Interaction of elementary atoms with matter

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The physics of elementary atoms, i.e., Coulomb bound states such as positronium, pionium, etc., is briefly presented. The formulas for cross sections of elementary-atom interactions with an external field are evaluated in the nonrelativistic and Born approximations. The numerical calculations of the total, elastic, and excitation cross sections are performed for seven elementary atoms and five typical targets. Atomic form factors are discussed in detail.

I. INTRODUCTION

The history of elementary atoms, i.e., the Coulomb bound states of two elementary particles,[1] started soon after the discovery of the positron. In 1934, Mohorovičic[2] suggested the existence of the bound state of an electron and a positron. Positronium was experimentally found by Deutsch,[3] who studied positron annihilation in gases. The existence of other elementary atoms was more or less obvious; however, the observation of the atoms of unstable particles was very difficult. Nemenov showed[4] that atom generation occurred in particle decays. For example, the kaon decay $K^0 \to \pi\mu\nu$ may provide an $A_{\pi\mu}$ atom. (A_{ab} denotes the elementary atom of positively charged particle a and negatively charged particle b. The atom of particle and antiparticle c and $\bar{c}$ is denoted by A_{2c}.) Indeed, in 1976 the $A_{\pi\mu}$ atom was discovered in the study of kaon decays.[5] Eight years later the ultrarelativistic positronium (with a Lorentz γ factor of 10^3–10^4) coming from the processes $\pi^0 \to e^+e^-\gamma$ was detected.[5] Then a very interesting proposition was made[7] to check the theory of relativity at such large gamma factors. It was suggested[8] that measurements of the atoms such as $A_{2\pi}$ and $A_{K\pi}$ could provide the unique information on meson-meson interactions at low energy. However, it was concluded in Ref. 8 that the registration of such atoms was beyond experimental possibilities. Recently the problem has been reviewed.[9] Nemenov has argued[9] that high-energy proton-proton or proton-nucleus collisions are the effective sources of elementary hadron atoms. Mesons abundantly produced in the collisions may form the atoms due to final-state interactions. It has been shown that the detection of these atoms is possible at currently available facilities.

For future experiments, knowledge of the interaction cross sections of elementary atoms with matter is needed. On the other hand, the problem is of theoretical interest. Nemenov has observed[10] that the destruction probability of ultrarelativistic positronium in matter may differ from the well-known exponential law since positronium collisions with atoms of matter are not independent. The point is that for a sufficiently fast elementary atom the characteristic time of its internal motion can be much longer than the time interval between successive collisions. Superpenetrability of ultrarelativistic positronium has been argued.[10] Lyuboshits and Podgoretsky have shown[11] that for targets thick when compared with the mean free path, the probability for observing the positronium in the bound state is inversely proportional to the target thickness and can be greater by some orders of magnitude than that coming from the exponential formula.

Dealing with elementary atoms coming from the decays of relativistic particles, we are faced with the well-known difficulty of the quantum field theory of the proper relativistic treatment of bound states. It should be stressed that this difficulty is of practical importance here, and different approaches can be confronted with the experimentally measurable process.

There are, to our knowledge, three papers[12–14] where the cross sections for elementary atom interactions with an external atomic field have been calculated. In one paper,[12] the old formulas of Bethe and Møller were used to study the interaction of $A_{\pi\mu}$. However, the author[12] has not noticed that these formulas, evaluated for the hydrogen atom, have been valid under the assumption that one particle of the atom has been much heavier than the other. This is, of course, not the case for the $A_{\pi\mu}$ atom. Therefore, the results of the paper[12] are incorrect. Dulian, Kotzinian, and Faustov[13] have employed the so-called pseudopotential technique for calculation of the dissociation cross section of relativistic positronium. As Dulian and Kotzinian have observed in their next paper,[14] the results of Ref. 13 were erroneous since the cross section was dependent on positronium helicity. The cross section calculated for zero helicity has been claimed to be correct and to be valid for all helicity states. This paper,[14] however, needs some comments. In this[14] paper the quantization axis of an atom orbital momentum has been chosen in a very unfortunate way. Usually this axis coincides with the momentum transfer vector. Then, there is a very simple selection rule. The transitions are allowed when the third magnetic quantum number m remains unchanged in the collision (see Appendix). To obtain the results independent of the choice of quantization axis, one has to summarize the transitions of different magnetic quantum numbers. However, if the quantization axis is chosen in the standard way the summation is trivial since there is only one nonvanishing transition probability. In the paper[14] the quantization axis is chosen along the beam axis. In such a case there is no simple selection rule, there

are complicated expressions for atomic form factors, and one has to summarize a few cross sections to obtain the result independent of the direction of the quantization axis.[15] Besides the problem presented above, the cross sections called break-up cross sections in Ref. 14 are, in fact, equal to the total cross sections. The integration over the relative momentum of the components of the ionized atom is equivalent to the summation over the complete set of wave functions if the plane waves are used for description of the ionized atom.

We intend to study systematically the whole problem of elementary-atom interaction with matter. In this paper we perform nonrelativistic calculations in the Born approximation. We discuss the total cross sections and the transitions to the discrete states. Dealing with nonrelativistic approximation, we want to elucidate peculiarities of the problem and to make the basis for relativistic calculations.

