# Post-collision-interaction effects in the ionization of helium by fast electrons at small momentum transfer

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(Received 29 July 1985)

Conspicuous effects from a weak post-collision interaction have been identified in triply differential cross sections for the ionization of helium by fast electrons at asymmetric kinematics. A classical correction to an accurate first Born approximation is in good agreement with experimental data, showing the important influence of correlation after the collision.

Ionization of atomic matter by fast, charged particle impact is a fundamental process in broad fields of pure and applied physics. The problem to calculate cross sections is well defined through exactly known forces and mechanics. Nevertheless, theoretical work for electron-impact ionization of even simplest atoms is generally not in satisfying agreement with experiment at intermediate and higher energies,[1,2] although a distorted-wave Born approximation[3] and an approximate second-order Born approximation[4,5] have improved simpler treatments. At lower energies the situation is even worse.[6] One may, however, not overlook that the process under consideration constitutes a difficult many-body Coulomb problem above the break-up threshold for three free charged particles (ion plus two electrons). In quantum mechanics no systematic approximation procedure is known to treat that type of problems. Neither Faddeev equations nor approximations to them can directly be used to describe atomic ionization by charged particle impact.[7] On the other hand, formally exact solutions of the many-body Coulomb problem have been found in terms of generalized power series;[8] in this formulation the incorporation of boundary conditions for ionization is difficult.

Fast collisions accompanied with small momentum transfer are usually treated within the framework of the first Born approximation[9,10] (BA) in which the triply differential cross section (TDCS) is given by

$$\frac{d^3\sigma}{d\Omega_a\, d\Omega_b\, dE_b}=\frac{4k_a k_a}{k_0 q^4}\left|\left\langle \psi_{\mathbf{k}_b}^{(-)}\left|\sum_n e^{i\mathbf{q}\cdot\mathbf{r}_n}\right|\phi\right\rangle\right|^2 . \qquad (1)$$

Here $\mathbf{k}_0$, $\mathbf{k}_a$, and $\mathbf{k}_b$ are the momenta of the incoming, the scattered, and the ejected electrons, and $E_0=\frac{1}{2}k_0^2$, and $E_a=\frac{1}{2}k_a^2$, and $E_b=\frac{1}{2}k_b^2$ are the energies in atomic units of the incoming, the scattered, and the ejected electrons, respectively. The momentum transfer is $\mathbf{q}=\mathbf{k}_0-\mathbf{k}_a$. The initial target state is $\phi$; the final continuum target state $\psi_{\mathbf{k}_b}^{(-)}$ satisfies incoming wave-boundary conditions. Exchange is here not relevant since small momentum transfer implies $E_a \gg E_b$. We stress that Eq. (1) cannot be considered as the first term of a converging perturbation-theory series. The second- and all higher-order Born approximations are actually known to diverge.[11]

The BA given by Eq. (1) predicts a TDCS cylindrically symmetric around the momentum transfer axis $\mathbf{q}$, with two maxima located at directions $\mathbf{q}$ and $-\mathbf{q}$, respectively (binary and recoil peak). A recent BA calculation[12] based on accurate helium wave functions has confirmed an earlier conjecture[13] that TDCS's for fast electrons ($E_0=600$ eV in this example) are understandable mainly within the BA. Results of this calculation are indeed in better but not in full agreement with experimental data.

The purpose of this communication is to show that for fast and asymmetric collisions at small scattering angles, departures from the first BA must be attributed to correlation effects between the two escaping continuum electrons. This conclusion is based on the fact that the interaction responsible for the energy transfer from the projectile electron to the target atom is a short-range interaction. Within a finite reaction volume the ionization process is adequately described by the exchange of one virtual photon, provided the incident energy is sufficiently high compared to binding energies, and the momentum transfer is sufficiently small. Outside this reaction volume, however, long-range Coulomb and polarization interactions allow for exchange of energy and angular momentum between the two escaping electrons before they arrive in the detectors. This particular post-collision interaction (PCI) may be regarded as a dynamical screening being known to control double escape at extremely low energies.[14] We remark that the first BA assumes static rather than dynamical screening, i.e., the fast electron is assumed to experience no force at all irrespective of the motion of the slow electron, and the slow electron is assumed to experience a singly charged ion irrespective of the fast electron's motion. The importance of PCI effects is expected to decrease for increasing incident energy $E_0$, but according to our analysis, these effects disappear only as slowly as $E_0^{-1/2}$ for $E_0\rightarrow\infty$ and $E_a \gg E_b$ such that they are still observable at several hundred electron volts incident energy.

