Surface-wave fluctuations and emission from an inhomogeneous magnetoactive plasma

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(Received 1 October 1984)

A theory has been developed for thermal fluctuations of electromagnetic surface waves propagating normal to a dc magnetic field along a plasma-plasma boundary. It is shown that for relatively weak magnetic fields the energy of these fluctuations can be emitted from the plasma. This emission is investigated in conditions which are typical for laser-produced plasmas when a spontaneously generated magnetic field exists. The spectral distribution obtained reveals both an amplitude and angular asymmetry which can be used for diagnostic purposes.

I. INTRODUCTION

The radiation emitted from a plasma in the presence of thermal fluctuations has been intensively studied for many years.[1] Most of the theoretical work concerning this problem deals with an infinite and/or homogeneous plasma. However, in the real experimental conditions the size of the plasma is always finite and, therefore, the effects of the boundaries have to be taken into account. One of the most important characteristics of the plasma boundaries with steep density gradients is the presence of surface waves as the eigenmodes of the system.[2] The theory of surface-wave fluctuations has been developed for both sharp[3] and diffused[4] plasma boundaries. As is well known, the phase velocity of a surface wave propagating along a plasma-vacuum or plasma-dielectric boundary is less than the speed of light. Consequently, their energy cannot escape from the boundary. However, a plasma-plasma boundary allows the phase velocity to exceed the speed of light for a certain range of the wavelengths. In that case, if the region of plasma which has the lower density has a finite density scale length, the energy of the surface waves can escape from the boundary region and be emitted in a form of an electromagnetic wave propagating in the low-density transparent plasma. Favorable conditions for the excitation of such radiating surface waves arise, for example, in a laser-produced plasma-density profile is steepened in the critical region.[5] Emission from such a plasma was studied in Ref. 6. However, in Ref. 6 the presence of a spontaneous dc magnetic field, which can be generated in a laser-produced plasma, has not been taken into account, although, at high intensities this magnetic field can reach several MG.[7] There are a few mechanisms for the generation of these fields. However, the geometry of our problem favors resonance absorption as the source.[8] Also, since a damping mechanism determines the correlation function of the fluctuating electromagnetic field and resonance absorption (i.e., linear conversion of the surface waves into the upper hybrid modes) dominates, the plasma is considered collisionless throughout this paper. In the initial stage of the laser-plasma interaction, the spontaneously generated field can be localized in the critical region, but subsequently it spreads well beyond this region due to both field diffusion and plasma flow. Furthermore, a similar problem can arise in the presence of an external magnetic field and be encountered, therefore, in other types of laboratory and space plasmas. For example, spontaneous magnetic fields have been measured in microwave plasma experiments.[9] Consequently, it is of considerable importance to develop a theory for the fluctuations of the surface waves in the presence of a dc-magnetic field.

In laser-produced plasma, in particular, direct measurement of the parameters of the critical region can be rather difficult. As a result, investigation of the plasma emission induced by these fluctuations can be used for diagnostic purposes. In this paper we present a theory of thermal fluctuations of the electromagnetic surface waves which propagate normal to a dc magnetic field. The spectral distribution of the radiation emitted from an inhomogeneous plasma is found for conditions typical of a laser-produced plasma.

In Sec. II we give a complete formulation of the problem. Then, in Sec. III, we calculate a correlation function for the fluctuating electromagnetic field in terms of the anisotropic electron temperature, the profile characteristics, and the dc magnetic field. In Sec. IV the spectrum of the plasma emission associated with thermal fluctuations of the surface waves is then found in the cases when the dc magnetic field is either localized within the critical region or extends over the whole space.