II. GENERAL FORMULAS

The matrix element for the atom interaction with an external field $U(\mathbf{r})$ in the Born approximation reads

$$S_{if}=-ie2\pi\delta(E_i-E_f)\int d^3r_1 d^3r_2 \psi_f^*(\mathbf{r}_1,\mathbf{r}_2) \times[U(\mathbf{r}_1)-U(\mathbf{r}_2)]\psi_i(\mathbf{r}_i,\mathbf{r}_2) , \quad (1)$$

where E_i and E_f are the energies of an initial and final state, and ψ_i and ψ_f describe the initial and final state of the atom. We use units where $c=\hbar=1$ and e^2 equals the fine-structure constant $e^2\cong\frac{1}{137}$. In the nonrelativistic approximation the atom's wave functions can be expressed as follows:

$$\psi_{i,f}(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{V}}e^{-i\mathbf{P}_{i,f}\cdot\mathbf{R}}\varphi_{i,j}(\mathbf{r}) , \quad (2)$$

where $\mathbf{P}_{i,f}$ is the initial (final) atom momentum and $V^{-1/2}$ is the normalization coefficient related to the volume of the system. The plane wave in the formula (2) describes the motion of a center of mass of the atom while $\varphi_{i,f}$ is the wave (Coulomb) function of the relative motion of atom components;

$$\mathbf{R}=\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2}{M}, \quad \mathbf{r}=\mathbf{r}_1-\mathbf{r}_2 ,$$

where $m_{1,2}$ and M are the masses of the atom components and the atom as a whole. It is assumed that $m_1+m_2=M$ because of a small value of binding energy of the atom. Substituting (2) into (1), one finds, after simple operations,

$$S_{if}=-\frac{ie}{V}2\pi\delta(E_i-E_f)\tilde{U}(\mathbf{q})[F_{if}(\eta\mathbf{q})-F_{if}(\xi\mathbf{q})] ,$$

where

$$\mathbf{q}=\mathbf{P}_f-\mathbf{P}_i, \quad \xi=m_1/M, \quad \eta=-m_2/M, \quad \xi-\eta=1 ,$$

and

$$\tilde{U}(\mathbf{q})=\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}U(\mathbf{r}) ,$$

$$F_{if}(\mathbf{q})=\int d^3 e^{i\mathbf{q}\cdot\mathbf{r}}\varphi_f^*(\mathbf{r})\varphi_i(\mathbf{r}) .$$

The atomic form factors F_{if} are discussed in detail in the Appendix. The cross section of the atom excitation from the state denoted by the quantum numbers (n,l,m) to the (n',l',m') state is

$$d\sigma_{nlm}^{n'l'm'}=\frac{1}{2\pi v^2}|\tilde{U}(\mathbf{q})|^2|F_{nlm}^{n'l'm'}(\eta\mathbf{q})-F_{nlm}^{n'l'm'}(\xi\mathbf{q})|^2 q\,dq , \quad (3)$$

where v is the initial atom velocity in the rest system of the external field, $\mathbf{q}$ is the momentum transfer, and $q\equiv|\mathbf{q}|$. The values of the minimal and maximal momentum transfer are the following ($P_i\equiv|\mathbf{P}_i|$):

$$q_{\min}=P_i-[P_i^2+2M(\epsilon_n-\epsilon_{n'})]^{1/2} ,$$
$$q_{\max}=P_i+[P_i^2+2M(\epsilon_n-\epsilon_{n'})]^{1/2} , \quad (4)$$

where $\epsilon_n,\epsilon_{n'}$ are the binding energies of the initial and final atom states. When $m_1=m_2$, formula (3) can be rewritten in the form

$$d\sigma_{nlm}^{n'l'm'}=\frac{1-(-1)^{l-l'}}{\pi v^2}|\tilde{U}(\mathbf{q})|^2|F_{nlm}^{n'l'm'}(\tfrac{1}{2}\mathbf{q})|^2 q\,dq ,$$

because

$$F_{nlm}^{n'l'm'}(-\mathbf{q})=(-1)^{l-l'}F_{nlm}^{n'l'm'}(\mathbf{q}) .$$

It is seen that the cross section differs from zero (in the Born approximation) when $l-l'$ is an odd number. In other words, the interaction occurs when parities of the initial and final states differ.[16] In particular, the positronium does not interact elastically.

The cross section of ionization of the atom initially being in the (n,l,m) state reads

$$d\sigma_{nlm}^{\text{ion}}=\frac{V}{(2\pi)^4 v^2}|\tilde{U}(\mathbf{q})|^2 \times|F_{nlm}^{\mathbf{p}}(\eta\mathbf{q})-F_{nlm}^{\mathbf{p}}(\xi\mathbf{q})|^2 q\,dq\,d^3p , \quad (5)$$

where $\mathbf{p}$ is the momentum of the atom components in the atom's center of mass. For the ionization process the formulas for $q_{\min}$ and $q_{\max}$ coincide with (4), however, $\epsilon_{n'}$ has to be substituted for by $\mathbf{p}^2/2\mu$, where μ is the reduced mass of the atom.

