Squeezing via optical bistability

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The spectrum of squeezing in the output field for the optically bistable system of an atomic medium in a coherently driven cavity is analyzed. Good squeezing is attainable only in certain limits of atomic parameters and cavity detuning.

I. INTRODUCTION

There has been recent interest in the generation of squeezed states using cavities. The initial works of Milburn and Walls[1,2] on parametric oscillation and dispersive bistability and Lugiato and Strini[3,4] on parametric oscillation and absorptive bistability focused attention on the squeezing in the internal cavity mode. They obtained the result that the squeezing attainable did not exceed much more than 50% in any of the proposed schemes. However the recent realization by Yurke[5] and Collett and Gardiner[6] and Gardiner and Savage[7] that the relevant calculation relates to the external field outside the cavity has lead to more promising results. Of particular interest is the recent work of Collett and Walls[8] which indicates perfect squeezing to be possible in the resonant external mode at the critical points for dispersive optical bistability in a single-ended cavity. This result is particularly important given that dispersive bistability has been experimentally realized.[9] However, the work of Collett and Walls used a macroscopic nonlinear polarizability model[10] which ignored absorption due to the medium. Reid and Walls[11] and Bondurant *et al.*[12] have shown loss to be important in reducing the squeezing attainable in four-wave-mixing systems. In a microscopic treatment modeling the medium as N two-level atoms, Reid and Walls[13] show the effects of additional atomic fluctuations present at higher intensities to also be significant in reducing the squeezing attainable.

In this paper, we similarly model the medium as an ensemble of N two-level atoms. Thus the full effects of loss and spontaneous emission is analyzed. Using the techniques developed by Collett and Gardiner[6] and Collett and Walls[8] we calculate the squeezing spectrum for the field outside the cavity. Good squeezing is possible in the resonant external mode for certain limits of atomic parameters and cavity detunings. The effect of collisional damping on the squeezing is also analyzed.

II. THE MICROSCOPIC MODEL

We follow the procedure of Drummond and Walls[14] by describing the interaction of the single-mode radiation field with an ensemble of two-level atoms via the following Hamiltonian in the electric dipole and rotating-wave approximations.

$$\begin{aligned}
H &= \sum_{j=1}^{5} H_j \,, \\
H_1 &= \hbar\omega_c a^\dagger a + \sum_{i=1}^{N} \frac{\hbar\omega_a}{2}\sigma_{zi} \,, \\
H_2 &= i\hbar\sum_{i=1}^{N} (g a^\dagger \sigma_i^- e^{-ikr_i} - g a \sigma_i^+ e^{ikr_i}) \,, \\
H_3 &= \sum_{i=1}^{N} \Gamma_a \sigma_i^+ + \Gamma_a^\dagger \sigma_i^- + \Gamma_P \sigma_i^z \,, \\
H_4 &= \Gamma_F a^\dagger + \Gamma_F^\dagger a \,, \\
H_5 &= i\hbar(a^\dagger \epsilon e^{-i\omega_I t} - a\epsilon^* e^{i\omega_I t}) \,,
\end{aligned} \tag{1}$$

a and $a^\dagger$ are the boson field operators and $\sigma_i^+,\sigma_i^-,\sigma_{zi}$ are the Pauli spin operators describing the atoms. ω_c and ω_a are the cavity and atomic resonance frequencies, respectively. g is the electric dipole constant coupling the single-mode interferometer field with the atomic medium. Γ_F is the field reservoir operator. Γ_P and Γ_a are reservoir operators coupled to the atoms describing phase-damping processes and radiative decay, respectively. ϵ is the external driving field of frequency ω_I.