II. FORMULATION OF THE PROBLEM

Let us consider a collisionless plasma with a steep density gradient across the critical region (see Fig. 1). The plasma is assumed to be homogeneous for $x > a$ and weakly inhomogeneous in a region $x < 0$. Thus, we intro-

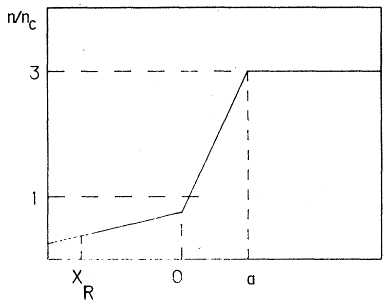

FIG. 1. Schematic representation of a plasma density profile.

duce two characteristic scale lengths of the plasma inhomogeneity $L = n\,|\,dn/dx\,|^{-1}_{x<0} \gg l = n\,|\,dn/dx\,|^{-1}_{0<x<a}$. Such a density profile allows for existence of the surface eigenmodes which propagate along the boundary layer $0<x<a$. An external dc magnetic field $\mathbf{B}_0=(0,0,B_0(x))$ is assumed to occupy either only a transition layer $0<x<a$ or, alternatively, the whole space. We will investigate the surface waves which propagate perpendicularly to the magnetic field. In that case the electromagnetic field of the waves has the following components: $\mathbf{E}=(E_x,E_y,0)$ and $\mathbf{B}=(0,0,B)$. Notice that the complementary electromagnetic wave $\mathbf{E}=(0,0,E)$, $\mathbf{B}=(B_x,B_y,0)$ does not permit a solution in the form of a surface mode. In order to study the fluctuations of these waves we start from the Fourier-Laplace transform of the electromagnetic quantities:

$$\mathbf{E}(x,k,\omega+i\Delta)=\int_0^{\infty}dt\int_{-\infty}^{\infty}dy\,\mathbf{E}(x,y,t)\times\exp\{i[(\omega+i\Delta)t-ky]\}\,,$$

$$\mathbf{B}(x,k,\omega+i\Delta)=\int_0^{\infty}dt\int_{-\infty}^{\infty}dy\,\mathbf{B}(x,y,t)\times\exp\{i[(\omega+i\Delta)t-ky]\}\,.$$

Then, Maxwell's equations lead to the following set of equations:

$$\frac{d}{dx}\left[\frac{1}{\alpha}\frac{dB}{dx}\right]-\frac{\omega^2}{c^2}\left[\frac{n^2-\alpha}{\alpha}+n\frac{c}{\omega}\frac{d}{dx}\left[\frac{\beta}{\alpha}\right]\right]=-i\frac{\omega}{c}\left[\frac{d}{dx}\left[\frac{\eta_1}{\alpha}\right]+k\frac{\eta_2}{\alpha}\right],\qquad(1)$$

$$E_y=-i\frac{c}{\omega}\left[\frac{1}{\alpha}\frac{dB}{dx}-k\frac{\beta}{\alpha}B+i\frac{\omega}{c}\frac{\eta_1}{\alpha}\right],\qquad(2)$$

$$E_x=-\frac{n}{\alpha}B-\frac{\omega}{c}\frac{\eta_2}{\alpha}+\frac{i}{kc}B(t=0)\,,\qquad(3)$$

where

$$\alpha=\frac{\epsilon^2-g^2}{\epsilon}\,,\quad \beta=\frac{g}{\epsilon}\,,\quad n=\frac{kc}{\omega}\,,\qquad(4)$$

$$\eta_1=\frac{4\pi e n_0}{\omega^2-\Omega^2}\left[\left[1+\frac{\Omega}{\omega}\beta\right]v_y(t=0)-i\left[\beta+\frac{\Omega}{\omega}\right]v_x(t=0)\right]+\frac{\beta}{\omega}E_x(t=0)-\frac{i}{\omega}E_y(t=0)\,,$$

$$\eta_2=\frac{4\pi e n_0}{\omega^2-\Omega^2}\left[\left[\frac{\Omega}{\omega}+\beta\right]v_y(t=0)-i\left[1+\frac{\Omega}{\omega}\beta\right]v_x(t=0)\right]+\frac{1}{\omega}E_x(t=0)-i\frac{\beta}{\omega}E_y(t=0)\,.\qquad(5)$$

Here ϵ and g are the components of the plasma permitivity tensor

$$\underline{\epsilon}=\begin{bmatrix}\epsilon & -ig & 0\\ ig & \epsilon & 0\\ 0 & 0 & \epsilon_{\parallel}\end{bmatrix},$$

$$\epsilon=1-\frac{\omega_p^2}{\omega^2-\Omega^2}\,,\quad g=\frac{\Omega}{\omega}\frac{\omega_p^2}{\omega^2-\Omega^2}\,,\quad \epsilon_{\parallel}=1-\frac{\omega_p^2}{\omega^2}\,.$$

$\omega_p^2=4\pi e^2 n_0/m$, $\Omega=eB/mc$, the electron plasma and cyclotron frequency, respectively, and m,e,n_0 are, respectively, the electron mass, charge, and density, and c is the speed of light. Finally, $v_x(t=0)$ and $v_y(t=0)$ are the initial values of x and y components of the electron velocity.

