is translated into a discrete spectrum of Rabi frequencies. The cavity damping broadens these spectral components[12] (or "dressed-atom" eigenenergies); when this broadening becomes comparable with the spacing between the Rabi frequencies the spectrum becomes continuous, the inversion collapses and never revives. The collapse, controlled only by the spread in Rabi frequencies, is much less affected by this broadening. In Fig. 1(a) we present the *undamped* evolution. In Figs. 1(b) and 1(c) we note that the revivals are significantly attenuated, even though they occur at times $t < \gamma^{-1}$, the cavity field lifetime. Our simple solution cannot be used to demonstrate the *complete* destruction of revivals as this requires a stronger damping than that permitted by our secular or underdamped approximation and would require a less restrictive approach to the solution of Eq. (7).

Cavity damping will significantly attenuate the revivals unless the cavity Q is such that the field damping rate γ is very much less than the one-photon Rabi frequency g. An elementary calculation shows that for a Rydberg-atom mm-wave transition, $\gamma / g \sim 10^{15} Q^{-1} (\Delta n V / n^7)^{1/2}$ where the cavity volume V is in m^3, n is the initial-state principal quantum number, and Δn the change in n in the transition. For the experiments of Haroche and co-workers,[1] $\gamma / g \approx 2$ whereas those of Meschede *et al.*[2] have $\gamma / g \approx 2 \times 10^{-3}$, and will be much less affected by the cavity dissipation. The observation of fully-quantum-electrodynamic strong-coupling collapses and revivals in the presence of dissipative couplings has a wider significance as an example of a quantum coherence phenomenon competing against an environmental damping.[13]

Note added in proof. Puri and Agarwal[14] have also studied the effects of dissipation in the JCM and its influence on revivals and other quantum features.

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