The applicability of the Born approximation has been widely studied in the past; see, e.g., Ref. 17. Here we use the practical and safe criterion that

$$\frac{\mathbf{P}_i^2}{2M}\gg-\epsilon_1 . \quad (6)$$

To calculate the cross section, we have to integrate the formulas (3) and (5) with respect to the momentum transfer. Because of the condition (6),

$$q_{\min}\cong\frac{\epsilon_{n'}-\epsilon_n}{v}\to 0 \quad \text{as } P_i\to\infty ,$$

$$q_{\max}\cong 2P_i\to\infty \quad \text{as } P_i\to\infty ,$$

so the lower limit of the integral can be shifted to zero, while the upper limit can be shifted to infinity, since the function under the integral strongly decreases when q goes to infinity. Then all cross sections found in the Born ap-

proximation can be written in the form

$$\sigma=\frac{1}{v^2}\tilde{\sigma}\ , \tag{7}$$

where $\tilde{\sigma}$ is the cross section independent of initial energy.

It is well known that the form factors satisfy the following sum rule:

$$\sum_f |F_{if}(\mathbf{q})|^2=1\ , \tag{8}$$

where the summation is performed over the complete set of final states. Considerations, which are very similar to those leading to Eq. (8), provide the sum rule for elementary atoms:

$$\sum_f |F_{if}(\eta\mathbf{q})-F_{if}(\xi\mathbf{q})|^2=2-2\,\mathrm{Re}F_{ii}((\xi-\eta)\mathbf{q})$$
$$=2-2F_{ii}(\mathbf{q})\ . \tag{9}$$

Let us observe that the result of the summation over the complete set of states is of the same form for all elementary atoms (does not depend on ξ and η). With the help of the sum rule (9) one can calculate the total cross section, where the incident energy dependence is that of the formula (7),

$$\sigma_{nlm}^{\mathrm{tot}}=\frac{1}{\pi v^2}\int_0^\infty dq\,q\,|\tilde{U}(\mathbf{q})|^2[1-F_{nlm}^{nlm}(\mathbf{q})]\ . \tag{10}$$

III. NUMERICAL RESULTS

To calculate the integrated cross sections, we have used two forms of the electrostatic potential.

(1) The screened Coulomb potential[18]

$$\tilde{U}(\mathbf{q})=\frac{4\pi Ze}{\mathbf{q}^2+b_z^{-2}}\ ,$$

where Ze is the charge of the target-atom nucleus and the screening length parameter is found on the basis of Thomas-Fermi model,

$$b_z^{-1}=m_e e^2 Z^{1/3}\ ,$$

where m_e is the electron mass.

(2) The so-called Thomas-Fermi-Molier potential[18]

$$\tilde{U}(\mathbf{q})=4\pi Ze\sum_{i=1}^{3}\frac{\alpha_i}{\mathbf{q}^2+\beta_i^2}\ ,$$

with

$$\beta_i=\frac{m_e b_i}{121}Z^{1/3}\ ,$$

and

$$b_1=6.0,\quad b_2=1.2,\quad b_3=0.3\ ,$$
$$\alpha_1=0.10,\quad \alpha_2=0.55,\quad \alpha_3=0.35\ .$$

Both forms of the potential have been used to check sensitivity of the results, however, the second potential is, of course, much more realistic.

The values of the cross sections $\tilde{\sigma}$ found by numerical integration of the formulas (3) and (10) are collected in Tables I–V. Seven elementary atoms and five target atoms of matter have been considered. The respective form factors are given in the Appendix. The upper numbers relate to the screened Coulomb potential, and the lower ones to the Thomas-Fermi-Molier potential. The zeros mean that the cross sections vanish in the Born approximation.

Let us discuss the content of the tables. It is seen that the results obtained for both potentials differ significantly for positronium while there are very small differences for pionium. The reason is the following. Both potentials are very similar for small (when compared with the screening length) distances from a nucleus, where the potentials are close to Coulombic. At large distances the potentials differ significantly. Because of a small value of the positronium binding energy (6.8 eV), collisions with large impact-parameter values contribute to the cross sections. Such collisions give no contribution for the interaction of pionium, the binding energy of which is 1.9 keV. Because

TABLE I. $\tilde{\sigma}$ (in cm^2) with C as the target atom.