Following the analysis of Drummond and Walls,[14] a master equation in the Markovian approximation may be derived. The method of Haken[15] was used to derive a Fokker-Planck equation in a normally ordered representation, and a scaling argument assuming large N is used to justify ignoring higher-derivative terms. Adiabatic elimination of atomic variables is facilitated by converting to equivalent Langevin equations. The final equation for the field variable in the normally ordered generalized P representation[14] is

$$\begin{aligned}
\dot{\alpha} &= \epsilon - \kappa(1+i\phi)\alpha - \frac{2C\kappa\alpha}{(1+i\delta)\Pi} + \Gamma(t) \,, \\
\dot{\alpha}^\dagger &= \epsilon - \kappa(1-i\phi)\alpha^\dagger - \frac{2C\kappa\alpha^\dagger}{(1+i\delta)\Pi} + \Gamma^\dagger(t) \,,
\end{aligned} \tag{2}$$

$\Gamma(t)$ is a stochastic term and the correlation properties of Γ, $\Gamma^\dagger$ are

$$\langle \Gamma(t)\Gamma^{\dagger}(t')\rangle = -d(X)\delta(t'-t) ,$$

$$d(X)=\frac{2C\kappa}{(1+X+\delta^2)^3}\left[(1-i\delta)^3 f + i\delta X(1-f)(1-i\delta)+\frac{X^2}{2}\right] , \quad (3)$$

$$\langle \Gamma(t)\Gamma(t')\rangle = \Lambda(X)\delta(t'-t) ,$$

$$\Lambda(X)=\frac{2C\kappa X}{(1+X+\delta^2)^3}\left[(1+\delta^2)(1-f) + X[2+\delta^2(1-f)]+\frac{X^2}{2}\right] .$$

We have defined

$$\Pi = 1+X/(1+\delta^2) , \quad X=\alpha\alpha^{\dagger}/n_0 , \quad n_0=\gamma_{\parallel}\gamma_{\perp}/4g^2 ,$$

where (4a)

$$\phi=(\omega_c-\omega_I)/\kappa \quad (4b)$$

is the cavity detuning,

$$\delta=(\omega_a-\omega_I)\gamma_{\perp} \quad (4c)$$

is the scaled atomic detuning,

$$C=g^2N/2\gamma_{\perp}\kappa \quad (4d)$$

is the cooperativity parameter, and

$$f=\gamma_{\parallel}/2\gamma_{\perp} \quad (4e)$$

is the relative degree of radiation and collisional damping. $\gamma_{\perp}=\gamma_{\parallel}/2+\gamma_{\text{col}}$, γ_{col} being the collisional damping rate, where $\gamma_{\perp}$ and $\gamma_{\parallel}$ are the atomic transverse and longitudinal relaxation rates, respectively, and κ is the cavity damping rate. (The adiabatic elimination has assumed $\gamma_{\perp},\gamma_{\parallel} \gg \kappa$.) The steady state deterministic solution is

$$Y=X\left[\left[1+\frac{2C}{(1+X+\delta^2)}\right]^2 + \left[\phi-\frac{2C\delta}{(1+X+\delta^2)}\right]^2\right] , \quad (5)$$

where we have introduced the scaled variables $Y=|\epsilon/\kappa|^2/n_o$. This equation has been analyzed by Drummond and Walls[14] and bistability exists under the following conditions.

(i) Absorptive bistability exists for $C>4$.

$$\delta=\phi=0 . \quad (6a)$$

(ii) Bistability exists for $C^2>27\delta^2/4$ $(C \gg 1)$.

$$\phi=0 , \quad \delta\neq 0 . \quad (6b)$$

(iii) Bistability exists for $C^2>27\phi^2/4$.

$$\phi\neq 0 , \quad \delta=0 . \quad (6c)$$

(iv) Bistability exists for $C>4|\delta\phi|$ $(C\gg 1)$.

$$\delta\phi<0 . \quad (6d)$$

(v) Bistability exists for $2C>\delta\phi$ $(C\gg 1)$.

