Scalar and tensor polarizabilities of ^2D Rydberg states in Rb

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Two-photon Doppler-free spectroscopy has been used to measure Stark shifts of $n\,^2D$ Rydberg states of ^{85}Rb (for $n=13$–55). Scalar and tensor polarizabilities were determined and found to depend on $(n^*)^6$ and $(n^*)^7$, in agreement with theory. The resulting polarizabilities were used to obtain estimates of radial integrals for D-P transitions.

I. INTRODUCTION

Recently, we reported measurements of the low-field Stark effect of highly excited $n\,^2S$ states of Rb.[1] Stark spectra were obtained using Doppler-free two-photon spectroscopy, and frequency shifts of components arising from the transitions $n\,^2S_{F=3}\leftarrow 5\,^2S_{F=3}$, with $n=15$–80 were measured within 2 MHz. Scalar polarizabilities associated with these excited states were determined for the first time, and found to increase by a factor of $\sim 10^5$ at high n over the value for $n=15$.

In this paper, the results of new measurements of the low-field Stark effect for excited $n\,^2D$ states of Rb are presented. Stark shifts and splittings of components associated with transitions $n\,^2D_{3/2,5/2}\leftarrow 5\,^2S_{F=3}$ for $n=13$–55 were measured using the techniques described in Ref. 1. The corresponding values of scalar and tensor polarizabilities were obtained by means of an analysis based on accepted theory of the Stark effect. In similarity with the observed behavior of ^2S-state polarizabilities, it was found that ^2D-state polarizabilities obeyed relations of the form

$$\alpha = A(n^*)^7 + B(n^*)^6 . \tag{1}$$

Here α is the polarizability and n^* the effective quantum numbers of excited states. Four such relations were obtained in the present work to describe the dependences of scalar and tensor polarizabilities associated with the $J=3/2$ and 5/2 states. These four equations were extrapolated for low n^* values, and the results for $n=6$–8 ^2D were compared with the only polarizabilities reported[2–4] for Rb prior to the present work. Also, scalar and tensor polarizabilities were calculated for levels with $n\leq 28$ using computed radial integrals,[5] and compared with the present results. Finally, values obtained here for $n>28$ were used to estimate radial integrals for D-P transitions.

II. THEORY

At zero electric field the ^2D states of Rb are nondegenerate with respect to angular momentum. At low electric fields these states are isolated and, having definite parity, exhibit no linear Stark effect; their energy shifts depend overwhelmingly on the square of the electric field.

Expressions for the Stark shifts $\Delta W^{J,|M|}$, of states having magnetic quantum number M and total angular momentum J at zero field, are obtained from the theory of the quadratic Stark effect.[6] Results specific to ^2D states of Rb are given below,[7]

$$\Delta W^{5/2,5/2} = -E^2(\alpha_0^{5/2}/2 + \alpha_2^{5/2}/2) , \tag{2a}$$

$$\Delta W^{5/2(3/2),3/2} = \tfrac{1}{2}\{ \underset{(+)}{-}\Delta_{FS} - AE^2/2 + [(\Delta_{FS} - BE^2/2)^2 \underset{(-)}{+} (6E^2\alpha_2^{5/2}/10)^2]^{1/2}\} , \tag{2b}$$

$$\Delta W^{5/2(3/2),1/2} = \tfrac{1}{2}\{ \underset{(+)}{-}\Delta_{FS} - CE^2/2 + [(\Delta_{FS} - DE^2/2)^2 \underset{(-)}{+} 6(E^2\alpha_2^{5/2}/10)^2]^{1/2}\} . \tag{2c}$$

The following identities have been used:

$$A = \alpha_0^{5/2} + \alpha_0^{3/2} + \alpha_2^{3/2} - \alpha_2^{5/2}/5 , \tag{3a}$$

$$B = \alpha_0^{5/2} - \alpha_0^{3/2} - \alpha_2^{3/2} - \alpha_2^{5/2}/5 , \tag{3b}$$

$$C = \alpha_0^{5/2} + \alpha_0^{3/2} - \alpha_2^{3/2} - 4\alpha_2^{5/2}/5 , \tag{3c}$$

$$D = \alpha_0^{5/2} - \alpha_0^{3/2} + \alpha_2^{3/2} - 4\alpha_2^{5/2}/5 . \tag{3d}$$