Our aim is to calculate the spectral distribution of the plasma emission associated with the thermal fluctuations. The set of the starting equations (1)—(3) is solved to obtain the fluctuating field $\mathbf{E}$, $\mathbf{B}$ in terms of $\mathbf{v}(t=0)$. The field correlation function then defines the relevant energy flux density

$$\mathbf{S}=\frac{c}{8\pi}\int\frac{d\omega\,dk}{(2\pi)^2}\langle\mathbf{E}(\omega,k,x)\times\mathbf{B}^*(\omega,k,x)\,|_{x=-\infty}+\text{c.c.}\rangle\,,\qquad(6)$$

where $\langle\ \rangle$ denote an ensemble average.

III. THERMAL FLUCTUATIONS OF THE SURFACE WAVES

At the beginning of this section we will briefly summarize a general formulation of a theory of hydrodynamic fluctuations in an inhomogeneous plasma. According to Ref. 1, the steady space-time correlations of the electric field components are

$$\langle E_i(\mathbf{r},t)E_j(\mathbf{r},t)\rangle = \int \frac{d\omega}{2\pi}\exp[-i\omega(t-t')]G_{ij}(\mathbf{r},\mathbf{r}',\omega) , \tag{7}$$

where

$$G_{ij}(\mathbf{r},\mathbf{r}',\omega) = \lim_{\Delta\to 0_+} 2\Delta\langle E_i(\mathbf{r},\omega+i\Delta)E_j^*(\mathbf{r}',\omega+i\Delta)\rangle . \tag{8}$$

The fluctuating field E is related (via the equation of motion) to the fluctuating current density

$$\mathbf{j}(\mathbf{r},t)=en_0\mathbf{v}(\mathbf{r},t)=e\int d\mathbf{v}\,\mathbf{v}\delta N_e(\mathbf{r},\mathbf{v},t) , \tag{9}$$

which, in turn, arises from thermal fluctuations of the electron phase micro density $\delta N_e(\mathbf{r},\mathbf{v},t)$. As shown by Klimontovich,[1]

$$\langle \delta N_e(\mathbf{r},\mathbf{v},t)\delta N_e(\mathbf{r}',\mathbf{v}',t)\rangle = \delta(\mathbf{r}-\mathbf{r}')\delta(\mathbf{v}-\mathbf{v}')f_e(\mathbf{r},\mathbf{v}) + g_{ee}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}',t) , \tag{10}$$

where $f_e(\mathbf{r},\mathbf{v})$ and $g_{ee}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}',t)$ are the electron distribution and pair correlation functions. Then, (9) and (10) lead to the following correlation at one instant between the velocity components

$$\langle v_i(\mathbf{r},t)v_j(\mathbf{r}',t)\rangle = \delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\frac{T^{(j)}(\mathbf{r})}{mn_o(\mathbf{r})} + \int d\mathbf{v}\,d\mathbf{v}'\frac{v_iv_j'g_{ee}(\mathbf{r},\mathbf{r}',\mathbf{v},\mathbf{v}',t)}{n_0(\mathbf{r})n_0(\mathbf{r}')} \tag{11}$$

in terms of the anisotropic electron temperature

$$T^{(j)}(\mathbf{r}) = \frac{1}{n_0(\mathbf{r})}\int d\mathbf{v}\, mv_j^2 f_e(\mathbf{r},\mathbf{v}) .$$

It was supposed here that in the ground state

$$\int d\mathbf{v}\,\mathbf{v}f_e(\mathbf{r},\mathbf{v})=0 ,$$

$$\int d\mathbf{v}\,v_iv_jf_e(\mathbf{r},\mathbf{v})=0 , \quad i\neq j .$$

In what follows we will calculate the Poynting vector $\mathbf{S}$ [Eq. (6)] by means of Maxwell's equation (1)—(3) and by relating the correlation of the fluctuating field (8) to the electron temperature via (11).