$$\delta\phi>0 . \quad (6e)$$

To simplify the analysis of the quantum statistics we adopt a linearized fluctuation procedure by expanding to first order about a stable steady-state solution α_0. Writing $\alpha=\alpha_0+\delta\alpha$ [where $X_0=|\alpha_0|^2/n_0$ is a solution to the state equations (5)], one has

$$\frac{\partial}{\partial t}\delta\boldsymbol{\alpha} = -A(\alpha_0)\delta\boldsymbol{\alpha}+D(\alpha_0)^{1/2}\boldsymbol{\epsilon}(t) , \quad (7)$$

where $\delta\boldsymbol{\alpha}\equiv(\delta\alpha,\delta\alpha^*)$ and $\epsilon(t)\equiv(\epsilon_1(t),\epsilon_2(t))$, where $\langle\epsilon_1(t)\epsilon_j(t')\rangle=\delta_{ij}\delta(t-t')$ and

$$\underline{A}=\begin{bmatrix} a & b \\ b^* & a^*\end{bmatrix} , \quad \underline{D}=\begin{bmatrix} -d(X_0) & \Lambda(X_0) \\ \Lambda(X_0) & -d^*(X_0)\end{bmatrix} ,$$

$$a=a_R+ia_I , \quad a_R=\kappa+\frac{2C\kappa(1+\delta^2)}{(1+X_0+\delta^2)^2} ,$$

$$a_I=\kappa\phi-\frac{2C\kappa\delta(1+\delta^2)}{(1+X_0+\delta^2)^2} ,$$

$$b=b_R+ib_I, \quad b_R=\frac{-2C\kappa\delta X_0}{(1+X_0+\delta^2)^2} ,$$

$$b_I=\frac{2C\kappa\delta X_0}{(1+X_0+\delta^2)^2} .$$

III. SQUEEZING SPECTRUM IN THE OUTPUT FIELD

Of particular interest to us is the spectrum of squeezing in the field outside the cavity. A method of calculation of this squeezing spectrum from the linearized drift and diffusion coefficients ($\underline{A}$ and $\underline{D}$) in the P representation (in which equal-time moments of the c numbers correspond to normally ordered moments of the operators) has been explained in Collett and Walls.[8] The spectrum matrix $\underline{S}(\omega)$ is defined as the Fourier transform of the two-time stationary correlation matrix $\underline{G}(\tau)$ where

$$(G(\tau))_{ij}=\langle\alpha_j(\tau),\alpha_i(0)\rangle \quad (8)$$

and we use the notation $\langle x,y\rangle=\langle xy\rangle-\langle x\rangle\langle y\rangle$. In fact the linearized spectrum $S(\omega)$ may be obtained directly as follows:[16]

$$\underline{S}(\omega)=(\underline{A}+i\omega\underline{I})^{-1}\underline{D}(\underline{A}^T-i\omega\underline{I})^{-1} . \quad (9)$$

To examine squeezing, we define the quadrature phases X_1 and X_2 as follows:

$$a=e^{i\theta}(X_1+iX_2) ,$$
$$a^{\dagger}=e^{-i\theta}(X_1-iX_2) . \quad (10)$$

The spectrum of squeezing in the output field is

$$:S_{1,2\ \mathrm{out}}(\omega): = \langle :X_{1,2\ \mathrm{out}}(\omega), X_{1,2\ \mathrm{out}}:\rangle$$
$$= \frac{\kappa}{2}\{S_{12}(\omega)+S_{21}(\omega) \pm [e^{2i\theta}S_{22}(\omega)+e^{-2i\theta}S_{11}(\omega)]\} . \quad (11)$$

Squeezing is attained when $:S_{1,2\ \mathrm{out}}(\omega):$ becomes negative, ideal squeezing corresponding to $:S_{1,2\ \mathrm{out}}(\omega):\to -0.25$.