Equations (2) are valid even when Stark shifts are comparable to the fine-structure spacing Δ_{FS} of the excited state. Stark shifts of components are seen to depend on the square of the electric field E, as well as on values of scalar and tensor polarizabilities $\alpha_0^{3/2}, \alpha_2^{3/2}$ and $\alpha_0^{5/2}, \alpha_2^{5/2}$ corresponding to the excited-state J value of 3/2 or 5/2, respectively. According to Eqs. (2), ^2D fine-structure doublets are split into five components, each being associated with a definite value of $|M|$. Although electric fields also mix states of $J=3/2$ and 5/2 having the same $|M|$, the observed Stark components were assigned their zero-field J values for the purpose of identification at other field values.

For very low electric fields such that $\Delta_{FS} \gg W^{J,|M|}$,

Eqs. (2) reduce to a more familiar form[6]

$$\Delta W^{J,|M|} = -\frac{E^2}{2}\left[\alpha_0^J + \alpha_2^J \frac{3M^2 - J(J+1)}{J(2J-1)}\right] . \quad (4)$$

In this limit, the overall Stark shifts of components having a given J are governed by a scalar polarizability α_0^J, while the splitting according to the value of M is governed by a tensor polarizability α_2^J.

The theory of the quadratic Stark effect also provides expressions for the polarizabilities. For ^2D states of alkali-metal atoms these may be written as

$$\alpha_0^{5/2} = e^2 \sum_{n'} \frac{4}{15} \frac{|R_{n'P}^{nD}|^2}{W^0 - W_{n'P\,3/2}^0} + \frac{2}{5} \frac{|R_{n'P}^{nF}|^2}{W^0 - W_{n'F}^0} , \quad (5a)$$

$$\alpha_0^{3/2} = e^2 \sum_{n'} \frac{2}{45} |R_{n'P}^{nD}|^2 \left[\frac{1}{W^0 - W_{n'P\,3/2}^0} + \frac{5}{W^0 - W_{n'P\,1/2}^0}\right] + \frac{2}{5} |R_{n'F}^{nD}|^2 (W^0 - W_{n'F}^0)^{-1} , \quad (5b)$$

$$\alpha_2^{5/2} = -e^2 \sum_{n'} \frac{4}{15} \frac{|R_{n'P}^{nD}|^2}{W^0 - W_{n'P\,3/2}^0} + \frac{2}{35} \frac{|R_{n'P}^{nF}|^2}{W^0 - W_{n'F}^0} , \quad (5c)$$

$$\alpha_2^{3/2} = -e^2 \sum_{n'} \frac{2}{45} |R_{n'P}^{nD}|^2 \left[\frac{-8/225}{W^0 - W_{n'P\,3/2}^0} + \frac{2/9}{W^0 - W_{n'P\,1/2}^0}\right] + \frac{2}{25} |R_{n'F}^{nD}|^2 (W^0 - W_{n'F}^0)^{-1} . \quad (5d)$$

Here $R_{n'L'}^{nL}$ are radial integrals between states having principal and angular momentum quantum numbers n,L and n',L', while W^0 and $W_{n'L'J'}^0$ are zero-field energies of the ^2D state and intermediate states, respectively. Only intermediate states (such as P and F), which are dipole-connected with a ^2D state contribute to its polarizability. Furthermore, in Rb, more than 99% of the polarizability (scalar or tensor) of a given ^2D state is contributed by neighboring intermediate states. Hence, for a given $n\,^2D$ state, the summations of Eqs. (5) (which in principle are taken over all allowed discrete and continuum intermediate states) can be truncated to include only the $(n+1)P$, $(n+2)P$, $(n-1)F$, and $(n-2)F$ states, with a decrease of only $\sim 1\%$ in the accuracies of calculated polarizabilities.

III. EXPERIMENT

Spectra of the transitions $n\,^2D_{3/2,5/2} \leftarrow 5\,^2S_{F=3}$ were obtained by inducing Doppler-free two-photon absorptions in Rb vapor at 5 mTorr pressure. Atoms having absorbed two photons from counter-propagating laser beams were ionized by thermal collisions, and detected with a thermionic diode detector. The laser light was modulated by a 12-Hz chopper, enabling synchronous detection of the resulting ions.