Equations (1)—(3) are now solved in three different regions: (1) $-\infty<x<0$, (2) $0<x<a$, and (3) $a<x<\infty$. The assumption of a weakly inhomogeneous plasma ($kL\gg 1$) in the region 1 allows the WKB approximation to be used:

$$B(x<x_r)=B(x=0)\left[\frac{\alpha_1[K(x)-k\beta(x)]}{\alpha(x)(\kappa_1-k\beta_1)}\right]^{1/2} \times\exp\left[-i\frac{\pi}{4}-i\int_{x_r}^{x}K(x)dx\right]b , \tag{12}$$

where

$$b=\exp\left[-\int_{x_r}^{0}\kappa(x)dx\right] ,$$

$$\kappa(x)=\frac{\omega}{c}[n^2-\alpha(x)]^{1/2} ,$$

$$K(x)=\frac{\omega}{c}[\alpha(x)-n^2]^{1/2} ,$$

and $\alpha_1=\alpha(x=0)$, $\beta_1=\beta(x=0)$, $\kappa_1=\kappa(x=0)$. The solution corresponding to region 3 is trivial and reads

$$B(x>a)=B(x=a)\exp[-\kappa_2(x-a)] , \tag{13}$$

where $\kappa_2=\kappa(x>a)$. In transition region 2, Eqs. (1)—(3) are solved by using an iterative procedure with $ka\ll 1$ as a small parameter. Up to the first order in ka the solution thus obtained has the following form:

$$B(0<x<a)=B(x=0)\left[1+k\int_0^x\beta\,dx + \frac{\kappa_1}{\alpha_1}\int_0^x\alpha\,dx\right]-i\frac{\omega}{c}\int_0^x\eta_1 dx . \tag{14}$$

The solutions (12)—(14) as well as the derivatives dB/dx are now matched at $x=0$ and $x=a$. It should be emphasized that the derivative dB/dx corresponding to region 2 must also be calculated up to the first order with respect to the small parameter ka, thus giving rise to some extra terms which would not appear if (14) were differentiated directly. Also, the terms which do not contribute to the correlation function (8) were omitted in (12)—(14). Following such a procedure one obtains

$$E_y(x=0)=-\frac{4\pi e}{\mathscr{D}}\int_0^a\frac{n_0}{(\omega^2-\Omega^2)\epsilon}\left[k+\frac{\kappa_2+k\beta_2}{\alpha_2}g\right] \times\left[-iv_x(t=0)+\frac{\Omega}{\omega}v_y(t=0)\right]dx , \tag{15}$$

where

$$\mathscr{D}=\mathscr{D}_0+i\mathscr{D}_l+i\mathscr{D}_r$$

is the surface wave dispersion function and

$$\mathscr{D}_0=1+\frac{\alpha_1(\kappa_2+k\beta_2)}{\alpha_2(\kappa_1-k\beta_1)} ,$$

$$\mathscr{D}_l=-\frac{C^2}{2}\left[1-\frac{\alpha_1(\kappa_2+k\beta_2)}{\alpha_2(\kappa_1-k\beta_1)}\right] , \tag{16}$$

$$\mathscr{D}_r=\frac{\alpha_1k^2}{\kappa_1-k\beta_1}\mathrm{Im}\int_0^a\left[\left[1+\frac{\kappa_2+k\beta_2}{k\alpha_2}g\right]^2\Big/\epsilon\right]dx ,$$

where $\beta_2=\beta(x=a)$, $\alpha_2=\alpha(x=a)$. Here $\mathscr{D}_0=0$ represents a dispersion relation for the surface waves propagating along a sharp boundary between two homogeneous magnetoactive plasmas. The term $\mathscr{D}_r$ represents dissipative losses due to linear conversion of the surface waves into the upper hybrid modes arising in the vicinity of the resonant point $\epsilon=0$. $\mathscr{D}_l$ corresponds to damping of the surface wave caused by energy leaking through the quasiclassical reflection point x_R $[\kappa(x_R)=0]$. The dispersion equation $\mathscr{D}_0=0$ has been investigated by Kaufman[10] for the case of a homogeneous dc magnetic field. In the limit of the potential surface waves the equation $\mathscr{D}_0=0$ has been analyzed in detail in Ref. 11. For the case of two homogeneous plasmas adjacent to an inhomogeneous transition layer, the analysis has been presented in Ref. 12.