	$2e$	$e\mu$	$e\pi$	2μ	$\mu\pi$	2π	πK
$\tilde{\sigma}_{100}^{\mathrm{tot}}$	3.4×10^{-19}	2.6×10^{-19}	2.6×10^{-19}	5.6×10^{-22}	4.4×10^{-22}	3.3×10^{-22}	1.4×10^{-22}
	4.6×10^{-19}	3.0×10^{-19}	3.0×10^{-19}	5.1×10^{-22}	4.0×10^{-22}	3.1×10^{-22}	1.3×10^{-22}
$\tilde{\sigma}_{100}^{\mathrm{el}}$	0	1.0×10^{-19}	1.0×10^{-19}	0	7.6×10^{-25}	0	3.4×10^{-24}
	0	9.9×10^{-20}	9.9×10^{-20}	0	7.5×10^{-25}	0	3.4×10^{-24}
$\tilde{\sigma}_{100}^{200}$	0	2.2×10^{-21}	2.2×10^{-21}	0	1.8×10^{-25}	0	7.4×10^{-25}
	0	4.2×10^{-21}	4.2×10^{-21}	0	1.8×10^{-25}	0	7.3×10^{-25}
$\tilde{\sigma}_{100}^{210}$	2.0×10^{-20}	5.2×10^{-21}	5.1×10^{-21}	2.8×10^{-22}	2.2×10^{-22}	1.6×10^{-22}	6.6×10^{-23}
	6.1×10^{-20}	1.6×10^{-20}	1.6×10^{-20}	2.5×10^{-22}	2.0×10^{-22}	1.5×10^{-22}	6.2×10^{-23}
$\tilde{\sigma}_{100}^{300}$	0	5.6×10^{-22}	5.6×10^{-22}	0	3.7×10^{-26}	0	1.5×10^{-25}
	0	1.0×10^{-21}	1.0×10^{-21}	0	3.7×10^{-26}	0	1.5×10^{-25}
$\tilde{\sigma}_{100}^{310}$	5.4×10^{-21}	3.7×10^{-21}	3.7×10^{-21}	4.7×10^{-23}	3.9×10^{-23}	2.8×10^{-23}	1.1×10^{-23}
	1.5×10^{-20}	3.8×10^{-21}	3.8×10^{-21}	4.3×10^{-23}	3.4×10^{-23}	2.6×10^{-23}	1.1×10^{-23}
$\tilde{\sigma}_{210}^{\mathrm{tot}}$	8.3×10^{-19}	8.5×10^{-19}	8.5×10^{-19}	8.0×10^{-21}	6.3×10^{-21}	4.7×10^{-21}	2.0×10^{-21}
	8.0×10^{-19}	7.5×10^{-19}	7.5×10^{-19}	6.5×10^{-21}	5.2×10^{-21}	4.0×10^{-21}	1.8×10^{-21}
$\tilde{\sigma}_{210}^{\mathrm{el}}$	0	4.7×10^{-19}	4.7×10^{-19}	0	2.4×10^{-23}	0	1.0×10^{-22}
	0	4.1×10^{-19}	4.1×10^{-19}	0	2.3×10^{-23}	0	9.9×10^{-23}

TABLE II. $\tilde{\sigma}$ (in cm^2) with Al as the target atom.

	$2e$	$e\mu$	$e\pi$	2μ	$\mu\pi$	2π	πK
$\tilde{\sigma}^{\text{tot}}_{100}$	1.0×10^{-18}	8.2×10^{-19}	8.2×10^{-19}	2.5×10^{-21}	2.0×10^{-21}	1.5×10^{-21}	6.5×10^{-22}
	1.5×10^{-18}	1.0×10^{-18}	1.0×10^{-18}	2.2×10^{-21}	1.8×10^{-21}	1.4×10^{-21}	6.1×10^{-22}
$\tilde{\sigma}^{\text{el}}_{100}$	0	3.4×10^{-19}	3.4×10^{-19}	0	3.5×10^{-24}	0	1.6×10^{-23}
	0	3.6×10^{-19}	3.6×10^{-19}	0	3.5×10^{-24}	0	1.6×10^{-23}
$\tilde{\sigma}^{200}_{100}$	0	4.8×10^{-21}	4.8×10^{-21}	0	8.3×10^{-25}	0	3.5×10^{-24}
	0	1.2×10^{-20}	1.2×10^{-20}	0	8.2×10^{-25}	0	3.4×10^{-24}
$\tilde{\sigma}^{210}_{100}$	4.0×10^{-20}	1.1×10^{-20}	1.0×10^{-20}	1.2×10^{-21}	9.6×10^{-22}	7.4×10^{-22}	3.0×10^{-22}
	1.5×10^{-19}	4.0×10^{-20}	3.9×10^{-20}	1.1×10^{-21}	8.6×10^{-22}	6.7×10^{-22}	2.8×10^{-22}
$\tilde{\sigma}^{300}_{100}$	0	1.3×10^{-21}	1.3×10^{-21}	0	1.8×10^{-25}	0	7.0×10^{-25}
	0	3.0×10^{-21}	3.0×10^{-21}	0	1.7×10^{-25}	0	7.0×10^{-25}
$\tilde{\sigma}^{310}_{100}$	1.1×10^{-20}	8.3×10^{-21}	8.2×10^{-21}	2.1×10^{-22}	1.8×10^{-22}	1.3×10^{-22}	5.3×10^{-23}
	3.9×10^{-20}	1.0×10^{-20}	1.0×10^{-20}	1.9×10^{-22}	1.5×10^{-22}	1.2×10^{-22}	4.8×10^{-23}
$\tilde{\sigma}^{\text{tot}}_{210}$	2.3×10^{-18}	2.4×10^{-18}	2.4×10^{-18}	3.6×10^{-20}	2.8×10^{-20}	2.1×10^{-20}	9.3×10^{-21}
	2.2×10^{-18}	2.2×10^{-18}	2.2×10^{-18}	2.8×10^{-20}	2.2×10^{-20}	1.7×10^{-20}	8.1×10^{-21}
$\tilde{\sigma}^{\text{el}}_{210}$	0	1.3×10^{-18}	1.3×10^{-18}	0	1.1×10^{-22}	0	4.7×10^{-22}
	0	1.2×10^{-18}	1.2×10^{-18}	0	1.1×10^{-22}	0	4.6×10^{-22}

TABLE III. $\tilde{\sigma}$ (in cm^2) with Cu as the target atom.