Two Pyrex cells containing ion detectors and Rb vapor were used. The Stark cell also housed a parallel-plate Stark electrode assembly,[1] and the reference cell contained a small Faraday cage attached to the detector which ensured zero fields for excited atoms and ions. In the Stark cell, electric fields as large as 10^3 V/cm and uniform to within 0.03% were produced without an electric discharge. Since the Stark electrode spacing was accurately known[1] [1.02445(8) cm] electric fields could be determined with relative accuracies of 0.1 to 0.4% by measuring applied voltages with a calibrated voltmeter.

The radiation from an Ar^+ laser was used to pump a frequency stabilized (~ 2 MHz jitter) and tunable R6G dye laser. Its output beam of 30–300 mW was focused and retro-reflected into both the Stark and reference cells to permit Doppler-free two-photon absorption, with a portion being directed into a 1.2-m confocal etalon. As the dye laser frequency was tuned through a two-photon resonance in Rb, signals from the Stark and reference cells were sensed with lock-in amplifiers and recorded on a three-pen chart recorder, along with transmission maxima from the etalon. In this way, Stark shifts and splittings of ^2D lines were recorded, and could be measured within 2 MHz for any value of the electric field produced in the Stark cell.

IV. OBSERVED SPECTRA AND STARK SHIFT MEASUREMENTS

A typical chart record illustrating the Stark shifts and splittings is shown in Fig. 1. Here, the ^2D fine-structure

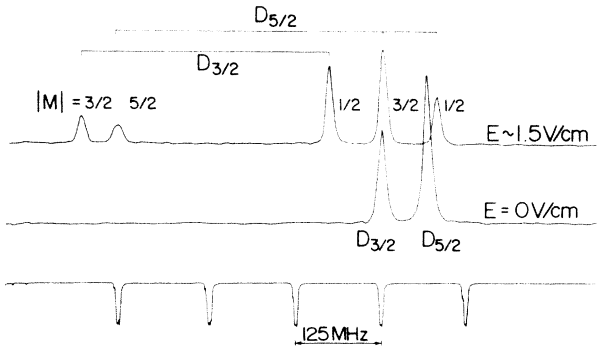

FIG. 1. Spectra of the $50\,^2D_{3/2,5/2} \leftarrow 5S_{F=3}$ transition in ^{85}Rb. The two fine-structure components at $E=0$ (middle trace) are split into five components by the electric field (top trace). The etalon fringes (bottom trace) provide a frequency scale.

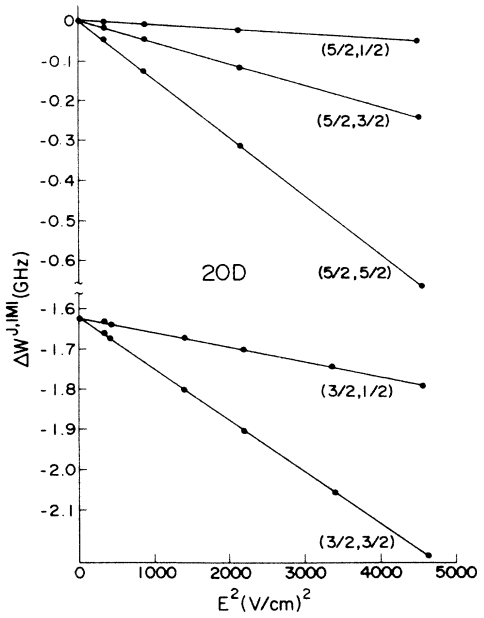

FIG. 2. Low-field Stark shifts of components J, $|M|$ for the transition $20\,^2D_{3/2,5/2} \leftarrow 5\,^2S_{F=3}$ plotted against E^2.