To optimize the squeezing for a particular frequency ω_0, we select the phase θ such that

$$e^{2i\theta} = \frac{S^*_{22}(\omega_0)}{|S_{22}(\omega_0)|} . \quad (12)$$

Thus

$$:S_{1,2\ \mathrm{out}}(\omega): = \frac{\kappa}{2}\left[S_{12}(\omega)+S_{21}(\omega) \pm 2\,\mathrm{Re}\left[\frac{S^*_{22}(\omega_0)}{|S_{22}(\omega_0)|}S_{22}(\omega)\right]\right] . \quad (13)$$

The work of Collett and Walls[8] adopted a macroscopic nonlinear polarizability model[10] for dispersive bistability, in which loss due to the medium was ignored. Ideal squeezing was shown to be attainable at the turning points of the state equation (5) for $\omega=0$, the spectrum at the critical point being a simple Lorentzian. We are presently interested in investigating the limit of atomic and cavity parameters corresponding to this ideal result. Selecting $\omega_0=0$ (to optimize for squeezing at $\omega=0$), we find

$$[S_{12}(\omega)+S_{21}(\omega)]/2 = [\Lambda(|a|^2+|b|^2+\omega^2)+2a_R(d_Rb_R+d_Ib_I)+2a_I(d_Ib_R-b_Id_R)]/|D|^2 ,$$
$$\mathrm{Re}\left[\frac{S^*_{22}(0)}{|S_{22}(0)|}S_{22}(\omega)\right] = \frac{\mathscr{R}^2+\mathscr{I}^2-\mathscr{R}d_R\omega^2-\mathscr{I}d_I\omega^2}{|D|^2(\mathscr{R}^2+\mathscr{I}^2)^{1/2}} , \quad (14)$$

where

$$\mathscr{R} = -2\Lambda(b_Ra_R+b_Ia_I)-d_R(a_R^2+b_R^2-b_I^2-a_I^2) -2d_I(b_Rb_I+a_Ra_I) ,$$
$$\mathscr{I} = -2\Lambda(a_Rb_I-a_Ib_R)-d_I(a_R^2-a_I^2+b_I^2-b_R^2) -2d_R(b_Rb_I-a_Ra_I) ,$$
$$|D|^2 = (|a|^2-|b|^2-\omega^2)^2+(2\omega a_R)^2 .$$

At the turning points of the state equation

$$\frac{d\sqrt{Y}}{d\sqrt{X}} = |a|^2-|b|^2 = 0 \quad (15)$$

and the squeezing spectrum simplifies to

$$:S_{1,2\ \mathrm{out}}(\omega): = \kappa\left[\Lambda+\frac{d_R\mathscr{R}+d_I\mathscr{I}}{(\mathscr{R}^2+\mathscr{I}^2)^{1/2}}\right]\Big/(4a_R^2+\omega^2) . \quad (16)$$

IV. PARTICULAR LIMITS

Firstly we write in our own notation the results of the macroscopic theory used by Collett and Walls.[8] One has

$$a_R=\kappa , \quad a_I=2|\bar{\epsilon}|+\phi ,$$
$$\Lambda=d_R=b_R=0 , \quad (17)$$
$$d_I=b_I=|\bar{\epsilon}| ,$$

where $|\bar{\epsilon}|$ is a dispersion parameter. We see by comparison with (7) that the model (17) neglects the loss (represented by $a_R-\kappa$) due to the medium and also nonideal atomic fluctuations (represented, for example, by a nonzero Λ and d_R) which will become significant at higher X_0 values. The assumptions (17) give ideal squeezing at the turning points as $\omega\to 0$, i.e., $:S_{1,2\ \mathrm{out}}(\omega):\to -\frac{1}{4}$ as in the result of Collett and Walls.[8] Thus the ideal squeezing requires both minimal atomic absorption (both in absolute terms and relative to the dispersion parameter b_I), i.e.,

$$a_R-\kappa \ll 1 , \quad (18a)$$
$$(a_R-\kappa)/b_I \ll 1 , \quad (18b)$$

and ideal fluctuation terms

$$\Lambda=d_R=0 ,$$
$$d_I=b_I . \quad (19)$$

The ideal fluctuation terms (19) are attained in the pure radiative limit ($f=1$) with large detuning and low X_0 as follows:

$$\delta \gg 1 ,$$
$$X_0/\delta^2 \ll 1 , \quad (20)$$
$$X_0^2/\delta^3 \ll 1 .$$

These conditions parallel those derived by Reid and Walls[11,13] who used an identical two-level atomic model to study squeezing produced via four-wave mixing. In the present case of optical bistability there will be an additional restraint on ϕ, the cavity detuning.