	$2e$	$e\mu$	$e\pi$	2μ	$\mu\pi$	2π	πK
$\tilde{\sigma}^{\text{tot}}_{100}$	3.1×10^{-18}	2.7×10^{-18}	2.6×10^{-18}	1.1×10^{-20}	9.0×10^{-21}	7.0×10^{-21}	3.2×10^{-21}
	4.7×10^{-18}	3.5×10^{-18}	3.4×10^{-18}	1.0×10^{-20}	8.2×10^{-21}	6.4×10^{-21}	2.9×10^{-21}
$\tilde{\sigma}^{\text{el}}_{100}$	0	1.2×10^{-18}	1.2×10^{-18}	0	1.7×10^{-23}	0	8.0×10^{-23}
	0	1.3×10^{-18}	1.3×10^{-18}	0	1.8×10^{-23}	0	8.0×10^{-23}
$\tilde{\sigma}^{200}_{100}$	0	1.0×10^{-20}	1.0×10^{-20}	0	4.1×10^{-24}	0	1.7×10^{-23}
	0	3.4×10^{-20}	3.4×10^{-20}	0	4.0×10^{-24}	0	1.7×10^{-23}
$\tilde{\sigma}^{210}_{100}$	7.7×10^{-20}	2.1×10^{-20}	2.1×10^{-20}	5.4×10^{-20}	4.4×10^{-21}	3.4×10^{-21}	1.5×10^{-21}
	3.8×10^{-19}	9.9×10^{-20}	9.8×10^{-20}	4.9×10^{-21}	3.9×10^{-21}	3.1×10^{-21}	1.3×10^{-21}
$\tilde{\sigma}^{300}_{100}$	0	2.8×10^{-21}	2.8×10^{-21}	0	8.7×10^{-25}	0	3.5×10^{-24}
	0	8.5×10^{-21}	8.6×10^{-21}	0	8.5×10^{-25}	0	3.5×10^{-24}
$\tilde{\sigma}^{310}_{100}$	2.2×10^{-20}	1.8×10^{-20}	1.8×10^{-20}	9.4×10^{-22}	8.3×10^{-22}	5.9×10^{-22}	2.6×10^{-22}
	9.9×10^{-20}	2.6×10^{-20}	2.5×10^{-20}	8.5×10^{-22}	6.8×10^{-22}	5.3×10^{-22}	2.3×10^{-22}
$\tilde{\sigma}^{\text{tot}}_{210}$	6.7×10^{-18}	6.9×10^{-18}	6.9×10^{-18}	1.6×10^{-19}	1.3×10^{-19}	1.0×10^{-19}	4.5×10^{-20}
	6.4×10^{-18}	6.5×10^{-18}	6.5×10^{-18}	1.2×10^{-19}	9.9×10^{-20}	7.8×10^{-20}	3.8×10^{-20}
$\tilde{\sigma}^{\text{el}}_{210}$	0	3.7×10^{-18}	3.7×10^{-18}	0	5.4×10^{-22}	0	2.3×10^{-21}
	0	3.5×10^{-18}	3.6×10^{-18}	0	5.2×10^{-22}	0	2.3×10^{-21}

TABLE IV. $\tilde{\sigma}$ (in cm^2) with Ag as the target atom.

	$2e$	$e\mu$	$e\pi$	2μ	$\mu\pi$	2π	πK
$\tilde{\sigma}^{\text{tot}}_{100}$	5.9×10^{-18}	5.3×10^{-18}	5.3×10^{-18}	2.8×10^{-20}	2.2×10^{-20}	1.8×10^{-20}	8.1×10^{-21}
	9.4×10^{-18}	7.2×10^{-18}	7.2×10^{-18}	2.5×10^{-20}	2.0×10^{-20}	1.6×10^{-20}	7.4×10^{-21}
$\tilde{\sigma}^{\text{el}}_{100}$	0	2.4×10^{-18}	2.4×10^{-18}	0	4.6×10^{-23}	0	2.1×10^{-22}
	0	2.8×10^{-18}	2.8×10^{-18}	0	4.6×10^{-23}	0	2.1×10^{-22}
$\tilde{\sigma}^{200}_{100}$	0	1.6×10^{-20}	1.6×10^{-20}	0	1.1×10^{-23}	0	4.5×10^{-23}
	0	6.1×10^{-20}	6.1×10^{-20}	0	1.1×10^{-23}	0	4.5×10^{-23}
$\tilde{\sigma}^{210}_{100}$	1.1×10^{-19}	3.2×10^{-20}	3.1×10^{-20}	1.3×10^{-20}	1.1×10^{-20}	8.5×10^{-21}	3.7×10^{-21}
	6.3×10^{-19}	1.7×10^{-19}	1.6×10^{-19}	1.2×10^{-20}	9.7×10^{-21}	7.6×10^{-21}	3.3×10^{-21}
$\tilde{\sigma}^{300}_{100}$	0	4.3×10^{-21}	4.3×10^{-21}	0	2.3×10^{-24}	0	9.2×10^{-24}
	0	1.6×10^{-20}	1.6×10^{-20}	0	2.2×10^{-24}	0	9.1×10^{-24}
$\tilde{\sigma}^{310}_{100}$	3.3×10^{-20}	2.8×10^{-20}	2.7×10^{-20}	2.3×10^{-21}	2.1×10^{-21}	1.5×10^{-21}	6.7×10^{-22}
	1.7×10^{-19}	4.4×10^{-20}	4.3×10^{-20}	2.1×10^{-21}	1.7×10^{-21}	1.3×10^{-21}	5.7×10^{-22}
$\tilde{\sigma}^{\text{tot}}_{210}$	1.3×10^{-17}	1.3×10^{-17}	1.3×10^{-17}	3.9×10^{-19}	3.2×10^{-19}	2.5×10^{-19}	1.2×10^{-19}
	1.2×10^{-17}	1.2×10^{-17}	1.2×10^{-17}	2.9×10^{-19}	2.4×10^{-19}	1.9×10^{-19}	9.4×10^{-20}
$\tilde{\sigma}^{\text{el}}_{210}$	0	6.8×10^{-18}	6.8×10^{-18}	0	1.4×10^{-21}	0	6.1×10^{-21}
	0	6.8×10^{-18}	6.7×10^{-18}	0	1.3×10^{-21}	0	5.9×10^{-21}