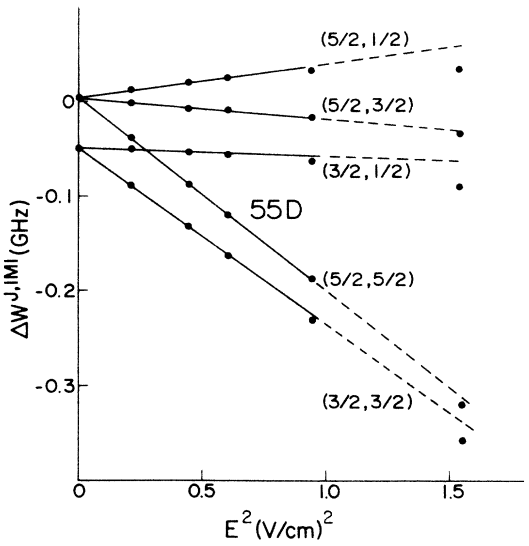

FIG. 3. Low-field Stark shifts of components, J, $|M|$ for the transition $55\,^2D_{3/2,5/2} \leftarrow 5\,^2S_{F=3}$ plotted against E^2. Only the data from $E=0$ to approximately 1.0 over which there is a quadratic dependence on E were used to evaluate the polarizabilities.

doublet (middle-trace) is seen to split into five components (top trace) in an electric field. Three components labeled with the zero-field J value 5/2 have $|M|$ values of 1/2, 3/2, 5/2, and the two of $J=3/2$ have $|M|$ values of 1/2 and 3/2. The frequency scale with markers every 125 MHz provided by transmission maxima from the etalon are displayed in the bottom trace. All of the observed Stark spectra exhibited linewidths of 4—20 MHz, due to pressure broadening, with the larger widths for states near $n=20$ which have been shown to have higher rates of self-broadening.[8]

At a given electric field, the shift of each component

TABLE I. Scalar and tensor polarizabilities α_0^J and α_2^J of ^2D states of Rb, in MHz/(V/cm)^2, determined from measured Stark shifts and calculated using radial integrals given by Gounand.

nD	$\alpha_0^{5/2}$(obs)	$\alpha_0^{5/2}$(calc)[a]	$\alpha_2^{5/2}$(obs)	$\alpha_2^{5/2}$(calc)[a]
13	0.00704(6)	0.00757(36)	0.0047(1)	0.0047(3)
15	0.0198(1)	0.0199(10)	0.0151(2)	0.0154(8)
20	0.145(2)	0.143(9)	0.149(5)	0.151(7)
25	0.66(1)	0.65(5)	0.81(1)	0.84(4)
30	2.33(2)		3.2(1)	
35	6.3(2)		10.4(2)	
37	9.6(4)		15.3(4)	
40	16.7(4)		26.2(9)	
45	38(2)		64(3)	
50	72(4)		134(5)	
55	139(8)		254(10)	
nD	$\alpha_0^{3/2}$(obs)	$\alpha_0^{3/2}$(calc)[a]	$\alpha_2^{3/2}$(obs)	$\alpha_2^{3/2}$(calc)[a]
13	0.00781(3)	0.00801(34)	0.00269(6)	0.00285(14)
15	0.0214(2)	0.0213(11)	0.0088(2)	0.0094(4)
20	0.160(3)	0.155(9)	0.089(3)	0.094(4)
25	0.74(3)	0.71(5)	0.50(2)	0.52(3)
30	2.6(1)		1.95(5)	
35	7.4(2)		6.2(2)	
37	10.8(2)		9.2(3)	
40	18.5(7)		16.0(8)	
45	42(2)		39(3)	
50	85(3)		82(3)	
55	164(9)		163(10)	

[a]Errors quoted for the calculated polarizabilities are obtained by assuming a 0.5% error in the computed radial integrals from Ref. 5.

from its zero-field position was measured, and an average value was obtained from three or more spectral recordings. Measurements were corrected for thermal drift of the etalon (<1 MHz/min) by observing changes in the relative positions of etalon transmission maxima and reference cell spectra. For each J,M component, such Stark shift measurements were made for at least five different values of electric field. The field range for all components of a specific $n\,^2D$ doublet was chosen to include fields over which the associated 5/2,5/2 component shift was observed to depend quadratically on the field, as expected from Eq. (2). Graphs of Stark shift measurements versus E^2 are presented in Figs. 2 and 3, for levels $20\,^2D$ and $55\,^2D$. For level $20\,^2D$, the shifts of all five components vary as E^2 over the field range shown, while for $55\,^2D$, the quadratic dependence only holds up to $E=1.0$ V/cm.