To examine the results in more detail, we consider separately the five special cases listed in Eqs. (6).

(i) Absorptive bistability $\delta=\phi=0$. In this case we have $a_I=b_I=d_I=0$. The result for the appropriate quadratures is, for $\omega=0$

$$:S_{+\ \mathrm{out}}: = \frac{\kappa(\Lambda+d_R)}{(a_R-b_R)^2} ,$$
$$:S_{-\ \mathrm{out}}: = \frac{\kappa(\Lambda-d_R)}{(a_R+b_R)^2} . \quad (21)$$

At the critical point $a_R=-b_R$, and the expression for the spectrum simplifies to

$$:S_{+\,\mathrm{out}}: = \frac{\kappa(\Lambda + d_R)}{4a_R{}^2 + \omega^2} ,$$

$$:S_{-\,\mathrm{out}}: = \frac{\kappa(\Lambda - d_R)}{\omega^2} . \qquad (22)$$

It is apparent immediately that no squeezing is attained at the critical point. Only a very small amount of squeezing is possible, in S_-, below threshold. In fact the squeezing predicted here for the external field is less than that predicted for this system by Lugiato and Strini,[3] whose calculations focused on the internal field.

It is worthwhile to consider at this stage the system of two-photon absorptive bistability. Calculations for the internal cavity mode by Lugiato and Strini[4] and Reid and Walls[17] have shown a squeezing of just over 50% to be attainable in the lower branch. The model Hamiltonian for the two-photon example is obtained from Eq. (1) by replacing H_2 with the two-photon interaction

$$H_2 = i\hbar \sum_{i=1}^{N} [\bar{g}(a^\dagger)^2 \sigma_i^- e^{-ikr_i} - \bar{g} a^2 \sigma_i^+ e^{ikr_i}] . \qquad (23)$$

The method of linearization and calculation of the external squeezing spectrum may be followed along the lines of the one-photon example. Final calculations reveal the maximum squeezing possible in the external field to be less than 20%, considerably less than that predicted for the internal mode. Thus the systems of one- and two-photon absorptive optical bistability produce very little squeezing.

(ii) $\phi=0$, $\delta\neq 0$. This situation is similar to the absorptive case above. Bistability occurs as $X_0 \approx \delta^2$. That is, the mechanism for nonlinearity is saturation and for such values of X_0 the ideal fluctuation condition (20) is violated. At lower X_0 values (lower branch), loss dominates and acts to counter squeezing. Thus any squeezing attained in this situation is small.

(iii) $\phi\neq 0$, $\delta=0$. With $\delta=0$, nonideal fluctuation terms d_R and Λ are significant, hence limiting the squeezing attainable.

(iv) $\delta\phi<0$. This case can give squeezing, but is less favorable than the following case (v).

(v) $\delta\phi>0$. This case is the most favorable for squeezing. We have parameters δ, C, X_0 to vary and also ϕ the cavity detuning. The optimal δ, C, and X_0 are determined similarly to the case of degenerate four-wave mixing studied by Reid and Walls.[13] For good squeezing we require the ideal fluctuation conditions (20). In this limit, the small loss condition (18b) simplifies to $\delta/X_0 \ll 1$, and the order of magnitude for δ and X_0 is thus determined. In fact $\delta \geq 10^4$ gives near perfect squeezing, with the appropriate choice of C and ϕ. Condition (18a) for small loss simplifies to $2C \ll \delta^2$, and this places an upper limit on C for good squeezing. Also required is a large enough dispersion parameter $b_I \approx (2C/\delta)(X_0/\delta^2)$. Unless $2C/\delta \gg 1$, the value of X_0 required for this is such that the ideal fluctuation condition is no longer satisfied. Thus there is also a lower limit on C. The sensitivity to C is illustrated for $\delta \approx 100$ in Fig. 1. For $\delta \approx 10^4$, the optimal C is 2×10^6. We have for simplicity illustrated the case of pure radiative damping ($f=1$) only. Results are for the stable lower branch which, having a lower X_0 value, is more favorable to squeezing.