TABLE V. $\tilde{\sigma}$ (in cm^2) with Pb as the target atom.

	$2e$	$e\mu$	$e\pi$	2μ	$\mu\pi$	2π	πK
$\tilde{\sigma}^{\rm tot}_{100}$	1.3×10^{-17}	1.2×10^{-17}	1.2×10^{-17}	7.7×10^{-20}	6.3×10^{-20}	5.0×10^{-20}	2.4×10^{-20}
	2.1×10^{-17}	1.7×10^{-17}	1.6×10^{-17}	7.2×10^{-20}	5.8×10^{-20}	4.5×10^{-20}	2.1×10^{-20}
$\tilde{\sigma}^{\rm el}_{100}$	0	5.3×10^{-18}	5.3×10^{-18}	0	1.4×10^{-22}	0	6.4×10^{-22}
	0	6.7×10^{-18}	6.7×10^{-18}	0	1.4×10^{-22}	0	6.3×10^{-22}
$\tilde{\sigma}^{200}_{100}$	0	2.5×10^{-20}	2.5×10^{-20}	0	3.3×10^{-23}	0	1.4×10^{-22}
	0	1.2×10^{-19}	1.2×10^{-19}	0	3.2×10^{-23}	0	1.4×10^{-22}
$\tilde{\sigma}^{210}_{100}$	1.7×10^{-19}	5.3×10^{-20}	4.9×10^{-20}	3.6×10^{-20}	3.0×10^{-20}	2.4×10^{-20}	1.1×10^{-20}
	1.1×10^{-18}	2.9×10^{-19}	2.9×10^{-19}	3.3×10^{-20}	2.7×10^{-20}	2.2×10^{-20}	9.6×10^{-21}
$\tilde{\sigma}^{300}_{100}$	0	6.9×10^{-21}	6.8×10^{-21}	0	6.9×10^{-24}	0	2.8×10^{-23}
	0	3.0×10^{-20}	3.0×10^{-20}	0	6.7×10^{-24}	0	2.8×10^{-23}
$\tilde{\sigma}^{310}_{100}$	5.0×10^{-20}	4.5×10^{-20}	4.4×10^{-20}	6.4×10^{-21}	6.0×10^{-21}	4.2×10^{-21}	2.0×10^{-21}
	3.0×10^{-19}	7.8×10^{-20}	7.7×10^{-20}	5.9×10^{-21}	4.8×10^{-21}	3.8×10^{-21}	1.7×10^{-21}
$\tilde{\sigma}^{\rm tot}_{210}$	2.6×10^{-17}	2.7×10^{-17}	2.7×10^{-17}	1.1×10^{-18}	8.9×10^{-19}	7.1×10^{-19}	3.4×10^{-19}
	2.5×10^{-17}	2.6×10^{-17}	2.6×10^{-17}	8.0×10^{-19}	6.6×10^{-19}	5.3×10^{-19}	2.7×10^{-19}
$\tilde{\sigma}^{\rm el}_{210}$	0	1.4×10^{-17}	1.4×10^{-17}	0	4.3×10^{-21}	0	1.9×10^{-20}
	0	1.4×10^{-17}	1.4×10^{-17}	0	4.0×10^{-21}	0	1.8×10^{-20}

the Bohr radius decreases when the atom's reduced mass increases, the cross sections of positronium are much larger than these of pionium.

One observes that the most probable transitions from the ground state are those to the $2p$ states. More generally, the most important are the transitions to the p states. This property is connected with the behavior of the atomic form factors at small (when compared with the inverse Bohr radius) momentum transfer. If one expands the inelastic form factor in the series of powers of the momentum transfer, the linear term occurs for the transition to the p states only. Therefore, the excitations to the p states can proceed in collisions of very small momentum transfer. Because the transitions from the ground state to the $2p$ state are the most important, we have calculated the elastic and total cross sections when the atom's initial state coincides with the $2p$ state. It is seen that the ratio of the total cross sections for $1s$ and $2p$ states essentially depends on the atom's reduced mass. This ratio is about 0.5 for positronium and less than 0.1 for pionium.

