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Experimental evidence of three-mode quasiperiodicity and chaos in a single longitudinal, multi-transverse-mode cw CO_2 laser

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We report observation of oscillatory instabilities leading to chaos in a single longitudinal but multi-transverse-mode cw CO_2 laser. Particularly interesting is the observation of stable oscillation involving three transverse modes en route to chaos. Furthermore, these results are obtained for a predominantly homogeneously broadened lasing transition.

Deterministic chaotic behavior in a single-mode homogeneously broadened laser was first predicted by Haken[1] in 1975. More recently it has been shown that the presence of higher-order transverse modes in this system may also lead to chaotic output,[2,3] but here not restricted by the requirement of a bad cavity and prohibitively high gain as for the single-mode systems. Shih, Milonni, and Ackerhalt[2] find a period-doubling route to chaos in such a system, while according to Hollinger and Jung[3] chaos is approached by a Ruelle-Takens mechanism. Although some experimental evidence of transverse-mode induced pulsating instabilities have been reported,[4] data are scant and provide no evidence of chaotic behavior. Here we report on the observation of oscillatory instability leading to chaos in such a system, viz., a homogeneously broadened single longitudinal but multi-transverse-mode cw-CO_2 laser. Particularly significant we find, en route to chaos, a distinct and stable oscillation involving two and subsequently three transverse modes. Our observations, which confirm the predictions of Hollinger and Jung,[3] are thus of special interest against the background of recent discussion in the theoretical literature[5-7] concerning the possibility of three-frequency quasiperiodicity in a typical nonlinear dynamical system.

A commerical cw-CO_2 laser was used for this investigation. The 1.8-m-long Invar stabilized resonator cavity is provided with a piezoelectric tuning facility, tunable over more than a free spectral range (FSR) of ~ 85 MHz and comprises a plane grating and a concave (6-m radius of curvature) ZnSe output mirror of 60% reflectivity. The gain of the system typically lies within thrice the lasing threshold. Gaussian to multitransverse mode selection was achieved by standard intracavity aperture-control technique. The laser signal was recorded on a fast photoconductive detector and displayed on a Tektronix 7104 oscilloscope. The laser uses commercially available premixed gas and operates in the pressure region of 7 to 30 mb. The pressure linewidth for our system is calculated as ~ 3.2 MHz/mb based on well-documented data for the pressure-broadened coefficients of the constituent gases, while the Doppler broadening is approximately 50 MHz. For most of the operating pressures single longitudinal mode conditions were therefore ensured. This was confirmed experimentally by tuning the cavity length over a full FSR. In order to obtain absolutely no lasing over at least some tuning range, the aperture had to be shut down appropriately to suppress all other transverse modes except the TEM_{00} mode. With the aperture fully open lasing always occurred over the entire FSR. The claim of single longitudinal mode condition is also based on the observed pulsation frequencies, which were always much smaller than the adjacent mode separation.

The transverse nature of the emission was confirmed throughout by taking a spatial intensity scan of the laser beam by a pyroelectric array detector and also by the use of a fluorescent image plate. A Gaussian distribution was obtained for TEM_{00} operation, whereas with wide aperture this was degraded to some extent. On cavity tuning the distribution expanded spatially and became complex with no readily definable mode pattern, indicating an admixture of transverse modes. Similar observations have also been made by Hauck, Hollinger, and Weber[4] for a pulsed solid-state laser. A typical pyroelectric array recording is shown in Fig. 1.

A large variation in oscillation period (67 to 360 nsec; cavity round-trip time t_R is ~ 12 nsec) was found to occur with coarse cavity-length tuning across the FSR (Fig. 2). This is attributed to a change in gain seen by individual transverse modes on cavity tuning, thus bringing different modes into lasing; corroborated also by the transverse intensity pattern of the laser beam. This in turn changes the beating period.

Figure 3 shows an approach to chaos with fine cavity tuning about the value for which trace (a) of Fig. 2 [the same as trace (a) of Fig. 3] was obtained. Undamped periodic oscillation [trace (a)] as a result of beating between two modes first develops into quasiperiodic oscillation [trace (b)] involving the excitation of three transverse modes. On further cavity tuning, irregularities in the pulsating instabilities are clearly evident [trace (c)]. This ultimately develops to the chaotic behavior shown in trace (d). This behavior is completely consistent with the recent predictions of Hollinger and Jung.[3] These data were taken for an operating

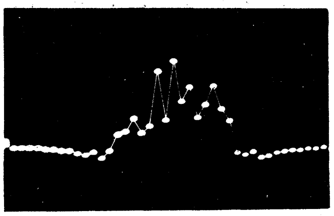

FIG. 1. A typical pyroelectric-array detector scan showing the complicated transverse nature of the CO_2 laser emission. Pitch of elements is 0.5 mm. For clarity, adjacent elements are joined by thin white lines.