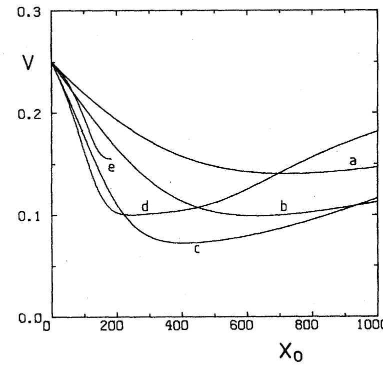

FIG. 1. Effect of cooperativity parameter on the squeezing. $V = :S_{\mathrm{out}}(0): + 0.25$ vs X_0 the scaled cavity mode intensity. Solutions correspond to the linearized result about the stable lower branch. $\delta=100$, $f=1$. a, $C=200$ ($\phi=3.6$); b, $C=400$ ($\phi=7.2$); c, $C=1100$ ($\phi=20.6$); d, $C=3000$ ($\phi=57.3$); e, $C=8000$ ($\phi=153$). The value of ϕ for each C has been optimized to give the best squeezing.

The squeezing attainable is also very sensitive to the cavity detuning ϕ. To obtain bistability, we require

$$\phi < 2C/\delta \ \text{ if } C \gg 1 . \qquad (24)$$

This behavior is illustrated in Fig. 2 with parameters $\delta=10^4$ and $C=2\times 10^6$. The transition to bistable behavior occurs at $\phi \approx 398.4$. The magnitude of squeezing attained for values of ϕ is shown in Fig. 3. As ϕ increases to $2C/\delta$, the value of X_0 for which the turning point occurs is decreased. Thus the squeezing attainable at the turning point improves, until one gets to the point of inflection ($\phi \approx 398.4$) where squeezing is optimal. Increasing ϕ further destroys optical bistability and the value of X for best squeezing is not improved. The term a_I increases and squeezing reduces.

The theory presented here assumes a single-cavity mode. This assumption is valid where the cavity detuning

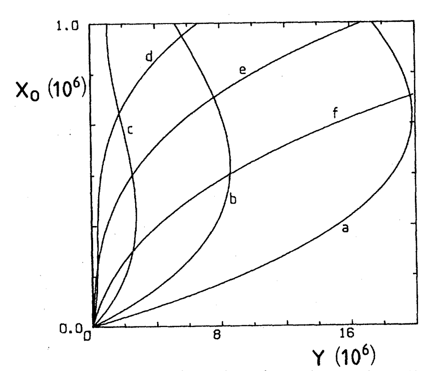

FIG. 2. Optical bistability in the dispersive limit. X_0 the steady-state cavity intensity vs Y the external driving field intensity. $\delta=10^4$. $C=2\times 10^6$. a, $\phi=392$; b, $\phi=394$; c, $\phi=396$; d, $\phi=398.4$; e, $\phi=400$; f, $\phi=402$.

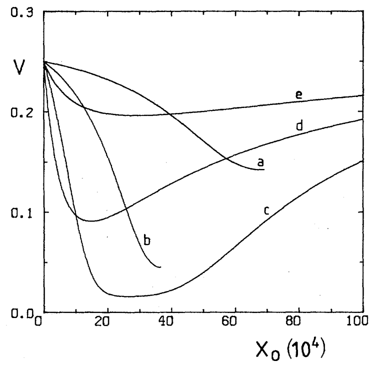

FIG. 3. Effect of cavity detuning on the squeezing. $V=:S_{out}(0):+0.25$ vs X_0. Solutions correspond to stable lower branch. $\delta=10^4$. $f=1$. $C=2\times10^6$. (The optimal choice of C for squeezing for $\delta=10^4$.) a, $\phi=392$; b, $\phi=396$; c, $\phi=398.4$; d, $\phi=400$; e, $\phi=402$.