Our last remark concerns the contribution of the excitation cross sections to the total cross sections. It is seen that for the atoms of small reduced mass, the ionization cross section gives the dominant contribution to the total cross section, while for atoms of heavier components the excitation cross sections are comparable with the total cross sections.

IV. DISCUSSION AND CONCLUSIONS

Let us discuss the results presented in this paper. The calculated cross sections have been found in the nonrelativistic approximation. However, the cross sections listed in Tables I–V coincide with those found in the relativistic approach for the atoms with $v=1$, if one neglects the magnetic part of interaction. In fact, the electric part of interaction gives the dominant contribution to the cross sections.

In our calculations the target has been represented by the external static potential. This means that target recoil effects have been neglected. On the other hand, the target-atom excitations have not been taken into account. For sufficiently heavy targets the cross sections for the processes associated with the target-atom excitation are much smaller than the cross sections without the excitation. The point is that the probability for the target-atom excitation is, loosely speaking, proportional to Z, while the interaction without excitation (coherent process) is proportional to Z^2. For light targets the incoherent processes can be important, particularly for the elementary atoms with small reduced mass. The total cross section for the positronium interaction with a carbon target is 2 times greater than that calculated in this paper if one takes into account the carbon target excitations.[19]

The processes of the elementary-atom interaction with an external field associated with photon radiation have been considered in our paper.[20] It has been shown that the cross sections for such processes are smaller by at least 5 orders of magnitude than those of the analogous processes without radiation.

We conclude as follows. The calculations presented in this paper provide a realistic basis for the estimation of elementary-atom mean free paths in matter, however, the extension of these calculations and some improvements are desirable.

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APPENDIX: ATOMIC FORM FACTORS

Let us calculate the form factor

$$F^{nlm}_{100}(\mathbf{q})=\int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\varphi^*_{nlm}(\mathbf{r})\varphi_{100}(\mathbf{r})\,, \tag{A1}$$

where φ_{100} and φ_{nlm} are the wave functions of the hydrogenlike atom in the ground state and the excited state denoted by the quantum numbers (n,l,m). We choose the quantization axis of the atom orbital momentum along

the momentum transfer vector. Then, the integral (A1) is nonzero for $m=0$ only. The wave functions present in (A1) are the following:[21]

$$\varphi_{100}(\mathbf{r})=\frac{1}{\sqrt{\pi}}e^{-r}\,,$$
$$\varphi_{nlm=0}(\mathbf{r})=\frac{1}{n^{l+2}(2l+1)!}\left[\frac{(n+l)!(2l+1)}{\pi(n-l-1)!}\right]^{1/2}\times(2r)^l e^{-r/n}F(-n+l+1,2l+2,2r/n)\times P_l(\cos\theta)\,, \tag{A2}$$

where F is the hypergeometric confluent function and P_l is the Legendre polynomial. The wave functions and the form factors calculated in this Appendix are written in atomic units, where the Bohr radius of the atom equals unity. Substituting (A2) in (A1) and performing integration with respect to the solid angle, one finds

$$F_{100}^{nlm}(\mathbf{q})=\delta_m^0\frac{2^{l+1}}{n^{l+2}(2l+1)!}\left[\frac{2\pi(2l+1)(n+l)!}{q(n-l-1)!}\right]^{1/2}\times e^{il\pi/2}\int_0^\infty dr\,r^{l+3/2}e^{-r(n+1)/n}j_{l+1/2}(qr)\times F(-n+l+1,2l+2,2r/n)\,, \tag{A3}$$

where $j_{l+1/2}$ is the Bessel function and δ_m^0 is the Kronecker symbol. Integration over the polar angle has been made with the help of the formula[22]

$$\int_{-1}^{+1}dx\,e^{iax}P_l(x)=\left[\frac{2\pi}{a}\right]^{1/2}e^{il\pi/2}j_{l+1/2}(a)\,.$$

Then we use the integral representation of the hypergeometric confluent function[21]

$$F(\alpha,\gamma,Z)=-\frac{1}{2\pi i}\frac{\Gamma(1-\alpha)\Gamma(\gamma)}{\Gamma(\gamma-\alpha)}\times\oint_C dt\,e^{tz}(-t)^{\alpha-1}(1-t)^{\gamma-\alpha-1}\,, \tag{A4}$$

where Γ is the Euler gamma function. For γ a positive integer number, C is the contour circulating the points $t=0$ and 1. Substituting (A4) in (A3), one can perform integration with respect to variable r using the formula[22]

$$\int_0^\infty dx\,x^{\nu+1}e^{-px}j_\nu(cx)=\frac{2p(2c)^\nu\Gamma(\nu+\frac{3}{2})}{\sqrt{\pi}(p^2+c^2)^{\nu+3/2}}\,.$$

Finally, we make the contour integration using the Cauchy formula. In this way one gets

$$F_{100}^{nlm}(\mathbf{q})=(-1)^{n-l-1}\delta_m^0 e^{il\pi/2}2^{2l+3}n^{l+1}(l+1)!\times\left[\frac{2l+1}{(n+l)!(n-l-1)!}\right]^{1/2}q^l\frac{d^{n-l-1}}{dx^{n-l-1}}\left[\frac{(n-2x+1)(1-x)^{n+l}}{[(n-2x+1)^2+n^2q^2]^{l+2}}\right]\Bigg|_{x=0}\,. \tag{A5}$$

Similar calculations of the form factor have been performed in Ref. 14, where the quantization axis has been chosen along the beam axis.