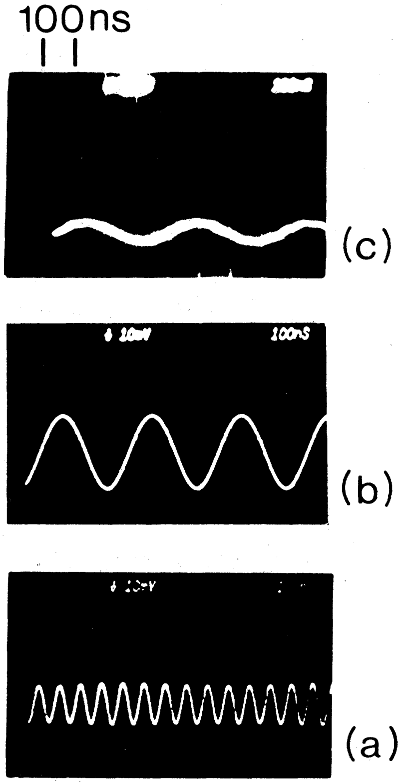

FIG. 2. Variation of the periods of the undamped oscillatory instabilities with cavity-length tuning—over an entire FSR of the cavity.

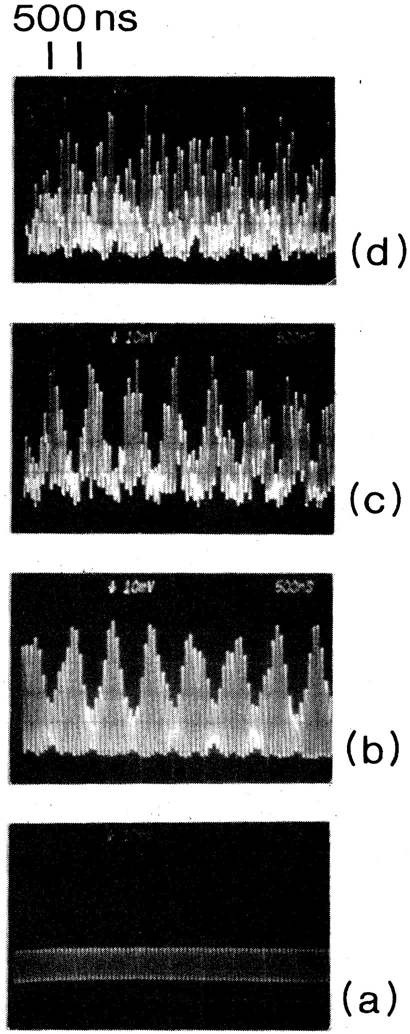

FIG. 3. Route to chaos with fine cavity-length tuning as a control parameter; trace (a)—undamped periodic oscillations as a result of beating between two transverse modes; trace (b)—quasiperiodicity involving the coupling of three transverse modes; trace (c)—onset of aperiodicity; and trace (d)—fully developed chaos. [Tuning range from trace (a) to trace (d) ~6 MHz.]

pressure of 20 mb. The amount of tuning required for going from trace (a) to trace (d) was about ~6 MHz compared to the cavity FSR of ~85 MHz.

The routes to chaos proceed via a Ruelle-Takens sequence involving initially two coupled transverse modes developing to chaos through the further coupling of a third mode. Newhouse, Ruelle, and Takens[5] had predicted that such quasiperiodic motion involving three frequencies should occur, though it can be destroyed by an arbitrarily small amount of perturbation. According to Grebogi, Ott, and Yorke,[6] such a scenario is rather common in a typical nonlinear dynamical system, and pertubation of some specialized form is needed to destroy the three-frequency quasiperiodicity. The numerical computation of Hollinger and Jung for the specific case of a laser system show evidence of such behavior. Our experimental observation of sustained three-mode oscillation en route to chaos confirm these predictions. Furthermore, the higher probability for observing quasiperiodicity than that of chaos, estimated here from the cavity tuning range over which one or the other is maintained, is qualitatively consistent with the predictions of Grebogi *et al.*[6] based on Lyapunov-exponent characterization of these two forms of attractors. We note here the observations of three-frequency quasiperiodicity recently reported in a Benard experiment with mercury in a magnetic field[8] and in the voltage spectrum of a ferroelectric barium-sodium-niobate crystal.[9] Similar observations are also briefly noted by Gioggia and Abraham[10] in an inhomogeneously broadened single-mode He-Ne laser, operating under the necessary bad cavity conditions: The effects observed may be ascribed to mode splitting.[11]