Because of the lack of an exact three-body Coulomb wave function of our purpose, we describe here the PCI extending a semiclassical treatment developed for symmetric collisions.[15] The long-range Coulomb interaction between the escaping electrons produces a deflection of their trajectories during their path from the reaction zone to the detectors. This deflection can be calculated classically,[15,16]

$$\vartheta_i(0)=\vartheta_i-\int_0^\infty dt\, r_a r_b r_{ab}^{-3}\sin\chi \int_t^\infty dt'[r_i(t')]^{-2} \tag{2}$$

for each electron $i=a,b$. Here $\vartheta_i(0)$ and $\vartheta_i=\vartheta_i(\infty)$ are the directions of a trajectory $r_i(t)$ at the boundary of the reaction zone ($t=0$) and at the detector ($t=\infty$), respectively, with respect to the incident momentum $\mathbf{k}_0$. The electron-electron separation is $r_{ab}=|r_a(t)-r_b(t)|$, and the angle $\chi$ is given by $\chi=\vartheta_a+\vartheta_b$. Energy exchange between the escaping electrons disregarded in Ref. 15 may be calculated from

$$E_i(0)=E_i-\int_0^\infty dt\, \dot{E}_i(t)\ , \tag{3}$$

where $E_i=E_i(\infty)$ is the energy of an electron observed in the detector, and $E_i(0)$ is its energy at the boundary of the reaction zone. As in Bethe's Eq. (1), we include for the ejected electron its potential energy in the ion field into $E_b(t)$, but we exclude the corresponding quantity from $E_a(t)$. In the following we denote the potential energy in central field approximation experienced by an electron by $-\varphi(r)/r$, where $\varphi(r)$ is a screening function. Since we expect nondominant PCI effects for fast collisions, we treat Eqs. (2) and (3) by iteration. We approximate exact trajectories in zeroth order by straight-line trajectories $r_i(t)$ and $\vartheta_i=\text{const}$, and find first-order energy corrections

$$E_a(0)\cong E_a+\left.\frac{\varphi(r_a)}{r_a}\right|_{t=0}-\int_0^\infty dt\, \dot{r}_a r_{ab}^{-3}(r_a-r_b\cos\chi)\ , \tag{4a}$$

$$E_b(0)\cong E_b-\int_0^\infty dt\, \dot{r}_b r_{ab}^{-3}(r_b-r_a\cos\chi)\ . \tag{4b}$$

We have evaluated the integrals (2), (4a), and (4b) approximately describing the fast electron as free particle and the slow electron as Coulomb particle, i.e.,

$$E_a=\tfrac{1}{2}\dot{r}_a^2\ ,$$

$$E_b=\tfrac{1}{2}\dot{r}_b^2-1/r_b\ ,$$

with initial conditions $r_a(0)=r_{0a}$ and $r_b(0)=r_{0b}$.

Experimental data observed at energies $E_a, E_b$ and at scattering angles $\vartheta_a,\vartheta_b$ may be compared with a BA calculation performed at energies $E_a(0), E_b(0)$, and scattering angles $\vartheta_a(0),\vartheta_b(0)$. Figure 1 shows the result of such a comparison at $E_0=600$ eV, $E_b=5$ eV, and $\vartheta_a=4°$. The solid curve is the result of the present modified BA. It turned out that the TDCS depends only weakly on the boundary values $r_{0a}$ and $r_{0b}$; in Fig. 1 they have the values $r_{0a}=3.5$ bohr, $r_{0b}=0.4$ bohr, and $\varphi(r_{0a})=1$. The agreement with experimental data[2] is reasonably good. We remark that the measured data have been normalized by extrapolation of the generalized oscillator strength to the optical limit. This procedure may have an uncertainty of a few percent. Except for an overall enhancement of the TDCS, the PCI effect produces a reduction of the binary-to-recoil peak intensity ratio with respect to the BA (ca. 10% in this example).

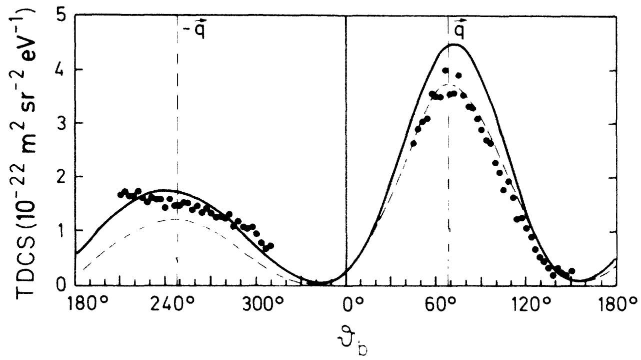


FIG. 1. Absolute triple differential cross section for $E_0=600$ eV, $E_b=5$ eV, and $\vartheta_a=4°$. The dots show experimental data from Ref. 2, the broken curve shows a Born approximation, and the solid curve shows the present calculation.

The reason for this is the following: The energy shift for the slow electron depends on its observation angle $\vartheta_b$,

$$E_b(t=\infty)-E_b(t=0)=f(\vartheta_b(t=\infty))\ . \tag{5}$$

At two angles $\vartheta_{b0}\cong 78°$ and $\vartheta'_{b0}\cong 274°$ the function $f$ in Eq. (5) has a zero,[16] i.e., no energy transfer occurs at these directions. The angle $\vartheta_{b0}$, however, is close to the direction of the binary peak (the direction of the momentum transfer is $\vartheta_q\cong 68°$), whereas the recoil peak is farther away from the second magic angle $\vartheta'_{b0}$. This explains why the BA works usually better in binary than in recoil direction. In recoil direction the relation $E_b(0)<E_b(\infty)$ implies the enhancement.