Another analytical formula for the atom form factor was found many years ago by Massey and Mohr.[23] Their result is also not very good for practical usage since the form factor (A1) is expressed through the Gegenbauer polynomials.

In Table VI we give explicit formulas for ten form factors of the lowest energy states.

From the formula (A5) it is easy to obtain the form factor in the limit $q\to\infty$ (momentum transfer much greater than the inverse Bohr radius),

$$F_{100}^{nlm}(\mathbf{q})\cong\delta_m^0 e^{il\pi/2}2^{2l+3}\frac{(l+2)l!}{(2l+1)!n^{l+2}}\times\left[\frac{(2l+1)(n+l)}{(n-l-1)!}\right]^{1/2}q^{-l-4}\,. \tag{A6}$$

TABLE VI. Form factors $F_{100}^{n10}(\mathbf{q})$.

	$n=1$	$n=2$	$n=3$	$n=4$
$l=0$	$\frac{16}{(4+q^2)^2}$	$2^{17/2}\frac{q^2}{(9+4q^2)^3}$	$2^4 3^{7/2}\frac{(16+27q^2)q^2}{(16+9q^2)^4}$	$\frac{2^{13}}{3}\frac{(768q^4+1056q^2+375)q^2}{(25+16q^2)^5}$
$l=1$		$i2^{15/2}3\frac{q}{(9+4q^2)^3}$	$i2^{11/2}3^3\frac{(16+27q^2)q}{(16+9q^2)^4}$	$i2^{11}5^{1/2}3\frac{(256q^2+352q^2+125)q}{(25+16q^2)^5}$
$l=2$			$-2^{17/2}3^{7/2}\frac{q^2}{(16+9q^2)^4}$	$-2^{19}\frac{(3+4q^2)q^2}{(25+16q^2)^5}$
$l=3$				$-i2^{18}5^{1/2}\frac{q^3}{(25+16q^2)^5}$

In contrast to formula (A5), formula (A6) allows a practical usage.

It is much more complicated to find the form factor in the limit $q \to 0$. This limit, which is often called a dipole approximation, can be realized by substituting the Fourier multiplier in formula (A1) by $1+i\mathbf{q}\cdot\mathbf{r}$. The master calculations of the atomic form factors were performed by Bethe,[24] who found a simple analytical formula. However, the atom states were described by parabolic quantum numbers. Bethe also found the form-factor module squared summarized over the parabolic quantum numbers n_1 and n_2 at fixed main quantum number n. Because this summation is equivalent to that over the orbital quantum number l, one can write[24]

$$\sum_{l=-n+1}^{n-1} |F_{100}^{nlm=0}(\mathbf{q})|^2 = \frac{2^8}{3} n^7 \frac{(n-1)^{2n-5}}{(n+1)^{2n+5}} q^2 + O(q^3) , \tag{A7}$$

where $n > 1$. Except for F_{100}^{100}, the form factors in the dipole approximation are

$$F_{100}^{nlm}(\mathbf{q}) = i \int d^3r\, \mathbf{qr} \varphi_{nlm}^*(\mathbf{r}) \varphi_{100}(\mathbf{r}) + O(q^2) . \tag{A8}$$

Let us now observe that the form factor (A8) is nonzero for $l=1$ only because of orthogonality of the Legendre polynomials. Thus, only the form factor with $l=1$ contributes to the sum in (A7). Finally, comparing formulas (A7) and (A5), one obtains the form factor in the small momentum transfer limit

$$F_{100}^{nlm}(\mathbf{q}) = i\delta_m^0 \delta_l^1 \frac{2^4}{3^{1/2}} n^{7/2} \frac{(n-1)^{n-5/2}}{(n+1)^{n+5/2}} q + O(q^2) .$$

To complete our discussion we give the formulas for the form factor F_{210}^{210} and the form factor which occurs in the considerations of atom ionization processes,

$$F_{210}^{210}(\mathbf{q}) = \frac{1-5q^2}{(1+q^2)^4} ,$$

$$F_{100}^{\mathbf{p}}(\mathbf{q}) = \frac{8\sqrt{\pi}}{[1+(\mathbf{q}-\mathbf{p})^2]^2} .$$

The ionized atom has been described by a plane-wave function. $\mathbf{p}$ is the atom component momentum in the center of mass of the atom.

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[1]We do not take into account a hydrogen atom. Later on we also do not consider the atom of a muon and a proton. The interaction with matter of very slow positronium which is formed when a positron from a β^+ decay is slowed down in a gas is also not discussed. For all these subjects there is a very extensive literature.

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[15]I am grateful to V. L. Lyuboshits and A. S. Pak for discussion on this point.

[16]One may ask whether this result depends on the internal parity of atom components, since the parity of two particles is the product of internal parities of the particles and the parity of relative motion which is $(-1)^{l+s}$. Because the same particles are in the initial and final states, the parity of the system can be changed only by the change of the parity of the relative motion. Spin is conserved in nonrelativistic approach.

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