Using (8), (11), and (15) one obtains a correlation function of the fluctuating electric field in terms of the electron temperature as follows:

$$G_0(\omega,k)=\frac{4\pi k^2}{|\mathscr{D}|^2}\int_0^a\left[1+\frac{\kappa_2+k\beta_2}{k\alpha_2}g\right]^2 \times\left[T^{(x)}+\frac{\Omega^2}{\omega^2}T^{(y)}\right]\delta(\epsilon)dx\ , \tag{17}$$

where $G_0(\omega,k)=G_{yy}(\omega,k,x=x'=0)$. In the limit $B_0=0$ this correlation function reduces to the result obtained in Ref. 6. Although both temperatures $T^{(x)}$ and $T^{(y)}$ correspond to directions perpendicular to the dc magnetic field B_0 they can, in general, substantially differ due to a resonance at $\epsilon=0$ where E_x significantly exceeds E_y. Since the imaginary part $\mathscr{D}_r$ originates from linear conversion of the surface waves into the upper hybrid modes it arises only if the point of resonance exists within the transition layer, i.e., for sufficiently weak dc magnetic field when

$$\Omega^2<\omega_{p2}^2-\omega_{p1}^2\ ,\quad \omega_{p2}=\omega_p(x=a)\ ,\quad \omega_{p1}=\omega_p(x=0)\ . \tag{18}$$

As seen from (17), in the opposite case G_0 vanishes.

IV. PLASMA EMISSION

The energy of the surface waves which propagate with a phase velocity exceeding the speed of light can be emitted from the transition region beyond the quasiclassical reflection point $x=x_R$ where $\kappa(x_R)=0$. Obviously, for $x_R<0$ the condition $|\Omega|<\omega_{p1}$ must be satisfied. This is a more restrictive condition on the strength of the magnetic field than that formulated by Uberoi and Rao[13] to provide $|n|<1$. The latter condition coincides with (18). Finally, we conclude that emission of the surface waves can take place only when the magnetic field is sufficiently weak $\Omega^2\ll\omega_{p1}^2$.

This condition allows analytical analysis of the dispersion relationship $\mathscr{D}_0=0$. In particular, such a condition is of interest in the context of laser produced plasmas. There, even for high intensity Nd:glass laser radiation where megagauss spontaneous dc magnetic fields can be generated, the ratio Ω^2/ω_{p1}^2 is still well below unity. In fact, in a laser-produced plasma a spontaneous dc magnetic field is initially localized at the critical region and then subsequently diffuses in both directions along the density gradient. As a result an antisymmetric $[B_0(x<0)=-B_0(x>a)]$ spatial distribution of the dc magnetic field is formed,[8,9] with respect to the critical surface. However, for sake of generality, we will also keep in our analysis the case with a homogeneous magnetic field.

The refractive index n can now be approximated as

$$n=n^0+n^1\left[\frac{\Omega}{\omega}\right]+n^2\left[\frac{\Omega}{\omega}\right]^2+\cdots\ , \tag{19}$$

where

$$n^0=n(B_0=0)=\pm\left[\frac{\epsilon_{\parallel 1}\epsilon_{\parallel 2}}{\epsilon_{\parallel 1}+\epsilon_{\parallel 2}}\right]^{1/2} \tag{20}$$

and $\epsilon_{\parallel 1,\parallel 2}=\epsilon_{1,2}(B=0)$. The equation $\mathscr{D}_0=0$ is then solved within the first-order approximation with respect to the small parameter Ω/ω, which leads to the following relationships for the perturbations $n^{1(s)}$ and $n^{1(as)}$ corresponding, respectively, to a homogeneous and antisymmetric dc magnetic field:

$$n^{1(s)}=\frac{1-(n^0)^2}{[-(\epsilon_{\parallel 1}+\epsilon_{\parallel 2})]^{1/2}}\ , \tag{21}$$

$$n^{1(as)}=\frac{\epsilon_{\parallel 1}^2+\epsilon_{\parallel 2}^2-\epsilon_{\parallel 1}\epsilon_{\parallel 2}(\epsilon_{\parallel 1}+\epsilon_{\parallel 2})}{(\epsilon_{\parallel 1}^2+\epsilon_{\parallel 2}^2)[-(\epsilon_{\parallel 1}+\epsilon_{\parallel 2})]^{1/2}}\ . \tag{22}$$

As can be seen, the perturbation $n^{1(s)}$ changes its sign at $(n^0)^2=1$, while $n^{1(as)}$ is always positive. However, since we are interested in plasma emission associated with the fluctuating surface waves, n must be less than unity and consequently, in this strongly nonpotential case, $n^{1(s)}>0$. In the limit $|\epsilon_{\parallel 2}|\gg\epsilon_{\parallel 1}$ both (21) and (22) converge to the same value

$$n^{1(s)}=n^{1(as)}=\frac{1-\epsilon_{\parallel 1}}{|\epsilon_{\parallel 2}|^{1/2}}\ . \tag{23}$$

It is worth emphasizing that when expanding n into the series (19) one has to ensure that the surface wave attenuation factors κ_1 and κ_2 are positive. This leads to the following restriction on the amplitude of the magnetic field corresponding to the homogeneous

$$\left[\frac{\Omega}{\omega}\right]^2<\frac{\epsilon_{\parallel 1}^3}{|\epsilon_{\parallel 2}|[1-(n^0)^2]}$$

and antisymmetric

$$\left[\frac{\Omega}{\omega}\right]^2<\frac{\epsilon_{\parallel 1}^3}{|\epsilon_{\parallel 2}|}\left[\frac{\epsilon_{\parallel 1}^2+\epsilon_{\parallel 2}^2}{\epsilon_{\parallel 1}^2+\epsilon_{\parallel 2}^2-\epsilon_{\parallel 1}\epsilon_{\parallel 2}(\epsilon_{\parallel 1}+\epsilon_{\parallel 2})}\right]^2$$

cases. Since $|\epsilon_{\|2}| > \epsilon_{\|1}$, we estimate that both restrictions are well satisfied if $|\Omega/\omega| < \epsilon_{\|1}$.

Now, once the condition for emission and the correlation function (17) are determined, one can represent the energy flux density (6) as

$$S_x(\omega,k) = -\frac{\omega\alpha_1 b^2}{4\pi(\kappa_1 - k\beta_1)} G_0(\omega,k) , \tag{24}$$

where the factor $\alpha_1 b^2/(\kappa_1 - k\beta_1)$ arises from transition from $G_0(\omega,k)$ to $G_{yy}(\omega,k,x=x'=-\infty)$. A spectral distribution I_ω can then be readily obtained from (17) and reads

$$I_\omega = \frac{\omega^2}{8\pi^3 c^2} S_x(\omega,k) ,$$

i.e.,

$$I_\omega(\theta) = \frac{\omega^2}{8\pi^3 c^2} \left[T^{(x)} + \frac{\Omega^2}{\omega^2} T^{(y)} \right]_{\epsilon=0} \frac{\mathscr{D}_r(\omega,\theta) b^2}{|\mathscr{D}(\omega,\theta)|^2} , \tag{25}$$

where θ is the angle between the observation direction and the x axis $[k=(\omega/c)\sin\theta]$. Since $I_\omega(\theta)$ must be real and positive, three conditions

$$n^2 - \alpha_1 > 0 , \quad n^2 - \alpha_2 > 0 , \quad \kappa_1 - k\beta_1 > 0$$

have to be satisfied. The condition which is the most restrictive then defines the threshold angles $\theta_t^{(+)} > 0$ and $\theta_t^{(-)} < 0$. Notice that, in general, the threshold angle $\theta_t^{(+)}$ corresponding to waves propagating in a positive y direction $(k>0)$ may differ from that $\theta_t^{(-)}$ corresponding to the counter-propagating waves $(k<0)$. Therefore, there is no radiation emitted in the cone defined by these two angles.