V. ANALYSIS AND RESULTS

For a given ^2D state, the measured Stark shifts of all five components were fitted simultaneously to Eqs. (2) using a least-squares fitting procedure. The sum of squares of differences between observed shifts and those calculated with Eqs. (2) were minimized by choosing optimum values of the four polarizabilities $\alpha_0^{5/2}$, $\alpha_2^{5/2}$, $\alpha_0^{3/2}$, and $\alpha_2^{3/2}$. The values of the scalar and tensor polarizabilities resulting from this analysis are listed in Table I. Quoted errors for these polarizabilities are twice the statistical errors determined with the fitting program. Also listed in this table are polarizabilities calculated with Eqs. (5) using computed radial integrals reported[5] for $n\leq 28$, and quantum defects determined from recent measurements.[9,10] The errors for these values were obtained by assuming relative errors of 0.5% in the computed radial integrals.

The experimental values of the polarizabilities were fitted to polynomials of the effective quantum number using a least-squares fitting program, in order to determine their n^* dependence. Both scalar and tensor polarizabilities corresponding to $J=5/2$ and 3/2 were best described by polynomials of the form given by Eq. (1), namely, $\alpha_i=A_i(n^*)^7+B_i(n^*)^6$ over the range of $n=13$–55, or $11.653\leq n^*\leq 52.653$. Coefficients of these polynomials are presented in Table II, along with the coefficients obtained from a similar analysis of polarizabilities calculated for $n\leq 28$ using computed radial integrals.[5]

When the fine-structure spacings of intermediate ^2P and ^2F states are neglected in the energy denominators of Eqs. (5), the following identities are obtained for ratios of the scalar and tensor polarizabilities:

$$\alpha_0^{5/2}/\alpha_0^{3/2}=1$$

and (6)

$$\alpha_2^{3/2}/\alpha_2^{5/2}=7/10\ .$$

For excited D states of Rb these relations are only approximate as the fine-structure splittings of intermediate ^2P states lead to significantly lower values of the ratios which vary slowly with n. Graphs of these ratios versus n display their monotonic variations (Figs. 4 and 5). At high n, the ratios tend towards constant values of 0.78(4) and 0.62(2), for the scalar and tensor polarizabilities, respectively. This behavior is expected since the n dependence of the energy denominators in Eqs. (5) is the same as that of the fine-structure spacings of the highly excited intermediate states, namely, $\Delta_{FS}\sim n^{-3}$. Thus, even though the fine-structure spacings become negligible at high n, their contributions to the energy denominators remain significant. Included in Figs. 4 and 5 are ratios resulting from the present measurements, and from those of Svanberg and co-workers,[2–4] as well as ratios calculated using the computed radial integrals of Gounand.[5]

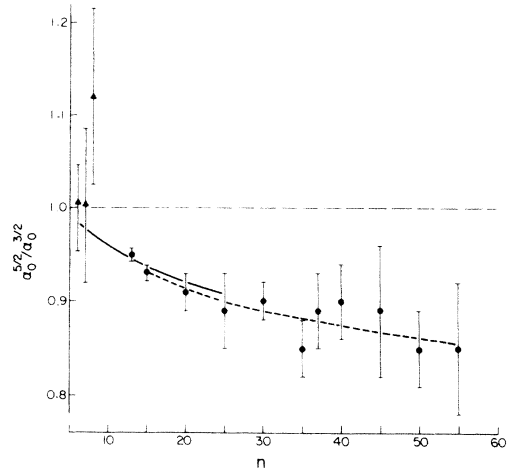

FIG. 4. Graph of the ratio $\alpha_0^{5/2}/\alpha_0^{3/2}$ plotted against n for $n\,^2D$ states of Rb. Included are the present data (●), those of Refs. 2, 3, and 4 for 6, 7, $8\,^2D$ (▲), the calculated values based on radial integrals of Ref. 5 (solid line), and those based on the parameters in Table II (dashed line).

TABLE II. Coefficients A and B of Eq. (1) for n^* scaling of polarizabilities, obtained from the present measurements, and when data for 6–$8\,^2D$ (Refs. 2–4) are included, as well as values calculated using computed radial integrals given in Ref. 5.