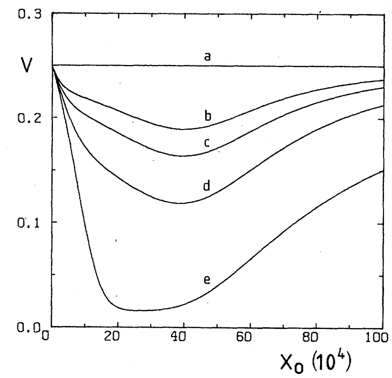

FIG. 4. Effect of collisional damping on the squeezing. $V=:S_{out}(0):+0.25$ vs X_0. $\delta=10^4$. $C=2\times10^6$. $\phi=398.4$. a, $f=0$; b, $f=0.85$; c, $f=0.9$; d, $f=0.95$; e, $f=1$.

ϕ is less than the cavity finesse F (the ratio of the separation between cavity modes to the transmission bandpass), related to the reflectivity R of the cavity mirrors by[18]

$$F=\frac{\pi\sqrt{R}}{1-R} . \tag{25}$$

Thus the above example giving an optimal squeezing (94%) for $\delta\sim10^4$ with $C\sim2\times10^6$ and $\phi=398.4$ would require a very high reflectivity ($R\gg0.99$) for consistency. However, with lower R values one can still satisfy the single-mode condition ($\phi<F$) and attain good squeezing by reducing C appropriately. For example, consider parameters $R=0.99$ and $\delta=10^4$. Taking $C=10^6$, the optimal ϕ is 198.4 (within the single-mode assumption) and a squeezing of 90% is still possible.

V. THE EFFECT OF COLLISIONAL DAMPING

The above results have assumed perfect radiative damping ($f=1$). The presence of phase-damping processes such as collisions will provide additional quantum fluctuation terms [as written in Eq. (3)] and will alter the squeezing attainable. To study the effect of collisional damping, we select the optimal atomic and cavity parameters δ, C, and ϕ of Fig. 3, and vary f from 1 (the ideal radiative damping case) to 0 (the pure collisional damping limit). The results shown in Fig. 4 are quite dramatic, a reduction of f from 1 to 0.9 being sufficient to significantly reduce the squeezing.

We wish to investigate whether the high values of C (and hence high atomic densities) required for good squeezing are compatible with a noncollisional damping limit. The following relations hold for an optical cavity filled with a two-level atomic medium.[19]

$$C=\frac{\alpha_0 L}{1-R} \text{ and } \alpha_0=\frac{3}{2\pi}\lambda^2\rho , \tag{26}$$

where α_0 is the absorption coefficient below saturation, L is the cavity length, R is the reflectivity of the mirrors, λ is the field wavelength, and ρ is the atomic density. Hence $\rho=2\pi C(1-R)/3\lambda^2 L$. Taking $C=10^6$, $1-R=10^{-2}$, $\lambda=6\times10^7$ m (for sodium) and a cavity length $L=0.5$ m, one finds $\rho\sim10^{11}/\text{cm}^3$. Resonant collisions between sodium atoms increase the collisional damping rate γ_{col} by 1100 MHz per $10^{16}/\text{cm}^3$ pressure.[20] A typical value for the radiative damping rate ($\gamma/2$) is 10 MHz. Hence an atomic density of $10^{11}/\text{cm}^3$ corresponds to $f=0.999$, sufficiently close to one to allow good squeezing.

VI. CONCLUSION

We have analyzed the squeezing attainable in the output field of a coherently driven cavity with a medium of two-level atoms. Steady-state solutions reveal both absorptive and dispersive bistability to be possible. The transmitted spectrum is calculated via a linearization procedure. One finds near perfect squeezing to be possible only in the limits of pure radiative damping and large atomic detuning, and with the cavity cooperativity parameter C appropriately optimized. The optimal value of the cavity detuning is that corresponding to the onset of bistability.

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