The expression (25) is now analyzed in three different regimes.

(a) dc magnetic field is localized within the transition layer. This corresponds to the initial stage of generation of the spontaneous dc magnetic fields in a laser-produced plasma.

(b) Antisymmetric dc magnetic field across the transition layer $[B_0(x<0) = -B_0(x>a)]$. This corresponds to the late stage of diffusion of the spontaneous magnetic field and convection of the magnetic lines frozen into the plasma expanding from the transition layer.

(c) A homogeneous dc magnetic field occupying the whole space.

For convenience, we have introduced a set of dimensionless variables as follows:

$$X = \frac{\omega^2}{\omega_{p1}^2} , \quad \alpha = \frac{\omega_{p2}^2}{\omega_{p1}^2} , \quad A = \frac{a\omega_{p1}}{c} ,$$

$$\delta = \frac{L\omega_{p1}}{c} , \quad \gamma = \frac{\Omega L_B}{c} , \quad \phi = \frac{\Omega}{\omega_{p1}} \Big|_{\epsilon=0} ,$$

$$\phi_1 = \frac{\Omega}{\omega_{p1}} \Big|_{x=0} , \quad \phi_2 = \frac{\Omega}{\omega_{p1}} \Big|_{x=a} ,$$

where $L_B = \Omega(d\Omega/dx)^{-1}_{\epsilon=0}$. In all calculations presented here the parameters α, A, δ, L_B were fixed as

$$\alpha = 4, \quad A = 3, \quad \delta = 30, \quad L_B = a/2 .$$

In Fig. 2(a) corresponding to case (a) $(\phi_1 = \phi_2 = 0)$, we present the spectral intensity I_ω as a function of θ and ϕ with X as a parameter. As can be seen the spectrum which is symmetric in absence of the magnetic field $(\phi=0)$ becomes asymmetric with increasing value of the