The characteristic of the instability phenomenon and notably the beat frequencies we observe here were found to be essentially invariant with operating pressure; at the highest pressure (30 mb) for lasing the homogeneous contribution toward line broadening was almost double that for inhomogeneous broadening, whereas at lowest pressure (7 mb) this ratio is ~0.5. Whereas spectral hole burning is probably the dominant effect for a multimode operation at low pressures, at higher pressures where this contribution is reduced by collisions such an operation may be maintained by the different transverse spatial-field structures associated with the various transverse modes: A detailed analysis should also account for the effect of spatial hole burning associated with our standing-wave laser cavity. The transverse-mode frequencies under passive conditions for our system are essentially independent of pressure. Furthermore, since the cavity linewidth for all operating pressure is much less than the gain bandwidth, mode-pulling effects are essentially the same. Consequently, the beat frequencies, as we observed, do not vary with pressure within the accuracy of our experimental measurement.

The frequencies observed suggest that the high-frequency oscillation arises from beating between adjacent transverse modes of the same axial index, whereas the low-frequency beating is associated with transverse modes having different axial-mode indices. In addition to such overlapping of the transverse modes with different axial-mode indices, the situation is far more complicated, since the frequencies of the various transverse modes are governed by their respective cavity losses, which are different, thus leading to nonequispaced mode spacing. In contrast, the well-documented observations and analysis on multimode systems generally involve equispaced longitudinal modes, which give rise to periodic pulsations under conditions of fixed phases for all the excited modes.

Earlier work by Hallas, Liu, and Abraham[12] on three-mode instability was on a purely inhomogeneously broadened system. Here we have demonstrated such effects in a homogeneously broadened system. The relative simplicity of such systems to the analysis of multimode instabilities is recognized as important. Our observation, we hope, will therefore prompt such investigation. Though the present study has been limited to a homogeneous to inhomogeneous broadening ratio of 2:1, we have recently observed similar results from a transversely excited atmosphere CO_2 laser, albeit pulse, where this ratio is $\sim 75{:}1$. Details of these results will be published elsewhere. Experimental efforts to obtain these effects under traveling wave (ring-cavity) conditions are currently in progress.

Finally we note that these effects were obtained under normal operating conditions of a standard commercial laser. Undesireable for many applications of such systems, these effects are eliminated by operation on the TEM_{00} mode alone, conventionally obtained using an intracavity aperture. However, in maximizing the efficiency of such an operation, design may be most effective by ensuring better matching between the TEM_{00} mode and the gain cross section.

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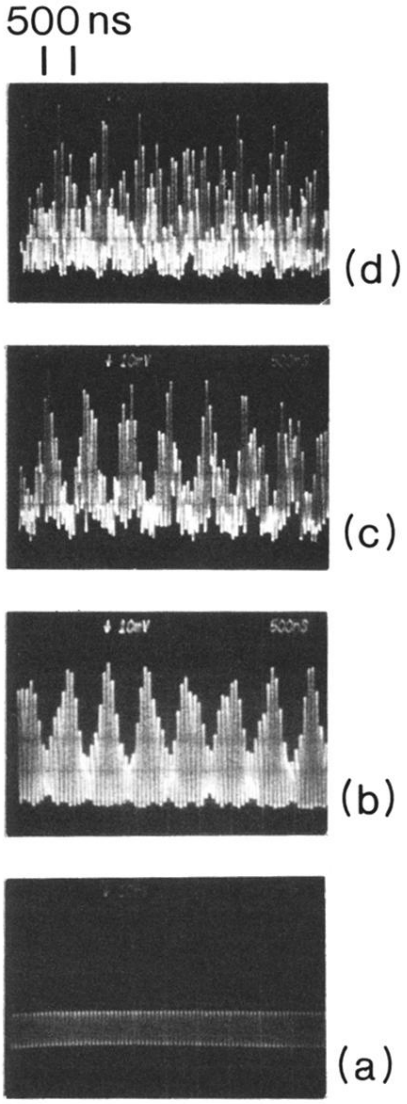

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