		$\alpha_0^{5/2}$	$\alpha_2^{5/2}$	$\alpha_0^{3/2}$	$\alpha_2^{3/2}$
$(n^*)^7$ coefft.	(present)	$7.2(3)\times10^{-11}$	$2.28(5)\times10^{-10}$	$9.1(2)\times10^{-11}$	$1.42(4)\times10^{-10}$
	(calc.)	$6(1)\times10^{-11}$	$2.4(1)\times10^{-10}$	$7(1)\times10^{-11}$	$1.54(5)\times10^{-10}$
	(incl. 6–$8\,^2D$)		$2.27(3)\times10^{-10}$		$1.4803(5)\times10^{-10}$
$(n^*)^6$ coefft.	(present)	$2.09(2)\times10^{-9}$	$-7.7(9)\times10^{-10}$	$2.06(4)\times10^{-9}$	$-5.7(6)\times10^{-10}$
	(calc.)	$2.34(15)\times10^{-9}$	$-9.6(1.5)\times10^{-10}$	$2.32(16)\times10^{-9}$	$-6.6(6)\times10^{-10}$
	(incl. 6–$8\,^2D$)		$-6.6(2)\times10^{-10}$		$-5.7(6)\times10^{-10}$
$(n^*)^5$ coefft.	(incl. 6–$8\,^2D$)		$-1.45(8)\times10^{-10}$		

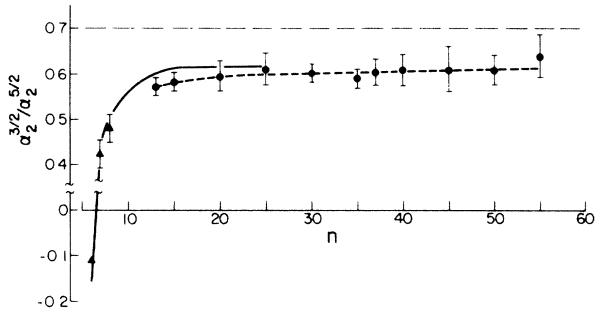

FIG. 5. Graph of the ratio $\alpha_2^{3/2}/\alpha_2^{5/2}$ plotted against n for $n\,^2D$ states of Rb. Included are the present data (●), those of Refs. 2, 3, and 4 for 6, 7, $8\,^2D$(▲), the calculated values based on radial integrals of Ref. 5 (solid line), and those based on the parameters in Table II (dashed line).

These results show that scalar and tensor polarizabilities of Rb ^2D states (either for $J=3/2$ or 5/2) are not linearly related owing to the ^2P-state fine-structure splitting. Furthermore, Eqs. (5) can be truncated to include only neighboring intermediate states. It follows from inspection of Eqs. (5) that, in principle, radial integrals corresponding to transitions from $n\,^2D$ to $(n+1)\,^2P$ and $n\,^2D$ to $(n+2)\,^2P$ states can be deduced from the four polarizabilities associated with each $n\,^2D$ level. However, the experimental errors of the present polarizabilities were such as to permit only the estimate of integrals for the $n\,^2D$ to $(n+1)\,^2P$ transitions. These values are listed in Table III along with values computed by Gounand.[5]

Intensity measurements of the five ^2D components were made for several of the observed spectra recorded at low fields (i.e., with $\Delta_{FS} > \Delta W^{J,|M|}$) using laser light polarized at 45° to the electric field. For the $35\,^2D_{3/2,5/2} \leftarrow 5\,^2S_{F=3}$ transitions, the average peak intensities at five different fields for the components $J, |M|$, namely, (5/2,5/2):(5/2,3/2):(5/2,1/2):(3/2,3/2):(3/2,1/2) were found to be 27(3):109(10):46(5):32(3):71(7). These results are in fair agreement with the values 31:100:65:36:62 calculated from the theory of two-photon absorption which neglects Stark mixing between states of the fine-structure doublet.[11]

VI. DISCUSSION

The only other evaluations of ^2D state polarizabilities of Rb were those made by Svanberg and co-workers[2–4] for 6, 7, and $8\,^2D$. Their values are compared in Table IV with predicted values obtained by extrapolating the n^*-scaling equations which resulted from measurements of the present work. It is seen that there is good agreement between predicted and measured scalar polarizabilities (with the predicted values being three times more precise than the earlier experimental values).