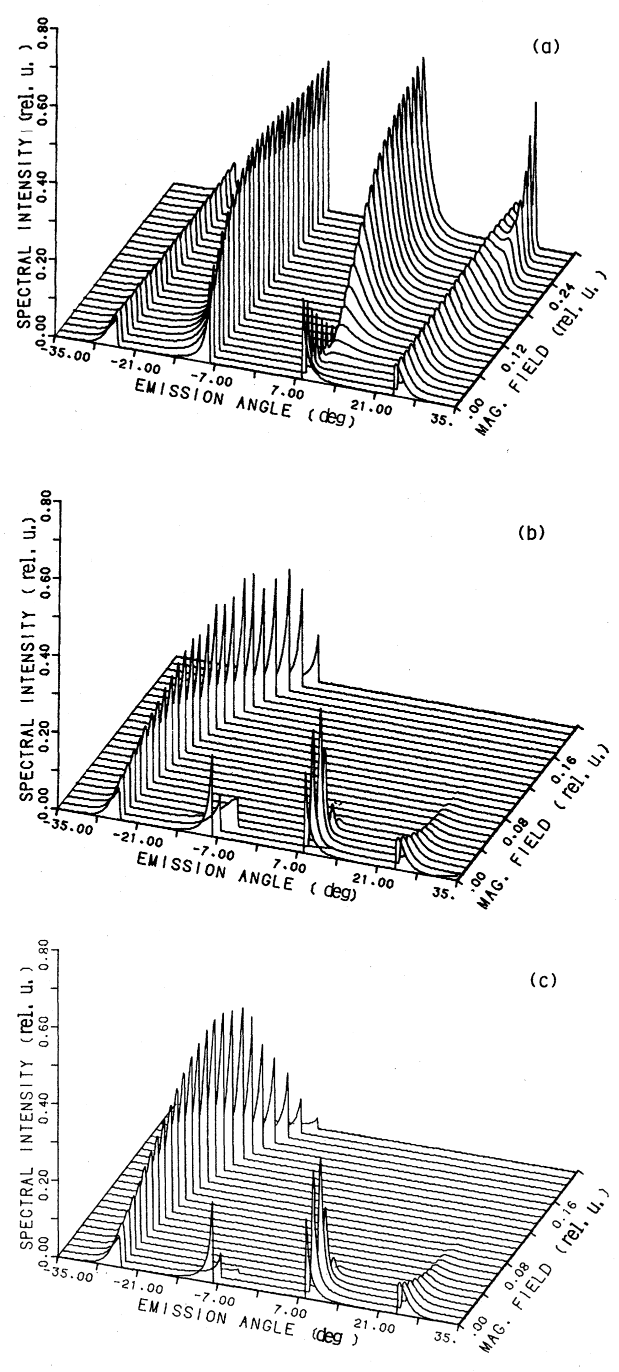

FIG. 2. A spectral intensity as a function of emission angle θ and normalized magnetic field ϕ. The central pair of the peaks corresponds to normalized frequency $X=1.02$ and the outer to $X=1.2$. (a) Magnetic field localized within the density step. (b) A homogeneous magnetic field extends over the whole space. (c) Antisymmetric magnetic field.

field. Notice that the dispersion relation $\mathscr{D}_0=0$ is the same as that for an isotropic plasma.[6] Thus the anisotropy of the spectrum originates from the magnetic field dependent damping factor $\mathscr{D}_r$. There are two types of asymmetry which appear in both the amplitude and the angular distributions. Initially, for a weak magnetic field, the amplitude asymmetry arises. Then, an increase in the magnetic field results in a double-peak structure for $\theta>0$ if $\phi>0$ (for $\theta<0$ if $\phi<0$). For large values of X a larger field is necessary for this effect to occur. Finally, a further increase in ϕ causes the left peak of the doublet to disappear which is followed by a simultaneous increase in the right peak.

Cases (b) and (c) are represented by Figs. 2(b) and 2(c), respectively. Since we consider only weak magnetic fields one expects a small difference between these two cases [see (21) and (22)]. In fact, for large density steps $|\epsilon_2| \gg \epsilon_1$, expressions (21) and (22) coincide, both tending to (23). For relatively large values of X Figs. 2(b) and 2(c) show practically no difference between the two cases.

Finally, it is worth emphasizing that the observed amplitude and angular asymmetry of the investigated emission can be used to diagnose the magnitude and spatial distribution of a dc magnetic field in a plasma where the conditions for excitation of the surface modes are met. Obviously, the characteristic parameters of the plasma density step can also be diagnosed as previously described in Ref. 6. Our estimates show that the peak value of I_ω for the parameters typical of a Nd:glass laser produced plasma ($T\approx 1$ keV) is $I_\omega=1$ MW/cm^2. Although the intensity of this radiation is low when compared to that of the backscattered radiation, it can be filtered out because of its frequency shift.

Throughout this paper we have neglected collisions. It can be shown that this is acceptable if $X-1\gg 10^{-3}$, which is easily fulfilled for the parameters corresponding to Fig. 2.

V. CONCLUSION

We have developed a theory of the fluctuations of the electromagnetic surface waves in a magnetoactive inhomogeneous plasma. The correlation function of these fluctuations is determined by both the dispersion characteristics and the damping mechanisms. In our collisionless model, linear conversion of the surface waves into the upper hybrid modes is the main dissipative process. It was demonstrated that the presence of a dc magnetic field introduces an asymmetry not only in the dispersion characteristics but also in the damping rates of the surface waves which are counter-propagating in a direction normal to a dc magnetic field. Further, the energy of these waves can be emitted from an inhomogeneous plasma only for a relatively weak magnetic field. Such emission has been investigated in detail for three cases with different geometry of the dc magnetic field. Although the magnetic field was assumed weak it still considerably affects the spectral distribution of the emitted radiation which results in both amplitude and angular asymmetry. Such a situation arises in a laser-produced plasmas where $\Omega^2/\omega_{p1}^2 \ll 1$. Measuring the emission characteristics thus allows the diagnosis of both the amplitude and spatial distribution of the magnetic field.

ACKNOWLEDGMENT

The authors are indebted to B. Luther-Davies for valuable discussions.

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