The PCI effect breaks the axial symmetry around the momentum tranfer; both binary and recoil peaks are shifted to larger angles. This effect is larger for the recoil peak ($\Delta\vartheta\cong 8°$) than for the binary peak ($\Delta\vartheta\cong 3°$), which is not in disagreement with the experimental observation.

We have also investigated the ratio of binary to recoil intensities as functions of the scattering angle $\vartheta_a$ and of the incident energy $E_0$. Figure 2 shows that ratio at $E_0=600$ eV and $E_b=5$ eV as a function of $\vartheta_a$. The broken curve was the result of an unmodified BA,[12] whereas the solid line is the result of the present calculation. The agreement with experimental data is very good. Figure 3 shows that ratio at a fixed angle $\vartheta_a=4°$ as a function of the incident energy $E_0$. Here we have made the plausible assumption that the initial value $r_{0a}$ is proportional to $k_a=(2E_a)^{1/2}$. We compare our calculation with measurements at $E_0=600$ eV,[2] $E_0=250$ eV,[17] and $E_0=150$ eV (Ref. 17) and find reasonable agreement. Note that the PCI effect produces here at lower energies recoil distributions larger than the binary peak.

The present approach should be regarded as guideline for further theoretical development. Such development should replace the classical treatment by a quantum mechanical one. The success of the classical model, however, indicates that a Wentzel-Kramers-Brillouin-(WKB)-type approximation will be useful to treat the correlation problem in the Coulomb zone. Multidimen-

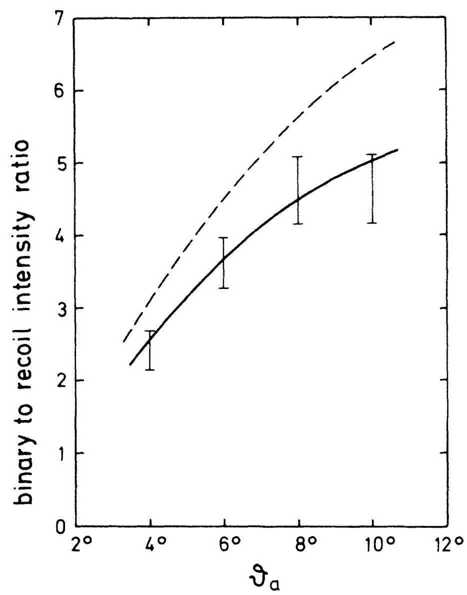


FIG. 2. Ratio of binary to recoil intensities in the maxima for $E_0=600$ eV and $E_b=5$ eV. Experimental data are from Ref. 2, the broken curve shows a Born approximation (Ref. 12), and the solid curve shows the present calculation.

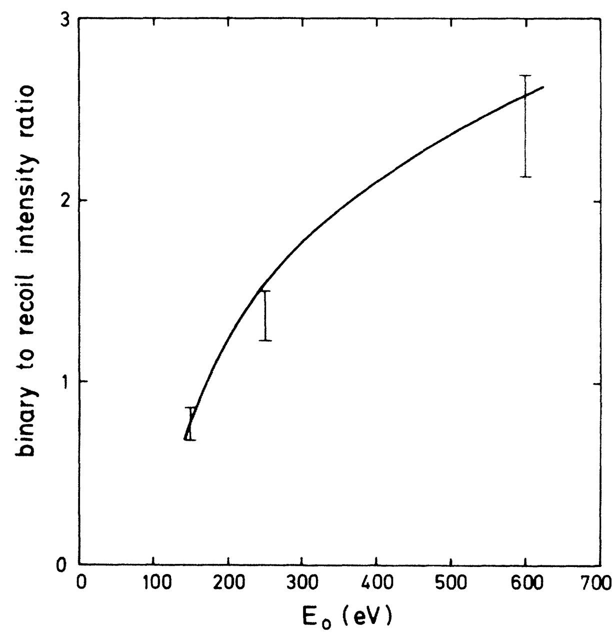


FIG. 3. Ratio of binary to recoil intensities in the maxima for $E_b=5$ eV and $\vartheta_a=4°$. Experimental data are from Refs. 2 and 17 (see text); the curve is the result of the present calculation.

sional WKB theories for nonseparable wave equations[18] are still in early stages and difficult to handle, but in our situation of a weak nonseparability, further simplifying approximations probably apply. Concluding, we remark that the same PCI corrections will be necessary at larger values of the momentum transfer and/or at lower incident energy where the BA must be replaced in the reaction zone by a more appropriate collision model.

Financial support by the Deutsche Forschungsgemeinschaft (SFB 91) is gratefully acknowledged.

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