The predicted tensor polarizabilities are less precise, however, because of a near cancellation of $(n^*)^6$ and $(n^*)^7$ terms in the n^*-scaling equations close to $n=6$, which makes the predicted values very sensitive to even small errors in the A and B coefficients of Eq. (1). With this qualification, there is general agreement between predicted tensor polarizabilities and the values reported in Refs. 2–4. The precision of the n^*-scaling coefficients for tensor polarizabilities was improved by including the measurements of Refs. 2–4 in the analysis, and the new coefficients are given in Table II. The resulting improvement is most striking for $\alpha_2^{3/2}$ while an additional term (n^{*5}) was necessary to describe the dependence of $\alpha_2^{5/2}$. These results are expected since the polarizabilities of low-lying excited states give information regarding lower power terms in the n^*-scaling equations which can be written in the following general form:

$$\alpha = \sum_{k=0}^{\infty} A_k (n^*)^{(7-k)} .$$

TABLE III. Comparison of radial integrals for $n\,^2D \leftrightarrow (n+1)\,^2P$ transitions derived from measured polarizabilities and calculated values reported in Ref. 5.

$n\,^2D \leftrightarrow (n+1)\,^2P$		$a_0^{-1} R^{n\,^2D}_{(n+1)\,^2P}$	
		obs.	calc.[a]
13	14	194(50)	183
15	16	274(75)	252
20	21	520(150)	472
25	26	800(300)	759
30	31	1200(400)	
35	36	1800(650)	
37	38	1800(800)	
40	41	2100(850)	
45	46	2700(1900)	
50	51	3500(1900)	
55	56	3800(2300)	

[a]Calculated values available for n up to 28.

TABLE IV. Comparison of scalar and tensor polarizabilities α_0^J and α_2^J, for ^2D states of Rb, in MHz/(V/cm)^2 reported in Refs. 2–4, and values based on the present measurements.

Level	α_0^J		α_2^J	
	Refs. 2–4	Present	Refs. 2–4	Present
$6\,^2D_{3/2}$	$2.5(1)\times10^{-5}$	$2.63(5)\times10^{-5}$	$-1.05(7)\times10^{-7}$	$10(9)\times10^{-7}$
$7\,^2D_{3/2}$	$8.4(6)\times10^{-5}$	$8.6(2)\times10^{-5}$	$5.0(3)\times10^{-6}$	$8(3)\times10^{-6}$
$8\,^2D_{3/2}$	$2.1(2)\times10^{-5}$	$2.34(5)\times10^{-4}$	$2.7(1)\times10^{-5}$	$3(1)\times10^{-5}$
$6\,^2D_{5/2}$	$2.4(1)\times10^{-5}$	$2.57(3)\times10^{-5}$	$9.4(5)\times10^{-7}$	$2(2)\times10^{-5}$
$7\,^2D_{5/2}$	$8.5(3)\times10^{-5}$	$8.3(1)\times10^{-5}$	$1.17(5)\times10^{-5}$	$7(6)\times10^{-5}$
$8\,^2D_{5/2}$	$2.4(1)\times10^{-4}$	$2.26(3)\times10^{-4}$	$5.6(3)\times10^{-5}$	$2.0(1.4)\times10^{-4}$
$9\,^2D_{5/2}$			$1.8(9)\times10^{-4}$	$5(4)\times10^{-4}$

Measured polarizabilities and their associated n^*-scaling coefficients are in reasonable agreement with calculations based on computed radial integrals. The observed differences between measured and calculated values are consistent with an assumed uncertainty of $\sim 0.5\%$ in the computed integrals.[5] Radial integrals of $n\,^2D$ to $(n+1)\,^2P$ transitions for $13 \leq n \leq 55$ deduced from the present measurements, also agree with the values for $n \leq 28$, computed by Gounand.[5]

VII. CONCLUSIONS

The scalar and tensor polarizabilities of excited $n\,^2D$ states of Rb over the range $n = 13$–55 have been determined for the first time, their accuracies being 1–10%. The derived values were found to depend on the effective quantum numbers according to relations of the form $\alpha = A(n^*)^7 + B(n^*)^6$. Values of polarizabilities obtained by extrapolation to low n^* agreed with the only existing values, those for 6, 7, and $8\,^2D$ levels, measured by Svanberg and co-workers. Polarizabilities calculated from computed radial integrals were in reasonable agreement with the values obtained here, and integrals derived for $n\,^2D \leftrightarrow (n+1)\,^2P$ transitions agreed with the available values up to an $n = 28$ computed by Gounand.

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